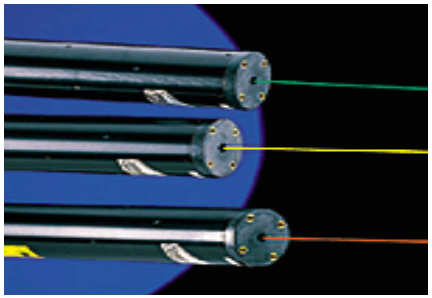
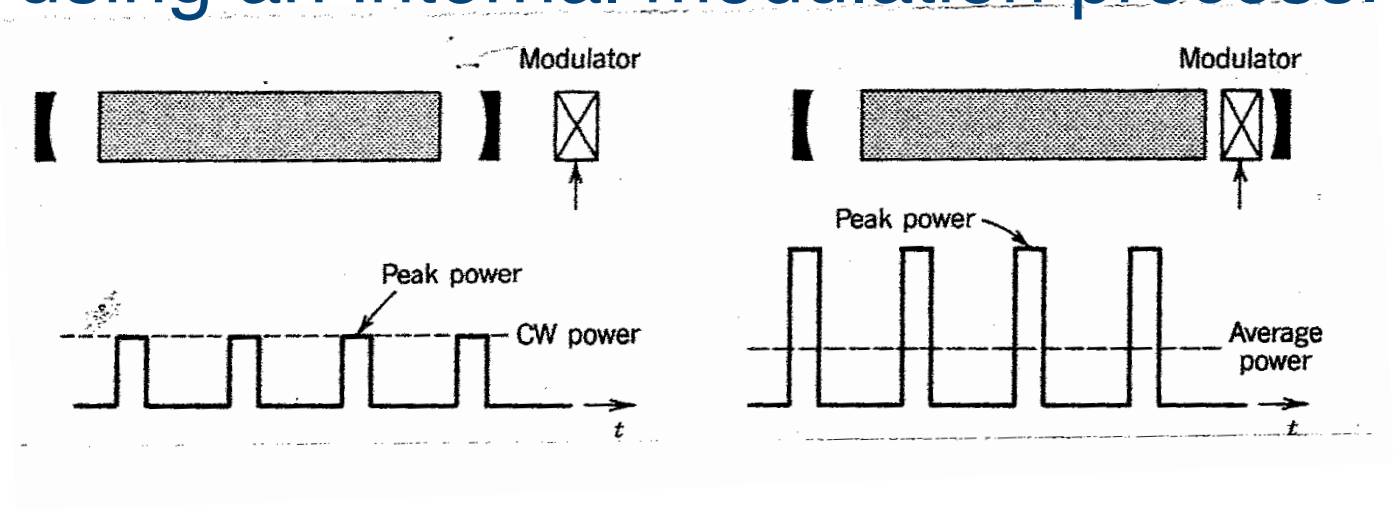
Quantum Electronics
Laser Physics
PS407

7. Pulsed Lasers

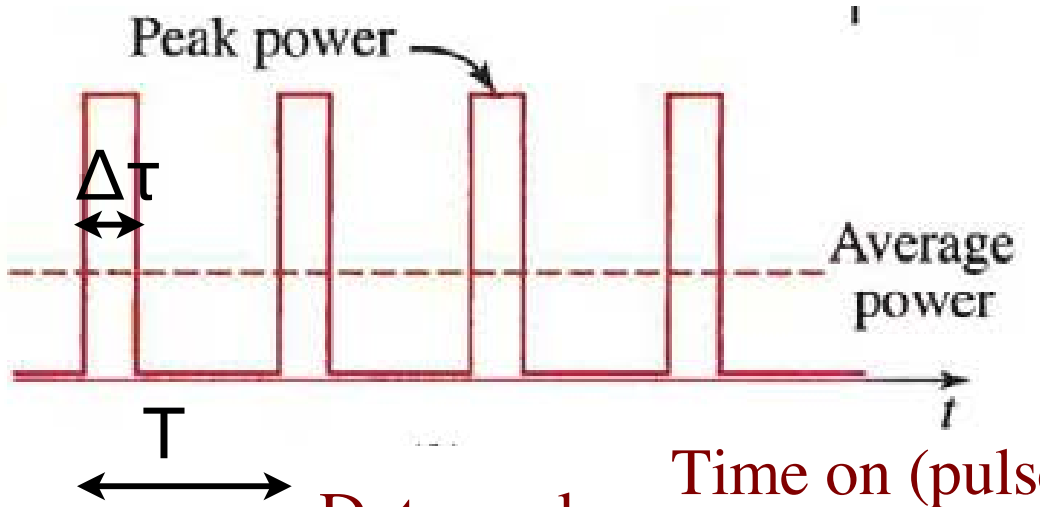


I. Pulsed lasers: Principles

- Most direct method: external switch or modulator applied to cw laser beam
  - Inefficient: most of the laser light is lost
  - Peak power can't exceed the steady-state power of cw laser
- Solution is to turn the laser itself on and off using an internal modulation process.



Duty Cycle %



T = period (s)

Δτ = pulse width (s)

$$\text{Duty cycle} = \frac{\text{Time on (pulse width)}}{\text{Period}} \times 100\%$$

$$\text{Peak Power (Watts): } P_i = \frac{\text{Pulse energy}}{\text{Pulse width}} = \frac{\varepsilon}{\Delta\tau}$$

$$\text{Average Power (Watts): } \bar{P} = \frac{\text{Pulse energy}}{\text{Period}} = \frac{\varepsilon}{T}$$

$$\frac{\text{Average Power}}{\text{Peak Power}} = \frac{\bar{P}}{P_i} = \frac{\Delta\tau}{T} \equiv \text{Duty cycle}$$

Duty Cycle %

100 mJ Nd:YAG laser, 10 ns pulse width, rep rate 50 kHz.

Peak Power (Watts): $P_i = \dfrac{0.1}{10 \times 10^{-9}} = 10 \text{ MW}$

Average Power (Watts): $\bar{P} = \dfrac{0.1}{20 \times 10^{-6}} = 5 \text{ kW}$

$$\frac{\text{Average Power}}{\text{Peak Power}} = \frac{\bar{P}}{P_i} = \frac{5 \text{ kW}}{10 \text{ MW}} = \frac{10 \times 10^{-9}}{20 \times 10^{-6}} \equiv 5 \times 10^{-4}$$

Duty cycle $= 5 \times 10^{-4} \times 100 \equiv 0.05\%$

I. Pulsed lasers: Principles

- Energy is stored during off-time and released during on-time
  - Energy stored in the form of light and periodically allowed to escape (cavity dumping; mode-locking)
  - Energy stored in the atomic system in the form of a population inversion and released by allowing oscillation (Q-switching)
  - Very high peak powers can be generated: Pulse widths: 10^{-9} s to 10^{-15} s (ns to fs)
- Gain is controlled by turning pump on and off (gain switching: max power output is cw

I. Pulsed lasers: Principles

- To model pulsing, steady-state solutions (see 5. LaserAmplifier) are inadequate
- Equations giving temporal behaviour of n (photon number density) and $N(t)$ (population inversion) must be used:

$$\frac{dn}{dt} = -\frac{n}{\tau_p} + NW_i$$

Photon loss due to leakage from resonator (τ_p = photon lifetime)

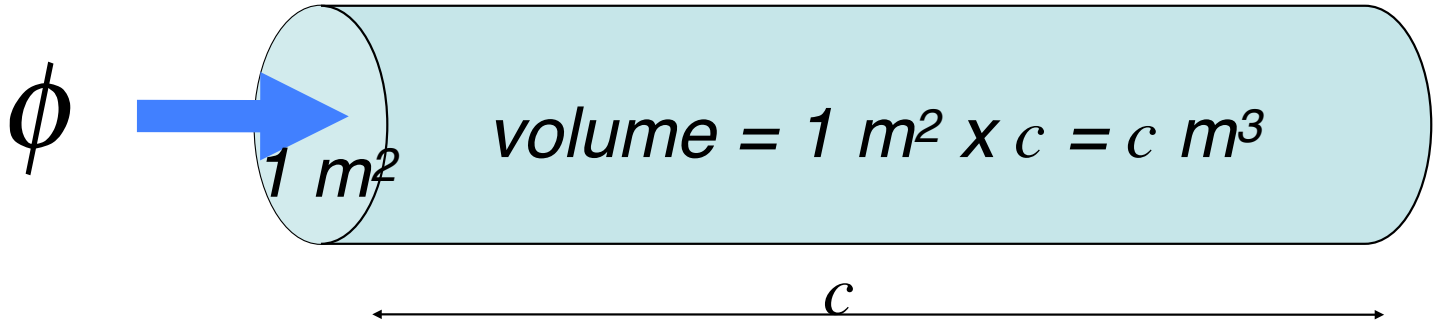
Net photon gain (stimulated processes)

I. Pulsed lasers: Principles

- Using previously established expressions:

$$W_i = \phi\sigma(\nu) \text{ and } N_t = \frac{\alpha_r}{\sigma(\nu)},\ \alpha_r = \frac{1}{c\tau_p}$$

- Photon flux is nc *(c speed of light).*



$$\phi = nc$$

$$\frac{dn}{dt} = -\frac{n}{\tau_p} + \frac{N}{N_t} \times \frac{n}{\tau_p}$$

I. Pulsed lasers: Principles

- The rate equation for N depends on the pumping scheme used (3- or 4-level)

3-level pumping scheme (Chap. 5)

$$\rightarrow \frac{dN_2}{dt} = R_2 - \frac{N_2}{t_{21}^{sp}} - W_i(N_2 - N_1)$$

$\tau_2 \approx t_{21}^{sp} =$ spontaneous rate between 1 and 2

R_2 is independent of N

$N_a = N_1 + N_2 \rightarrow N_1 = (N_a - N)/2$ and $N_2 = (N_a + N)/2$

$$\rightarrow \frac{dN}{dt} = \frac{N_0}{t_{21}^{sp}} - \frac{N}{t_{21}^{sp}} - 2W_i N = \frac{N_0}{t_{21}^{sp}} - \frac{N}{t_{21}^{sp}} - 2\frac{N}{N_t} \times \frac{n}{\tau_p}$$

I. Pulsed lasers: 3-level scheme

Set of coupled nonlinear differential equations

$$\begin{cases} \dfrac{dn}{dt} = -\dfrac{n}{\tau_p} + \dfrac{N}{N_t} \times \dfrac{n}{\tau_p} \\ \dfrac{dN}{dt} = \dfrac{N_0}{t_{21}^{sp}} - \dfrac{N}{t_{21}^{sp}} - 2\dfrac{N}{N_t} \times \dfrac{n}{\tau_p} \end{cases}$$

2 parameters:

N_0: small-signal population inversion

N_t: laser threshold

- Suggests two methods to "modify" the temporal behaviour of $n(t)$ and $N(t)$:
  - Modulate the value of N_0 and keep a fixed threshold: **Gain switching**
  - Modulate the laser threshold N_t value and keep a fixed pumping rate: **Q-switching** and **Cavity dumping**

- Same conclusions would be obtained for 4-level scheme

I. Pulsed lasers: Principles

- (Numerical) Solutions will determine the transient behaviour:

$$n(t) \text{ and } N(t)$$

Setting $\frac{dN}{dt} = \frac{dn}{dt} = 0 \Rightarrow$ steady-state solutions:

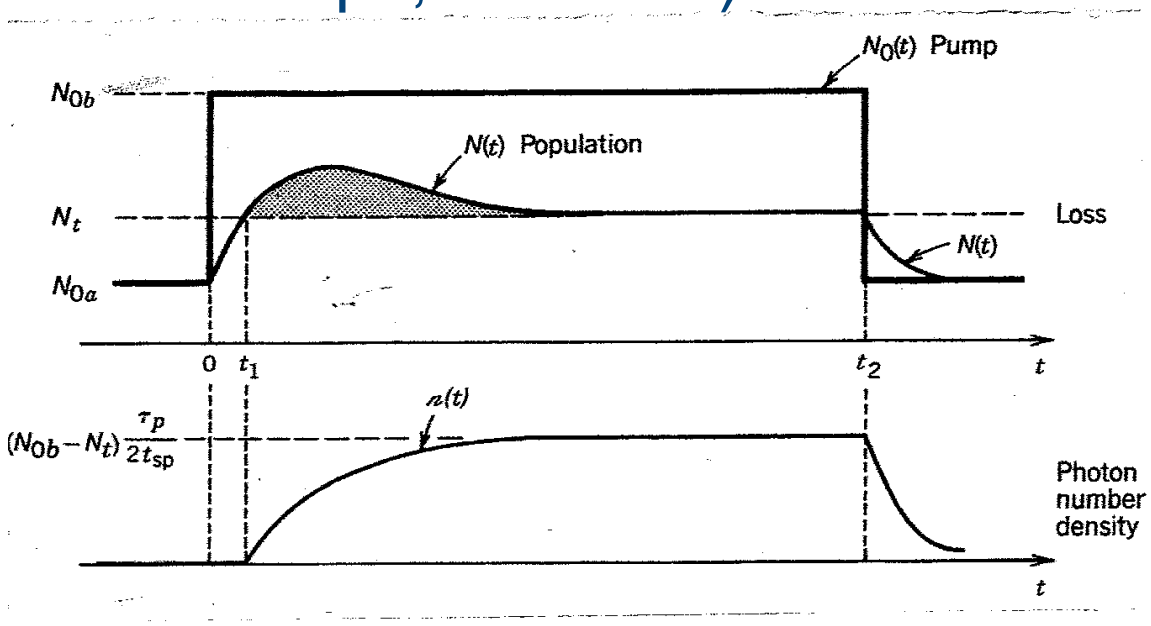
$$\begin{cases} N = N_t \\ n = (N_0 - N_t)\dfrac{\tau_p}{2t_{21}^{sp}} \end{cases}$$

$2t_{21}^{sp} \equiv \tau_S$: Saturation time-constant in 3-level scheme

(same as calculated in chap.5)

II. Pulsed lasers: Gain switching

- Laser pump is turned on and off:
  - During on-times, gain exceeds loss: oscillation is possible and laser light is produced
- Technique widely used to modulate semiconductor lasers: electric current is easily switched (several amps, 10's kHz)



•For t<0, population difference is below threshold

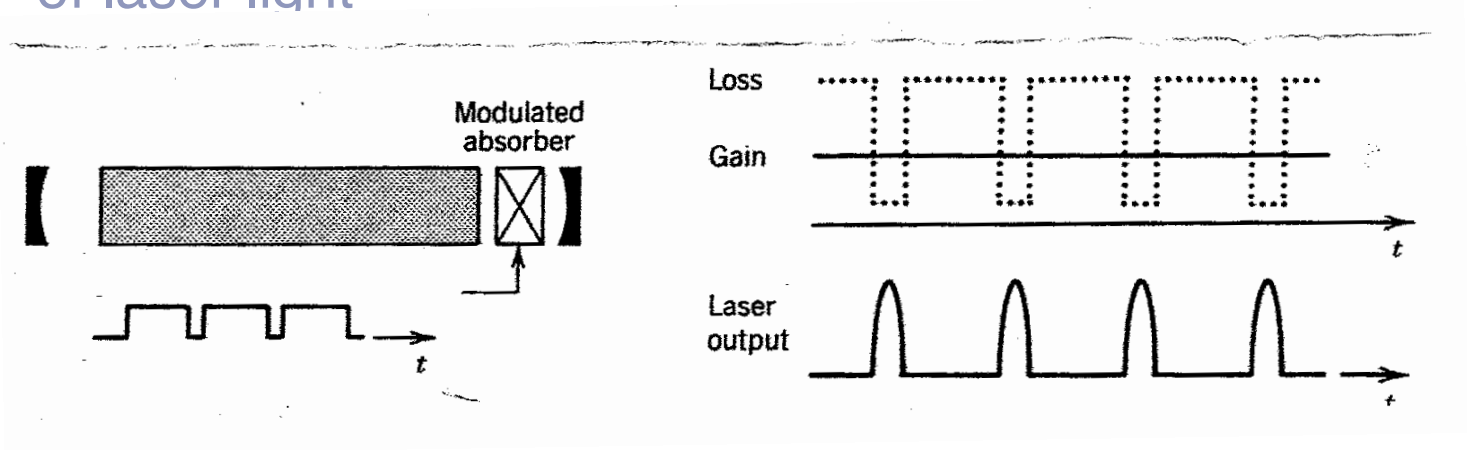
•At t=0, pump turned on to above threshold.

•Population inversion is above threshold at t_1 and then depletes.

•Pump turned off at t_2

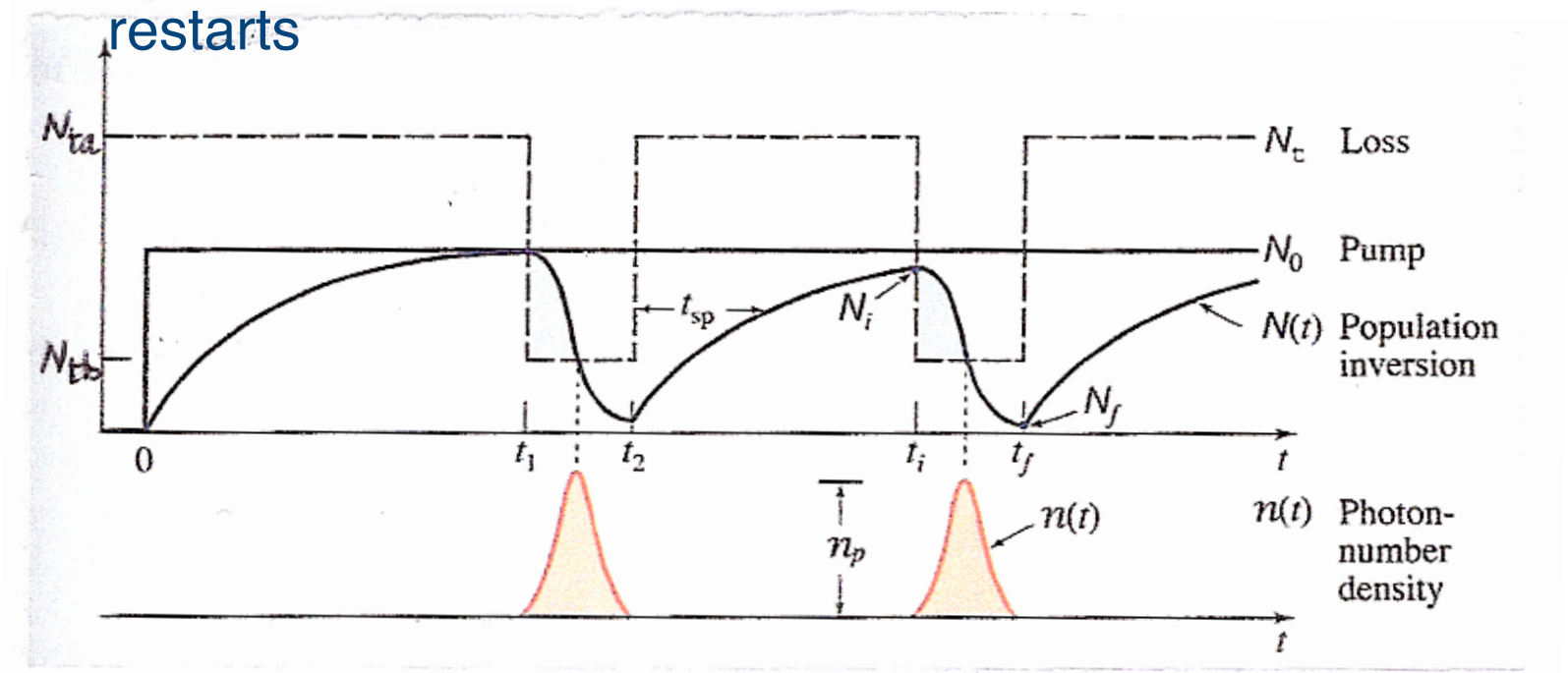
II. Pulsed lasers: Q- switching (Giant laser pulse)

- Laser output is turned off by increasing the resonator loss (spoiling the resonator quality factor Q) periodically with the help of a modulated absorber
- Q-switching = Loss switching
- Pump continues to deliver constant power at all times
  - Accumulated population difference during off-times
  - Losses are reduced (on-times), N_i is released: intense pulse of laser light



II. Pulsed lasers: Q- switching

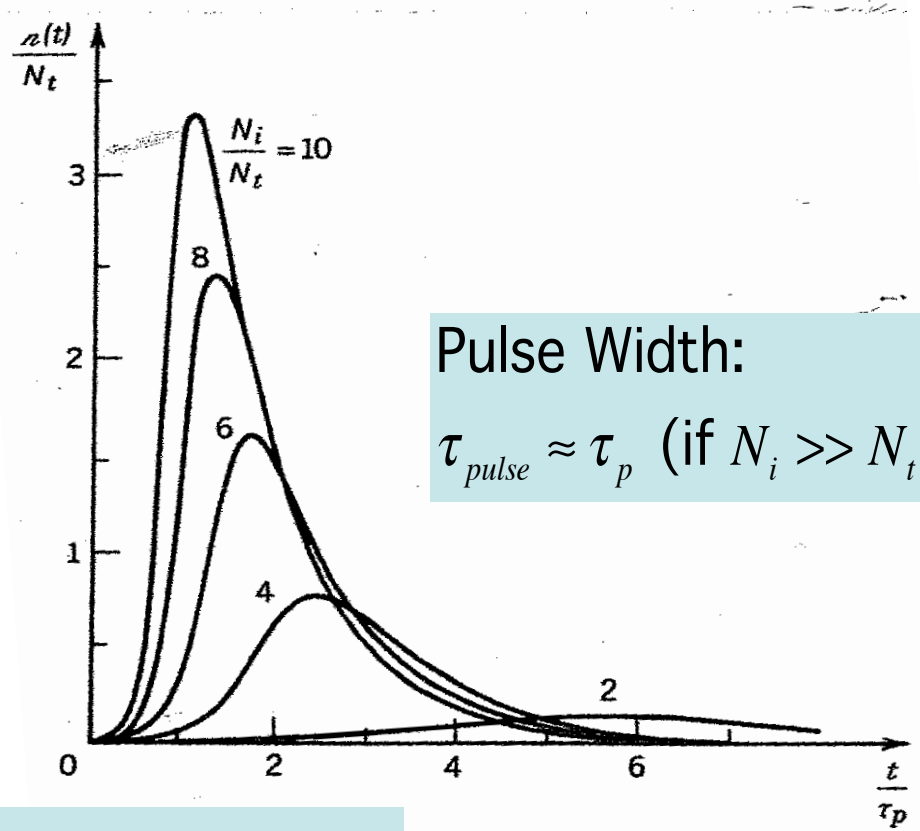
- At t = 0: pump is turned on. Loss is very high (high threshold N_t)
- At t_1 the loss is suddenly decreased, as soon as $N_t < N_0$ the oscillation begins: photon number increases sharply
- Due to gain saturation, $N(t)$ decreases and falls below loss level: oscillation stops
- At t_2: high loss is reinstated, long period of build up of $N(t)$ restarts



II. Pulsed lasers: Q- switching

- Numerical integration of the set of 2 equations provides all the characteristics of the output Q-switched pulse+pulse shape

n=0 @ N = Ni



Photon density during pulse:

$$n \approx \frac{1}{2} N_t Ln \frac{N}{N_t} - \frac{1}{2}(N - N_i)$$

Pulse output optical power:

$$P_O = h\nu A\phi_O = h\nu T \frac{c}{2d} Vn$$

(T = mirror transmittance)

V = Ad

Pulse Width:

$$\tau_{pulse} \approx \tau_p \quad (\text{if } N_i >> N_t)$$

Peak Pulse Power:

$$P_p = \frac{1}{2} h\nu T \frac{c}{2d} VN_i$$

Pulse Energy:

$$E = \int_{t_i}^{t_f} P_O \, dt = \frac{1}{2} h\nu T \frac{c}{2d} V\tau_p (N_i - N_f)$$

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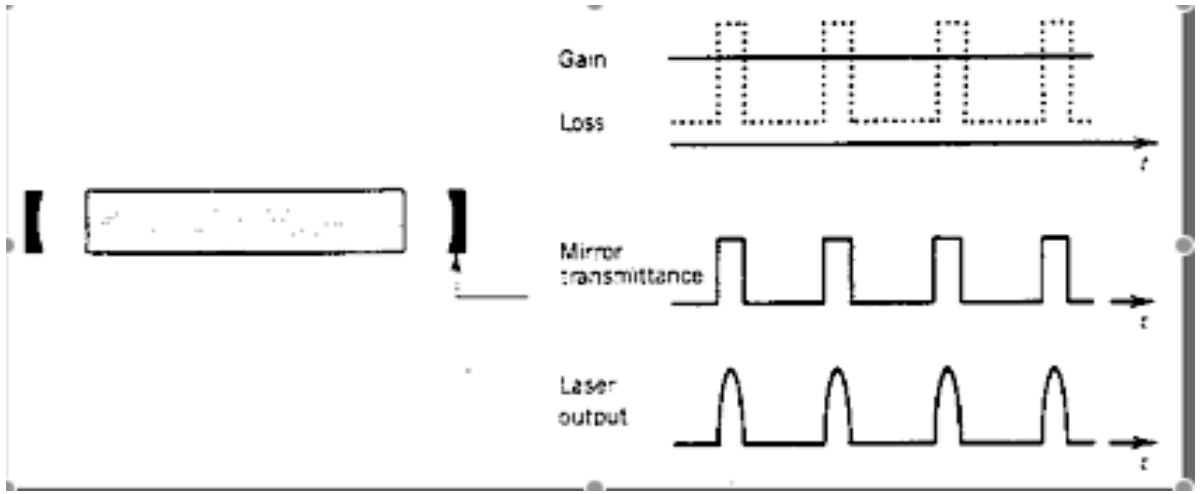
II. Q- switching: Giant pulse Ruby laser

- $N = 1.58 \times 10^{20}$ cm^{-3} (density of Cr^{3+}: active centres)
- Wavelength $\lambda_{laser} = 694.3$ nm
- $\alpha = 0.2$ cm^{-1} (absorption coefficient)
- $A = 1$ cm^2
- $d = 0.1$ m
- $n = 1.78$
- (1-R) = loss per pass = 20%
- Threshold $N_t = 8.8 \times 10^{18}$ cm^{-3}

- Peak power 8×10^8 Watts
- Pulse energy = 2 Joules
- Pulse width = 17.5 ns

II. Methods of pulsing: cavity dumping

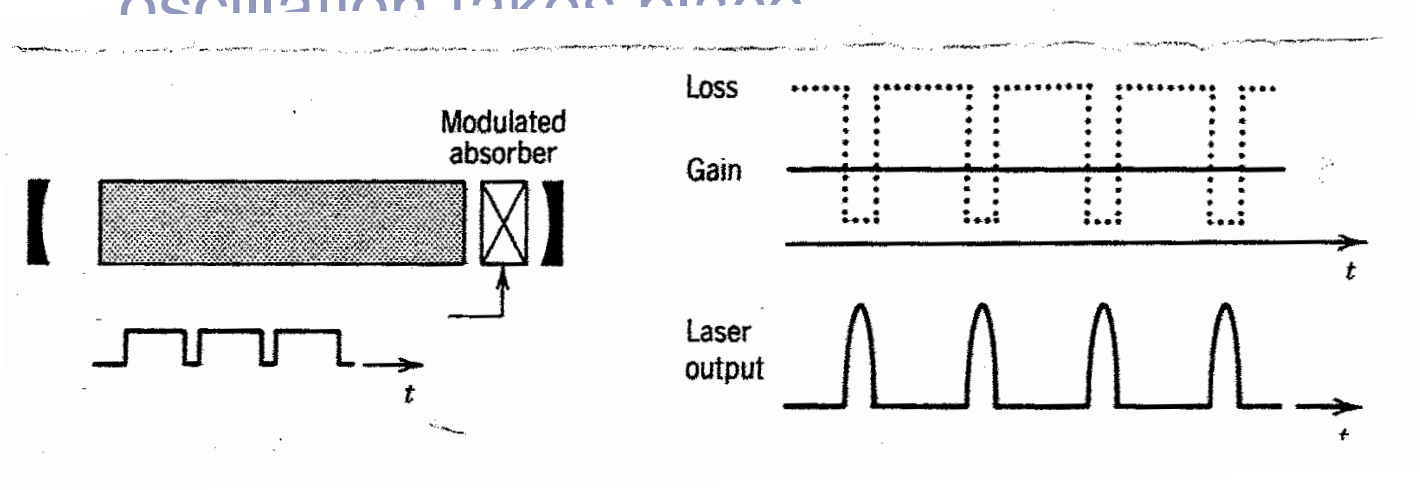
- Photons are stored rather than a population difference. Resonator loss is altered by modifying the output coupler transmittance
  - Optical losses are extremely low except for very brief interval during which mirrors are not parallel (eg rotating shaft)



Mirror is 100% transmitting when out of alignment

II. Passive Q-switching

- Passive Q-switching: Use of a saturable absorber -included inside the resonator.
  - Initially, high losses due to absorption: population inversion accumulates (with small photon flux)
  - When intensity becomes large: absorption saturates and the losses drop below gain: oscillation takes place

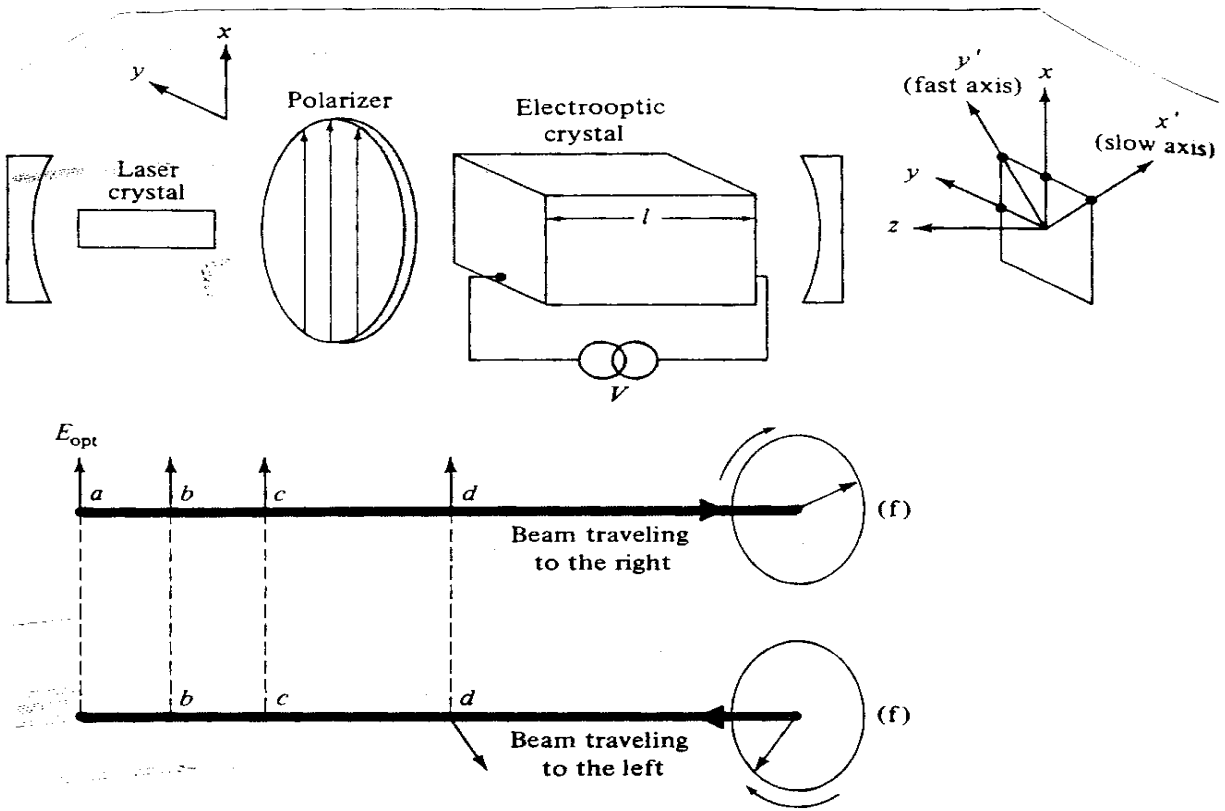


II. Active Q-switching

- Active Q-switching: Use of a voltage controlled gate inside the resonator.
  - An electro-optic crystal or a liquid Kerr cell can be used:
  - A linear polariser is placed after the amplifier crystal (linearly polarised along x)
  - The electro-optic crystal introduces a pi/2 phase shift (1/4 wave retarder): converts to circular polarisation
  - Next pass: another pi/2 phase shift converts to linear polarisation with a total phase difference of pi: linearly polarised along y:
  - Crossed-polarisers situation: light is blocked
  - Q-switching achieved by removal of voltage applied to electro-optic crystal

An electro-optic crystal: change of index of refraction resulting from the application of a DC or low-frequency E field. If the crystal is anisotropic, it will therefore change the state of polarisation of polarised light when the voltage is applied.

II. Q-switching with electro-optic crystal: voltage control



For beam traveling to right:

At point d,

$$E_{x'} = \frac{E}{\sqrt{2}} \cos \omega t, \quad E_{y'} = \frac{E}{\sqrt{2}} \cos \omega t$$

The optical field is linearly polarized with its electric field vector parallel to x

At point f,

$$E_{x'} = \frac{E}{\sqrt{2}} \cos\left(\omega t + kl + \frac{\pi}{2}\right), \quad E_{y'} = \frac{E}{\sqrt{2}} \cos(\omega t + kl)$$

Circularly polarized

For beam traveling to left:

At point f,

$$E_{x'} = -\frac{E}{\sqrt{2}} \cos\left(\omega t + kl + \frac{\pi}{2}\right), \quad E_{y'} = -\frac{E}{\sqrt{2}} \cos(\omega t + kl)$$

Circularly polarized

At point d,

$$E_{x'} = -\frac{E}{\sqrt{2}} \cos(\omega t + 2kl + \pi), \quad E_{y'} = -\frac{E}{\sqrt{2}} \cos(\omega t + 2kl)$$

Linearly polarized along y

Electrooptic crystal used as voltage-controlled gate in Q-switching a laser.

III. Mode-locking

- Typically lasers oscillate on many different axial modes (frequency separation c/2d between these modes).
- These modes normally oscillate independently: “poor” temporal coherence.
- External means can be used to couple all the modes and lock their phases together.
- Full treatment uses Fourier analysis
- Alternative treatment consists in looking at the properties of a mode-locked pulse train:
  - Each mode is a plane wave traveling in the +z direction at velocity $c=c_0/n$

III. Mode-locking

The total optical field:

$$U(z,t)=\sum_q A_q \, exp\left[i2\pi\nu_q(t-z/c)\right]$$

$\nu_q=\nu_0+q\nu_F$ and $(q=0,\pm1,\pm2,\pm3,...)$

(It is assumed that the q = 0 mode coincides with ν_0).

The phase and arguments of the A_q are statistically independent in a laser.

Substituting:

$$U(z,t)=\sum_q A_q \, exp\left[i2\pi q\nu_F(t-z/c)\right]\times exp\left[i2\pi\nu_0(t-z/c)\right]$$

$\equiv$ Complex envelope $\times$ "constant" term (ν_0)

Complex envelope:

Periodic function of period $T_F=\dfrac{1}{\nu_F}=\dfrac{2d}{c}$

Periodic function of z of period $c\times T_F=2d$

III. Mode-locking

Consider ideal case $A_q = A$ and M modes:

$$q = 0, \pm 1, \pm 2, \ldots, \pm S \Rightarrow 2S + 1 = M$$

Complex amplitude becomes:

$$A\left(t - \frac{z}{c}\right) = A \sum_{q=-S}^{q=+S} \exp\left[2\pi i q \left(\frac{t}{T_F} - \frac{z}{cT_F}\right)\right]$$

Same as: $A\left(t - \frac{z}{c}\right) = A \sum_{q=-S}^{q=+S} x^q = A \frac{x^{S+1} - x^{-S}}{x - 1}$

$$= A \frac{x^{S+1/2} - x^{-S-1/2}}{x^{1/2} - x^{-1/2}}$$

Finally:

$$A(t - z/c) = MA \frac{\sin\left[M(t - z/c)/T_F\right]}{\sin\left[(t - z/c)/T_F\right]} \times e^{i \ldots\ldots}$$

$$\sin cx \equiv \frac{\sin(x\pi)}{x\pi}$$

The optical intensity: $I(t,z) = \left|A(t - z/c)\right|^2$

$$I(t,z) = M^2 A^2 \frac{\sin^2\left[M(t - z/c)/T_F\right]}{\sin^2\left[(t - z/c)/T_F\right]}$$

III. Mode-locking

- The power of the mode-locked pulse is emitted in the form of a train of pulses with period: $T_F = \frac{2d}{c}$

- The light in the mode-locked laser can be regarded as a single narrow pulse of photons reflecting back and forth between the resonator mirrors.

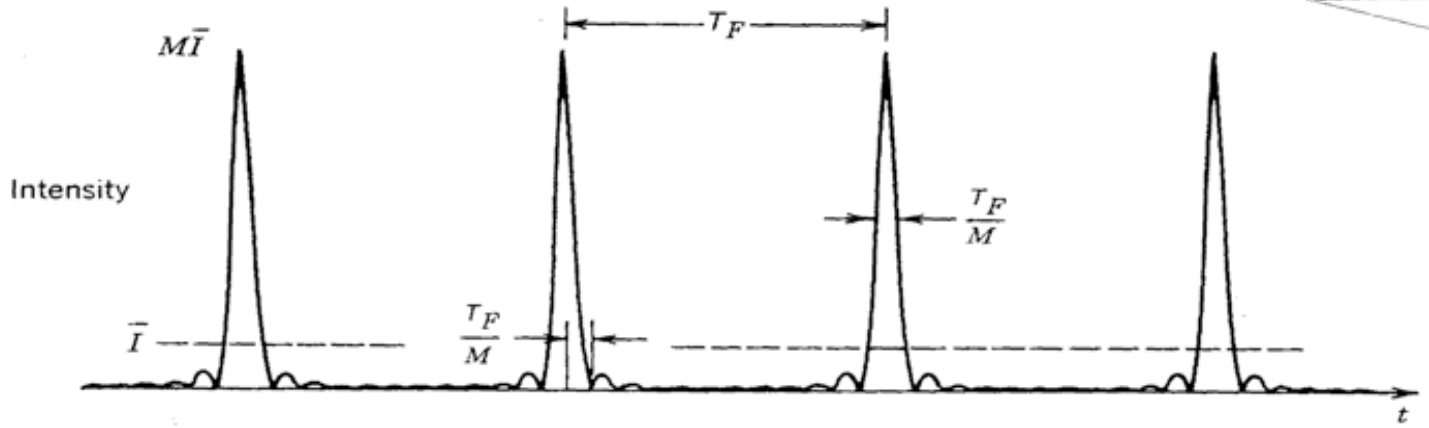
Average intensity: $\bar{I} = M|A|^2$

Peak intensity: $I_p = M^2|A|^2 = M\bar{I}$

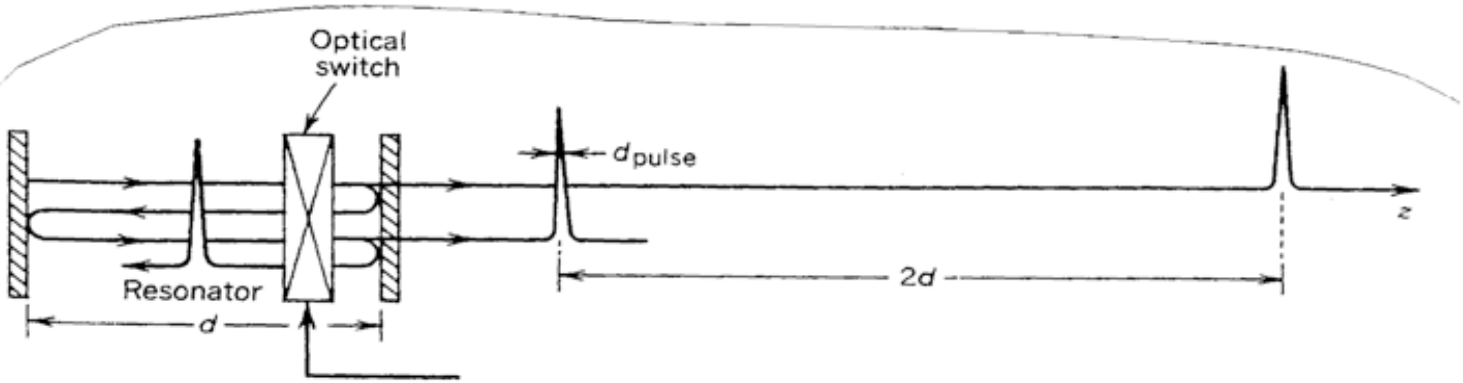
M-times larger than mean intensity ($\bar{I} \equiv$ incoherent addition)

Individual pulse width is defined as the time from the peak to the first zero: $\tau_{pulse} = \frac{T_F}{M}$

III. Mode-locked pulse



Intensity of the periodic pulse train resulting from the sum of M laser modes of equal magnitudes and phases. Each pulse has a width that is M times smaller than the period T_F and a peak intensity that is M times greater than the mean intensity.



The mode-locked laser pulse reflects back and forth between the mirrors of the resonator. Each time it reaches the output mirror it transmits a short optical pulse. The transmitted pulses are separated by the distance $2d$ and travel with velocity c. The switch opens only when the pulse reaches it and only for the duration of the pulse. The periodic pulse train is therefore unaffected by the presence of the switch. Other wave patterns, however, suffer losses and are not permitted to oscillate.

III. Mode-locking

- Because the atomic linewidth can be quite large, very narrow mode-locked pulses can be generated:

$$M \approx \frac{\Delta\nu}{\nu_F} \quad (\Delta\nu \text{ is the atomic linewidth})$$

$$\tau_{pulse} = \frac{T_F}{M} \approx \frac{1}{\Delta\nu}$$

- Laser energy is condensed in a packet that travels back and forth with a distance between each pulse of:

$$cT_F = 2d$$

- The spatial length of a pulse = its duration x c:

$$d_{pulse} = \frac{T_F}{M} \times c = \frac{2d}{M}$$

III. Mode-locking

Example: the Nd:glass laser

$\lambda_0 = 1.06\ \mu m$; $n = 1.5$; $\Delta\nu = 3\times10^{12}$ Hz

$$\tau_{pulse} \approx \frac{1}{\Delta\nu} = 0.33 \text{ ps} = 330 \text{ fs}$$

$$d_{pulse} = c\times\tau_{pulse} = 67\ \mu m$$

Resonator length $d = 0.1$ m

Mode separation $\nu_F = \frac{c}{2d} \approx 1$ GHz

Mode number $M \approx 3{,}000$

Peak intensity $= 3{,}000 \times$ average intensity

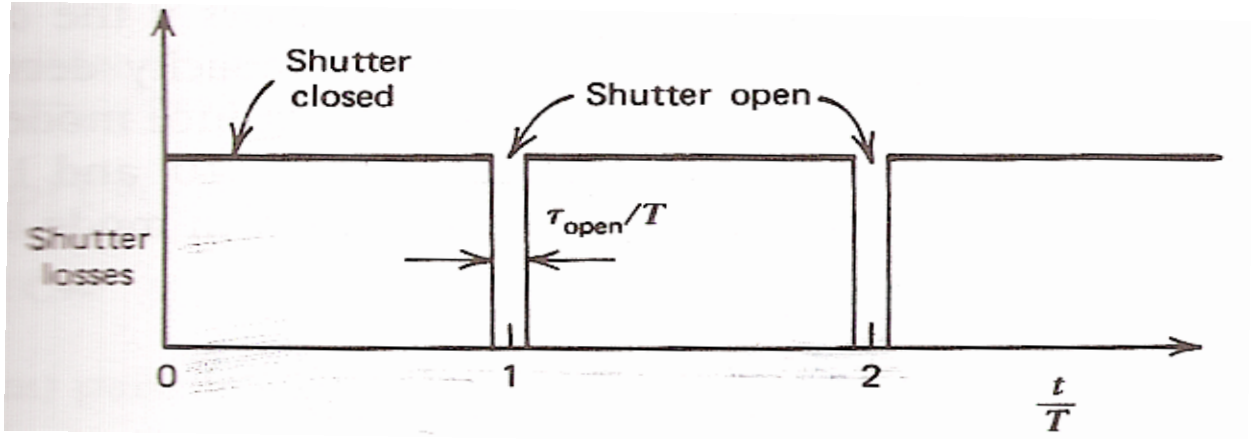
- Mode-locking works well for amplifiers with broad linewidths, ie most solid state lasers

- Gas lasers have narrow linewidths: mode-locking not so efficient for the generation of ultrashort pulses.

III. Mode-locking

- From Fourier analysis: mode-locking can be achieved by modulating the losses or gain at a frequency equal to the intermode frequency ν_F

- Loss modulation = thin shutter inside resonator:
  - Closed most of the time: high losses α_r
  - Open every 2d/c for the duration of τ_{pulse}



- Only a mode-locked pulse arriving at the shutter position when it is open will survive and not be attenuated by it.

III. Mode-locking

- Optical oscillators containing saturable absorbers tend to spontaneously mode-lock (passive mode-locking):
  - Absorption saturates for the high peak intensity of the mode-locked pulse: becomes transparent for a brief instant.

Methods of mode-locking: acousto-optical (Bragg) loss modulation

- Periodic loss is introduced by Bragg diffraction of a portion of the laser intensity from a standing acoustic wave

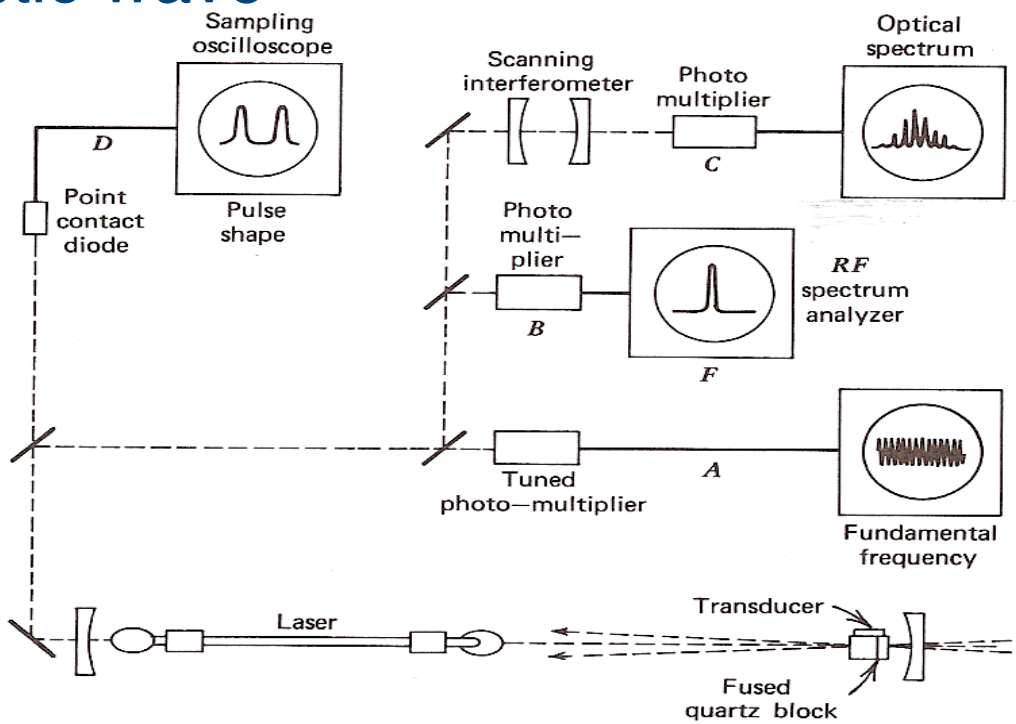


FIGURE 20.11 Experimental setup for laser mode locking by acoustic (Bragg) loss modulation. Parts *A*, *B*, *C*, and *D* of the experimental setup are designed to display the fundamental component of the intensity modulation, the power spectrum of the intensity modulation, the power spectrum of the optical field $E(t)$, and the optical intensity, respectively. *Source:* Reference 8.

Methods of mode-locking:acousto-optical (Bragg) loss modulation

- Acoustic oscillation (strain) sets up a standing wave pattern on the crystal (modulation of the index of refraction)
- Equivalent to a phase diffraction grating:

$$S(z,t) = S_0 \cos\omega_a t \cos k_a z$$

Acoustic velocity $v_a = \dfrac{\omega_a}{k_a}$

Grating spatial period: $\dfrac{2\pi}{k_a}$

- Diffraction loss during one acoustic period has its peak value twice: loss modulation frequency is thus $2\omega_a$
- Mode-locking will occur for $2\omega_a = \omega_F$

Methods of mode-locking:synchronous gain modulation

- Mode locked argon ion laser pumps a dye laser
- Pump pulses are synchronised exactly to the pulse repetition rate of the dye laser
- Dye laser gain medium is pumped once in each round-trip time period: pumping pulse and mode-locked pulse overlap spatially and temporally in the dye cell.
- 30 fs, mJ laser energy

Methods of mode-locking:synchronous gain modulation

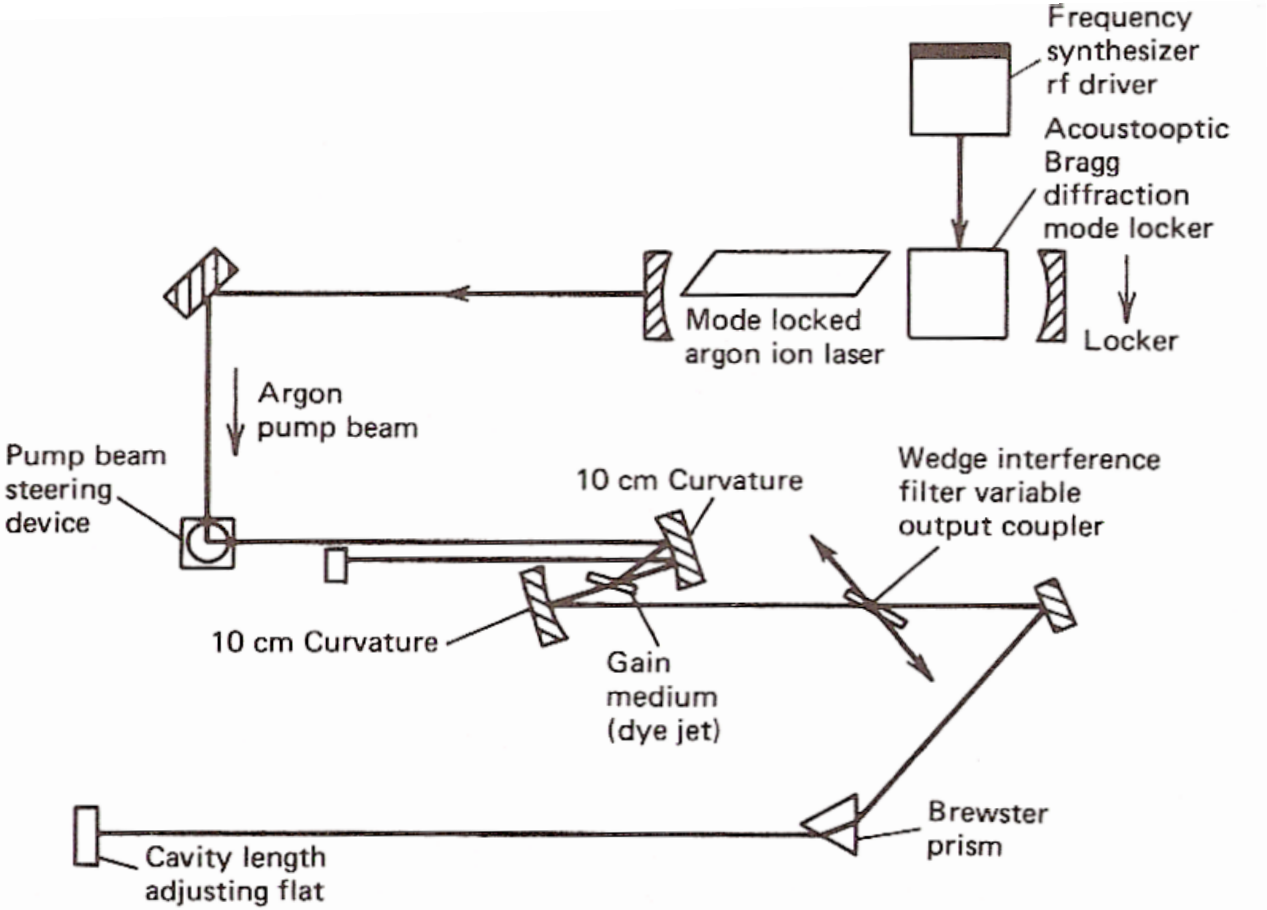


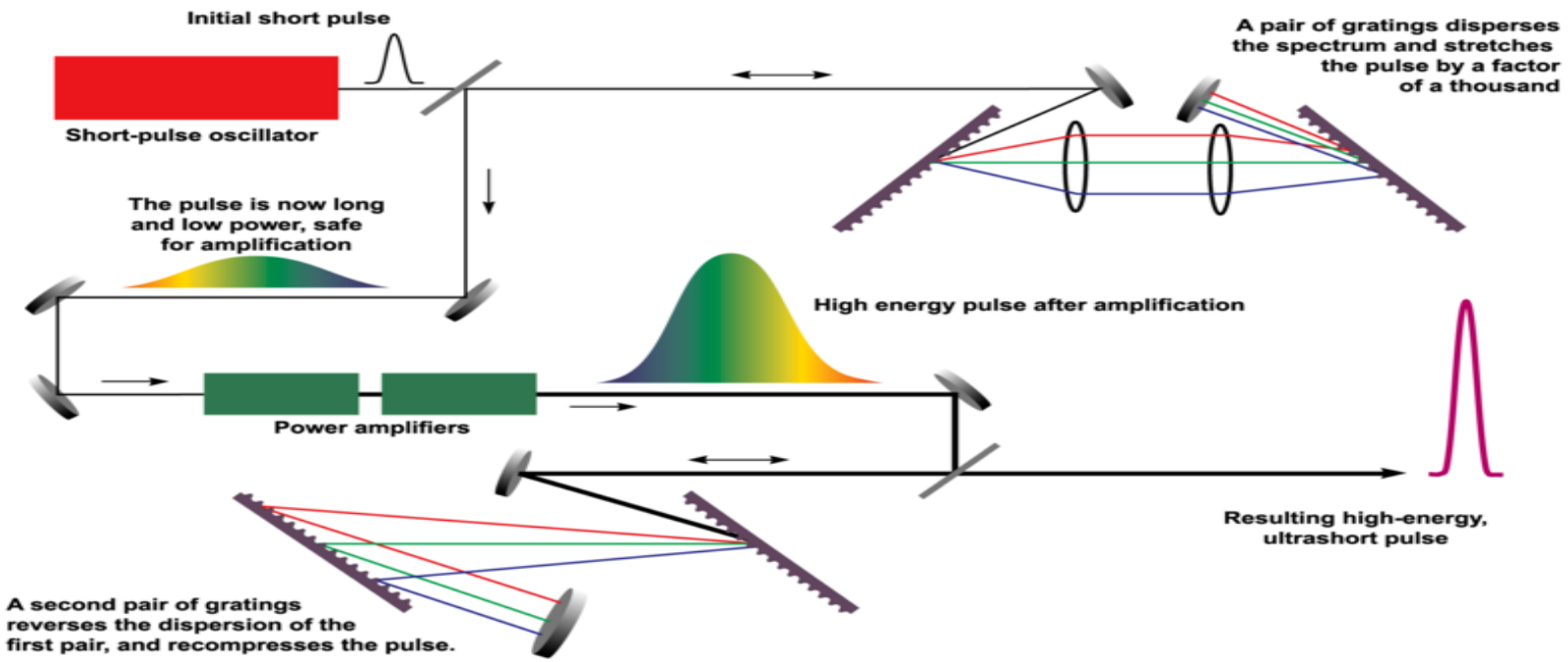
FIGURE 20.15*a* Synchronously mode-locked dye laser configuration. *Source:* T. L. Koch, The California Institute of Technology, Pasadena, CA.

Mode-locking:limitations

- Mode-locking cannot be used to produce even shorter pulses (higher intensities) with solid state amplifiers
- Non linear effects (e.g., optical Kerr effect, self-waveguiding, self-focusing,...) become too important and severely distort beam + lead to optical damage.

- Solution: stretching and frequency-coding of the pulses using the technique of “chirped pulsed amplification” CPA.
- “True” femto lasers, can produce intensities of 10^{15}-10^{18} Wcm^{-2}: Ti-sapphire lasers

Chirped pulse amplification: CPA- femto lasers



- The main points of CPA
  - Generate an ultrashort pulse – Short Pulse Oscillator,
  - Stretch the pulse by positive dispersion – Stretcher,
  - Amplify the pulse without damage to the laser – Ultra Broadband Amplifier,
  - Compress the pulse with negative dispersion – Compressor

Chirped pulse amplification: CPA- femto lasers