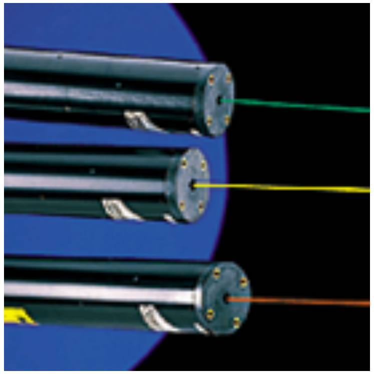
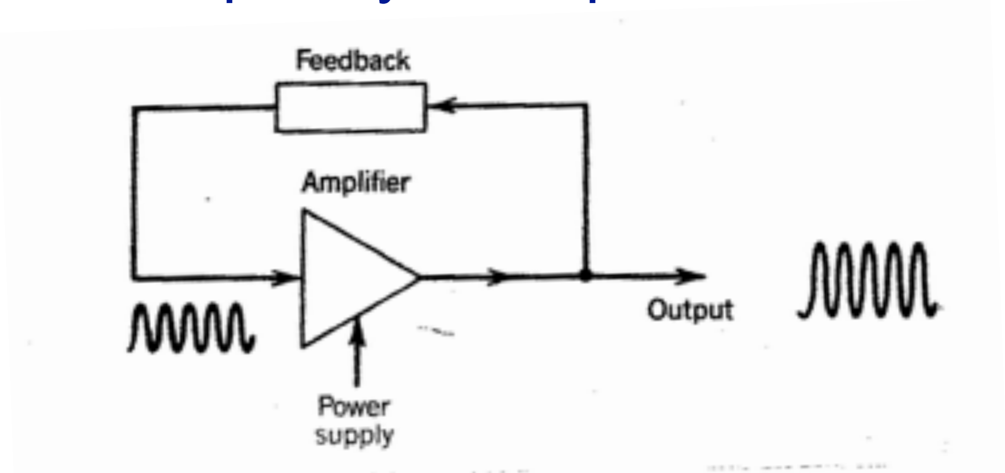
Quantum Electronics
Laser Physics
PS407

6. Theory of the Laser Oscillation



I. Laser oscillator: Overview

- Laser is an optical oscillator.
- Resonant optical amplifier whose output is fed back into its input with matching phase (coherent) -positive feedback.
- Unstable situation due to gain saturation of amplifier: limits further growth of signal.
- Steady-state situation is reached: output signal is created at frequency of amplifier

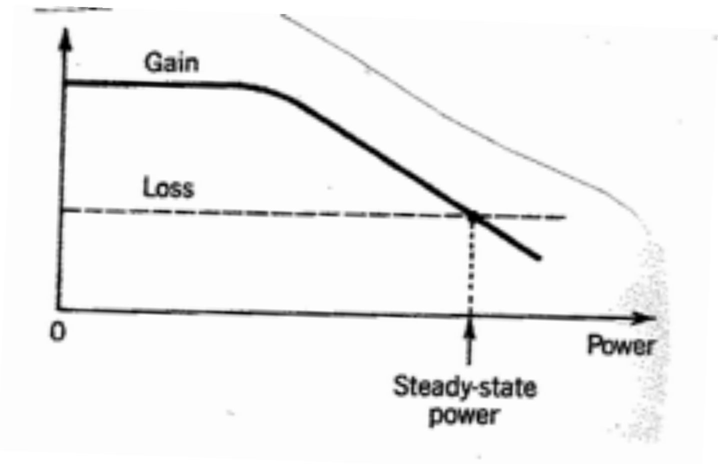


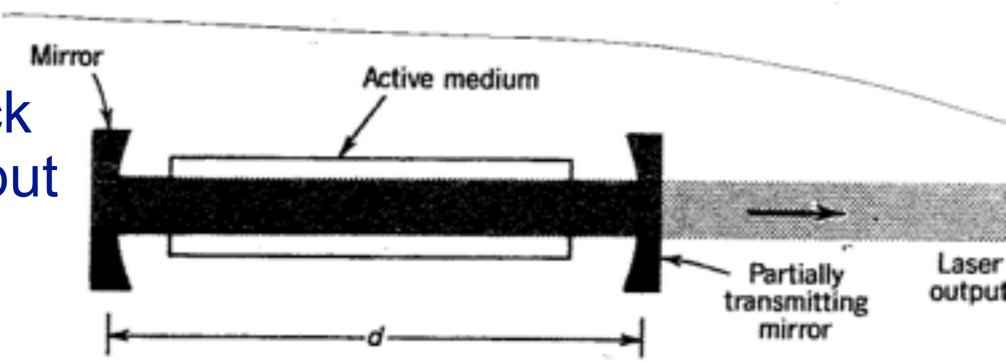
I. Laser oscillator: Overview

- Two conditions must be satisfied simultaneously (frequency-dependent):
  - Amplifier gain must be greater than the loss in the feedback system: net gain must be incurred in a round trip
  - Total phase shift must be a multiple of 2pi: input phase matches the output phase.
- The oscillation begins by amplifying noisy frequency components (spontaneous photons). The system is unstable and power grows, amplifier saturates and gain drops.
- Stable condition obtains when the reduced gain is equal to the loss per cycle: gain just compensates the losses and steady-state oscillation is achieved.
- Losses comprise the useful output (laser light is extracted via an output coupling scheme).

I. Laser oscillator: Overview

- Required components for laser action:
  - Amplifier with gain saturation mechanism
  - A feedback system
  - A frequency-selection mechanism
  - An output coupling scheme
- Pumped active medium: freq. dependent amplifier with gain saturation.
- Optical Resonator: Feedback + frequency selection + output coupling.
- Oscillation conditions achieved at the resonance frequencies of the oscillator.





II. Review of laser amplifier

- Laser amplifier is a narrowband coherent amplifier.
- Stimulated emission from atomic level whose population is inverted.
- Bandwidth determined by linewidth (to include broadening mechanisms).
- Characterised by its gain per unit length: governs the rate at which the photon flux increases.
- Phase shift per unit length.

$\gamma(\nu) \equiv$ gain coefficient

$I = h\nu\phi$ Optical intensity

$$\gamma(\nu) = \frac{\gamma_0(\nu)}{1+\phi/\phi_S(\nu)} : \text{gain per meter}$$

$$\text{with } \frac{1}{\phi_S(\nu)} = \tau_S \sigma(\nu)$$

$$\mathrm{N} = \frac{\mathrm{N}_0}{1+\phi/\phi_S(\nu)}$$

$$\text{with } \gamma_0(\nu) = N_0 \times \sigma(\nu)$$

$$\varphi(\nu) = \left(\frac{\nu-\nu_0}{\Delta\nu}\right)\gamma(\nu) : \text{phase shift}$$

II. Review of optical resonator

- Optical feedback is achieved by placing the amplifying medium in an optical resonator (length d).
- Phase shift per unit length is equal to wavenumber k:

$$\Delta\varphi = \frac{2\pi}{\lambda} d = \frac{2\pi}{\lambda_0} nd = k_0 nd \rightarrow k_0 = \Delta\varphi / nd$$

- Resonator is lossy due to scattering and absorption: distributed loss coefficient per unit length:

Overall distributed-loss written as:

$$\exp(-2\alpha_r d) = R_1 R_2 \exp(-2\alpha_S d)$$

$$\alpha_r = \alpha_S + \frac{1}{2d} Log \frac{1}{R_1 R_2}$$

α_r is the loss per unit length,

$c\alpha_r$ is the loss per unit time

The photon lifetime:

$$\tau_p = \frac{1}{c\alpha_r}$$

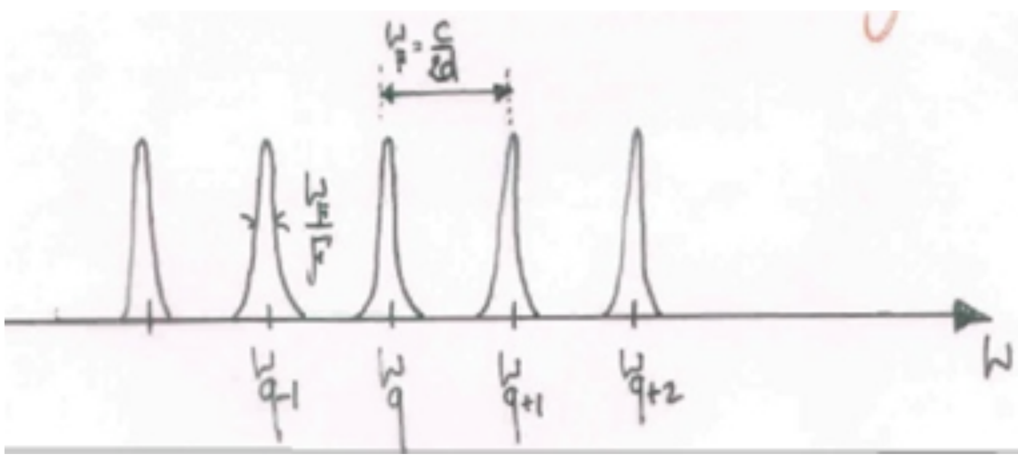
II. Review of optical resonator

- The resonator can only sustain frequencies that correspond to a round-trip phase shift of $q \times 2\pi$:

Mode frequencies: $\nu_q = q\nu_F,\ q = 1,2,3,\ldots$

$\nu_F = \dfrac{c}{2d} \equiv$ spacing between the modes.

$\delta\nu \approx \dfrac{\nu_F}{F},\ F \equiv$ finesse of resonator



III. Gain condition and laser threshold

- Small-signal gain coefficient must be greater than loss coefficient: otherwise oscillation will never start.

$\gamma_0(\nu) > \alpha_r$ with $\gamma_0(\nu) = N_0 \sigma(\nu) \Rightarrow$

$$\gamma_0(\nu) = N_0 \left(\frac{1}{4\pi\varepsilon_0} \right) \frac{\pi e^2}{mc} \times f \times g(\nu) \Rightarrow$$

$$N_0 > \frac{\alpha_r}{\sigma(\nu)} = \frac{\alpha_r}{\left(\frac{1}{4\pi\varepsilon_0} \right) \frac{\pi e^2}{mc} \times f \times g(\nu)} \equiv N_t(\nu)$$

$N_t \equiv N_t(\nu) =$ Threshold population difference

Minimum pumping rate for initiation of laser oscillation

III. Gain condition and laser threshold

- Threshold condition can be expressed in terms of photon lifetime.
- Shows that threshold is lowest at frequency where cross-section is highest and directly proportional to linewidth:

$$N_t = \frac{1}{c\tau_p \sigma(\nu)} = \frac{1}{c\tau_p \left(\frac{1}{4\pi\varepsilon_0}\right)\frac{\pi e^2}{mc} \times f \times g(\nu)}$$

For Lorentzian lineshape: $g(\nu_0) = \frac{2}{\pi\Delta\nu}$ is max

$$N_t(\nu_0) = \frac{\pi\Delta\nu}{2c\tau_p \left(\frac{1}{4\pi\varepsilon_0}\right)\frac{\pi e^2}{mc} \times f}$$

III. Phase condition: laser frequencies

- Total phase shift after round trip must be a multiple of 2π:

(Phase lag due to 2d path)+(Phase shift due to amplifier) $= 2q\pi$

$$2 \times (\frac{2\pi}{\lambda_0} nd) + 2\varphi(\nu)d = 2q\pi$$

- If $\varphi(\nu)d$ is small $\Rightarrow \nu = \nu_q = q\left(\frac{c}{2d}\right)$ Cold resonator modes
- If $\varphi(\nu)d$ is not small $\Rightarrow$ "New" laser modes must be determined from knowledge of $\varphi(\nu) \Rightarrow$ "Frequency pulling"

III. Phase condition: laser frequencies

For Lorentzian lineshape phase shift is known:

$$\nu + \frac{c}{2\pi} \times \frac{\nu - \nu_0}{\Delta\nu} \gamma(\nu) = \nu_q$$

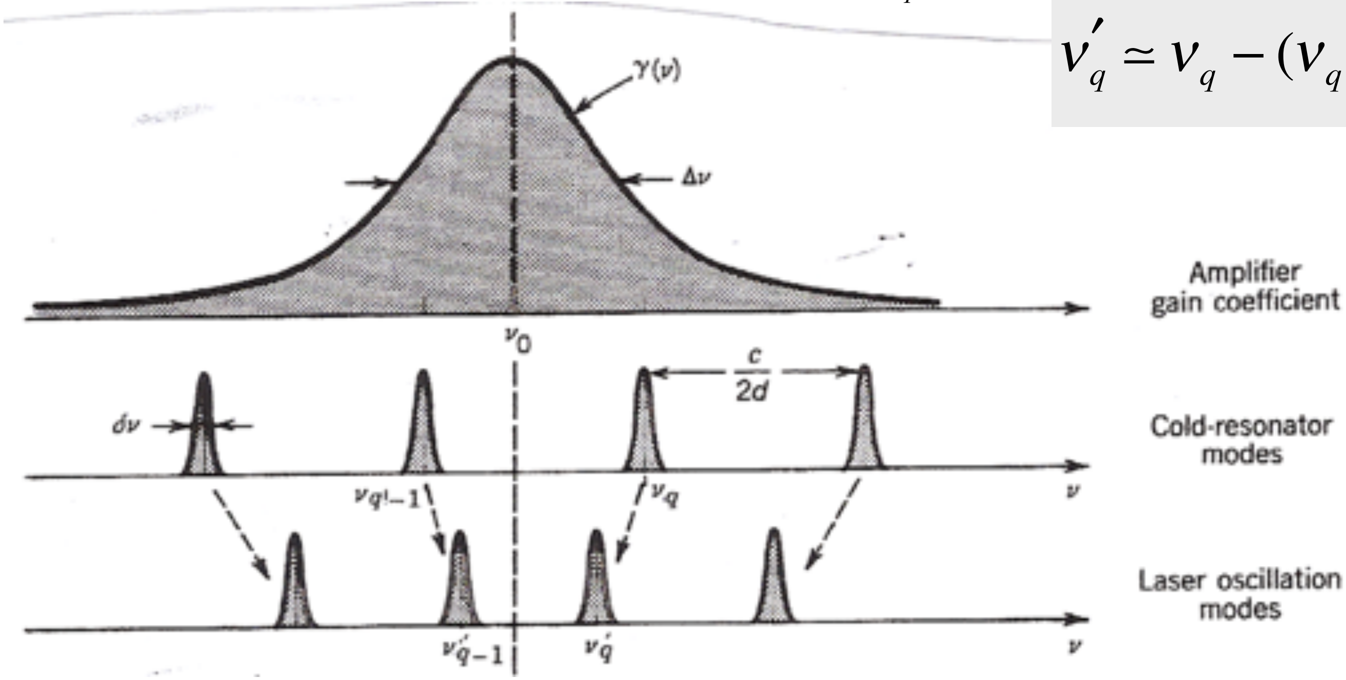
Nonlinear equation to be solved for

the new frequencies $\nu = \nu_q'$

corresponding to each cold resonator mode ν_q

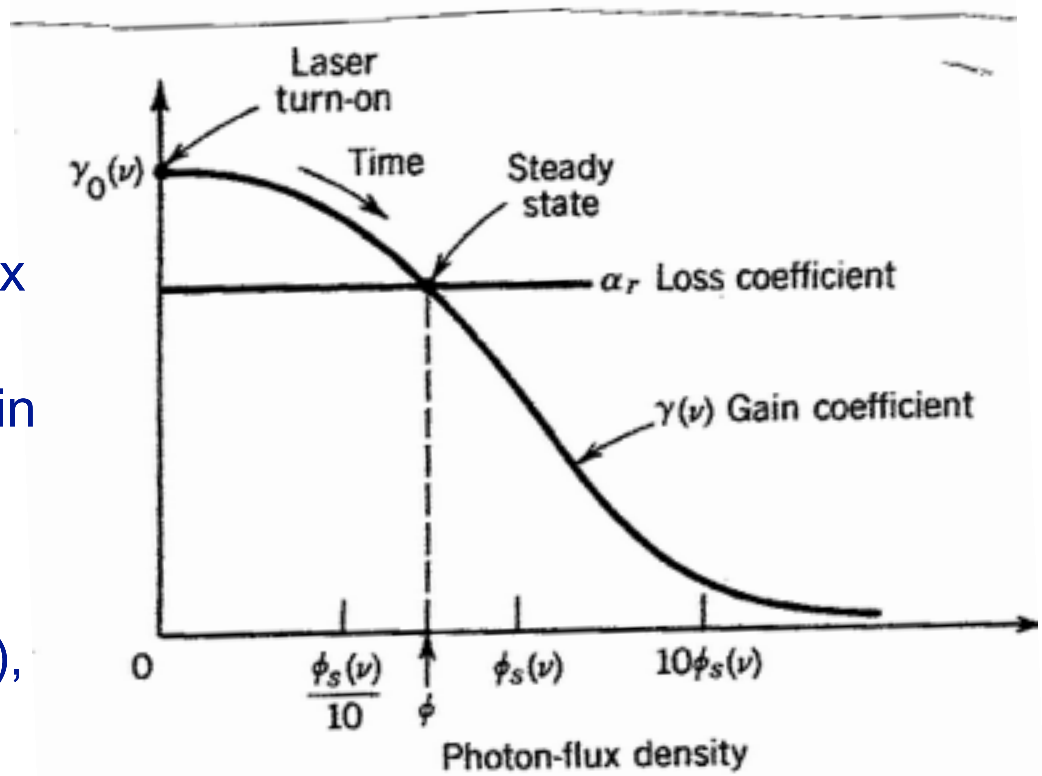
- New modes are slightly pulled towards the central frequency of the line:

$$\nu_q' \simeq \nu_q - (\nu_q - \nu_0)\frac{\delta\nu}{\Delta\nu}$$



IV. Characteristics of the laser output: Power

- When laser turned on: small-signal gain coefficient.
- Oscillation grows once phase condition is satisfied: increase in flux (gain gets smaller).
- When the saturated gain coefficient equals the loss coefficient: population inversion @ threshold value ($N = N_t$), no further growth and steady-state oscillation obtains at gain equals loss: **Gain clamping**.



IV.Laser output: Power

In steady-state (gain clamped at loss value):

$$\frac{\gamma_0(\nu)}{1+\phi/\phi_S(\nu)} = \alpha_r \Rightarrow \text{the output flux } \phi \text{ is:}$$

$$\phi = \phi_S(\nu)\left[\frac{\gamma_0(\nu)}{\alpha_r} - 1\right], \quad \gamma_0(\nu) > \alpha_r$$

$$\phi = 0, \qquad \gamma_0(\nu) \le \alpha_r$$

$\phi \equiv$ Photon flux density= mean number of photons crossing a unit surface area of the laser in both directions.

Since $\gamma_0 = N_0\sigma(\nu)$ and $\alpha_r = N_t\sigma(\nu) \Rightarrow$

$$\phi = \phi_S(\nu)\left[\frac{N_0}{N_t} - 1\right], \quad N_0 > N_t$$

$$\phi = 0, \qquad N_0 \le N_t$$

IV. Laser output: Power

- The output photon flux density:

$$\phi_0 = \tau \times \frac{\phi}{2}$$ where τ is the output coupler transmittance

- Corresponding optical intensity:

$$I_0 = \frac{h\nu\tau\phi}{2}$$

- Laser output power (explicitly calculated in terms of saturation flux density, threshold and small-signal population inversion, transmittance and beam size):

$$P_0 = I_0 \times A$$

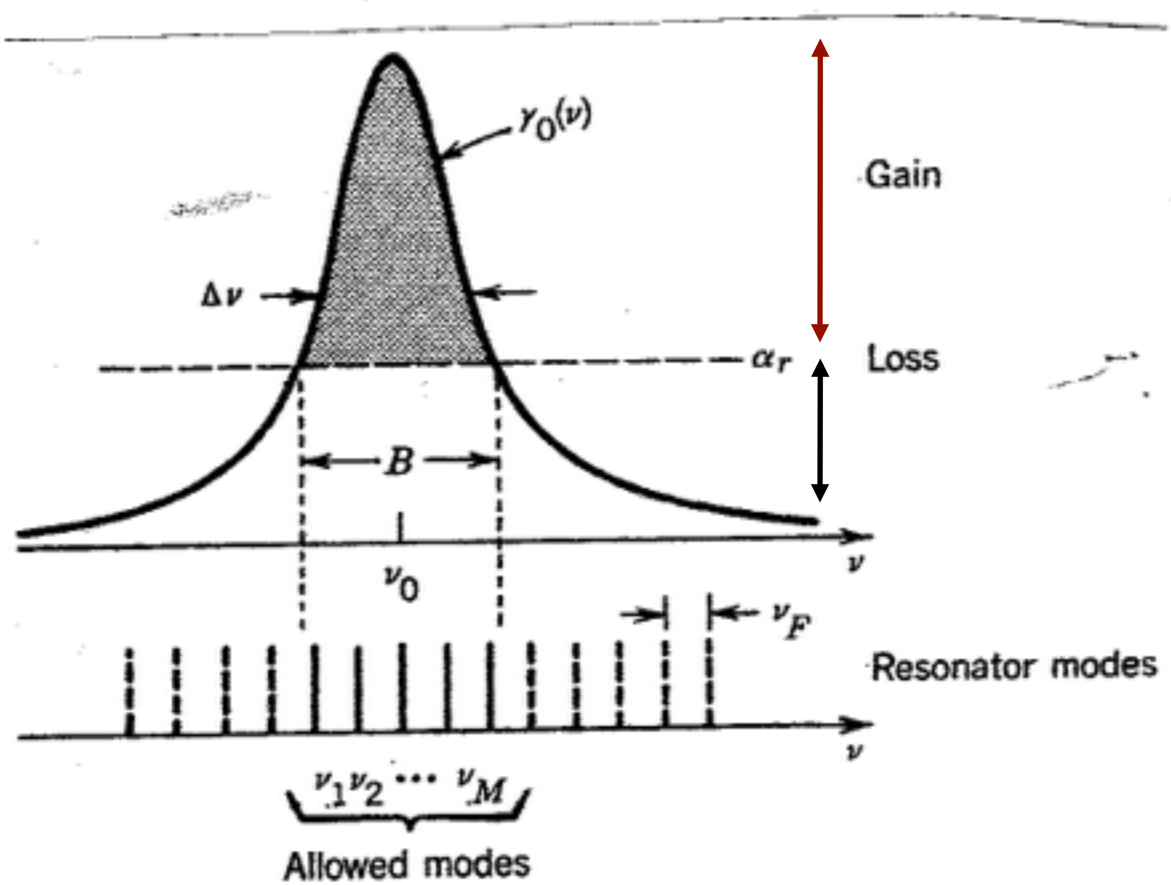
$A \equiv$ Cross-sectional area of laser beam

IV. Laser output: Spectral distribution

- Spectral distribution is determined by the atomic line shape and the resonator modes.
- Illustrated in the two conditions for laser oscillation:
  - gain condition: only frequencies for which the gain is greater than the losses will oscillate. The corresponding frequency band has width B (centered on the line centre).
  - phase condition: the oscillation must be at one of the resonator modal frequencies (either cold resonator or slightly modified for frequency pulling).
- Only a finite number of modes M are allowed:

$$M \approx \frac{B}{\nu_F}$$

IV. Laser output: Spectral distribution



- The approximate spacing between adjacent modes:

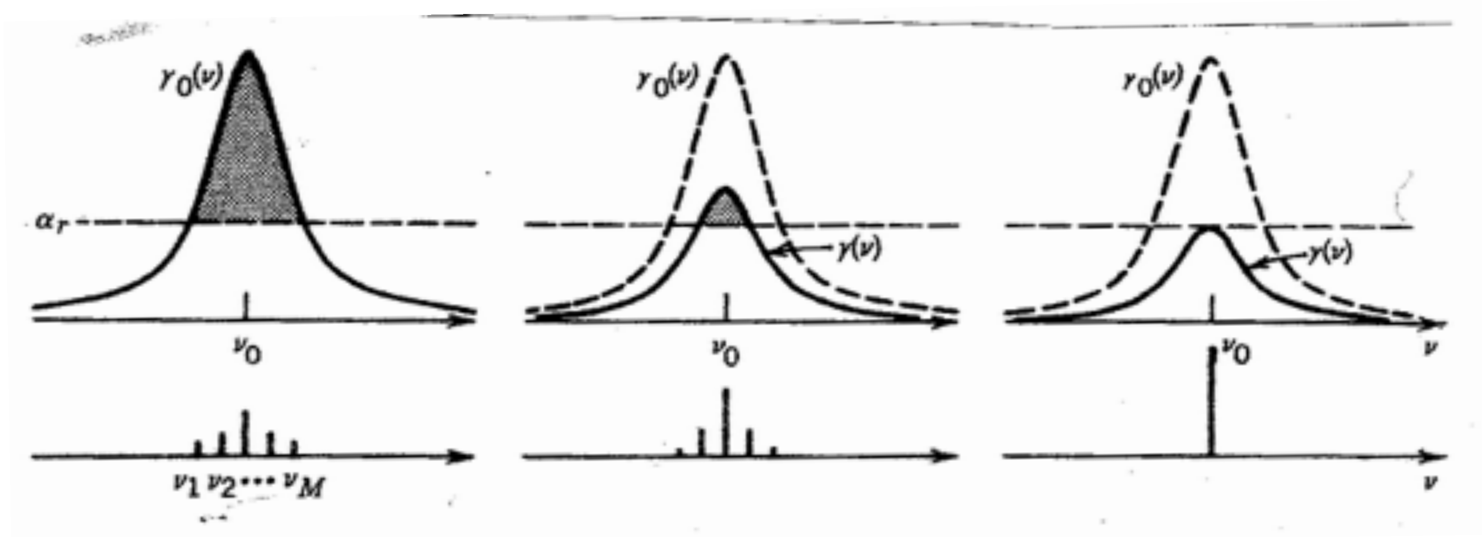
$$\nu_F = \frac{c}{2d}$$

- Of the M modes, the actual number that carry optical power depends on the line broadening mechanism.

IV. Laser output: Spectral distribution

- For homogeneously broadened lasers (e.g. solid state lasers), all the atoms in the amplifier have the same gain curve and "share" the initial population inversion.
- Photon flux densities $\phi_1, \phi_2, \ldots \phi_M$ are created in the M modes. Modes with frequencies nearer the line centre grow more quickly as they have higher gain.
- Competition between the modes: for distant modes the loss becomes greater than the gain, they lose power and can go sub-threshold => quenched.
- Ultimately, a single surviving mode could maintain a gain equal to the loss. It will have a frequency lying closest to the line centre: **Single mode operation.**

IV. Laser output: Spectral distribution



Saturated gain:

$$\gamma(\nu)=\frac{\gamma_0(\nu)}{1+\sum_{j=1}^{M}\phi_j/\phi_S(\nu_j)}$$

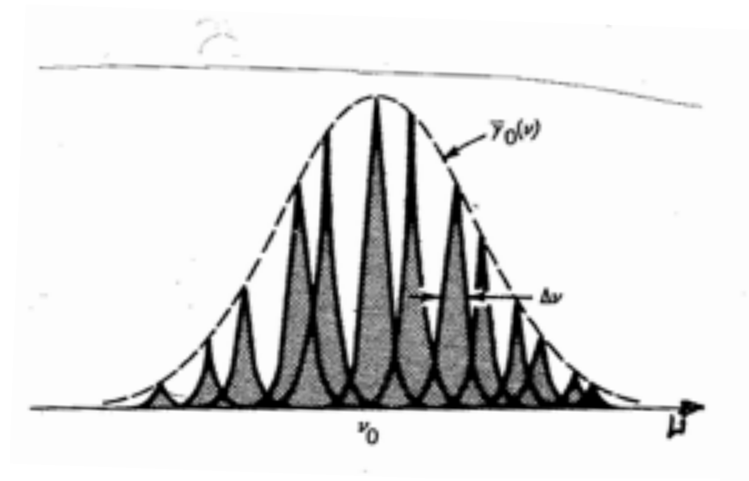
- In practice, homogeneously broadened lasers oscillate on multiple modes.
- Different modes occupy different spatial portions of the active medium
- In spatial regions where most central modes vanish, other modes can lase:

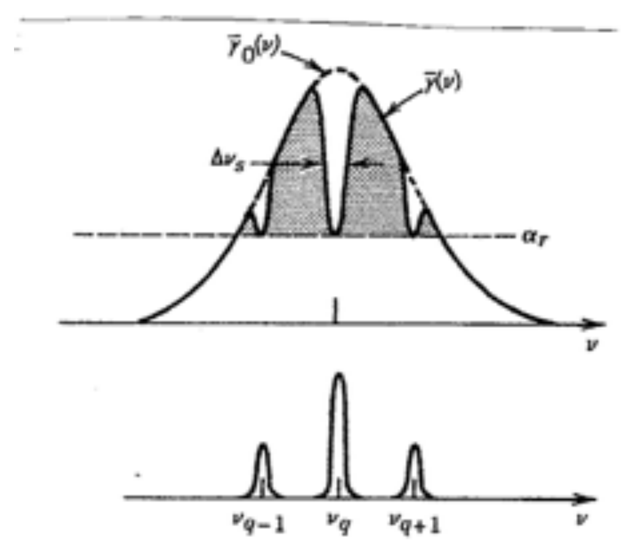
- **Spatial hole burning**

IV. Laser output: Spectral distribution

- For inhomogeneously broadened lasers (gas lasers): the active atoms have different gain curves associated with different properties or different environments (eg. temperature).

- The overall gain curve is a composite envelope of gains of the different lasing species.





IV. Laser output: Spectral distribution

- After the laser is turned on, modes with gain greater than the losses begin to grow. If the spacing between the modes is larger than the width of the atomic lineshapes, then different modes interact with different atoms/molecules
- Atoms whose lineshapes fail to coincide with a mode do not lase (do not “see” photons in the resonator). They remain unaffected and stay at the “small-signal” inversion.
- The lasing atoms deplete the population inversion and hence gain: Spectral hole burning.

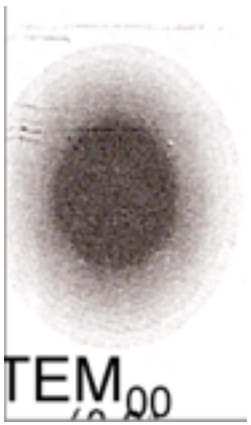
V. Laser output: Spatial distribution

- Spatial distribution depends on the geometry of resonator and shape/dimensions of active medium.
- Laser resonators with spherical mirrors:
  - Support Gaussian beam (TEM_{00})
  - Support a hierarchy of transverse modes $TEM_{l,m}$ (Hermite-Gaussian beams). Each (l,m) mode has an associated spatial distribution
  - For each axial mode q, all (l,m) transverse modes are allowed: $TEM_{q,l,m}$
  - Resonance frequencies of two sets of modes are slightly displaced (but modes still separated by c/2d):

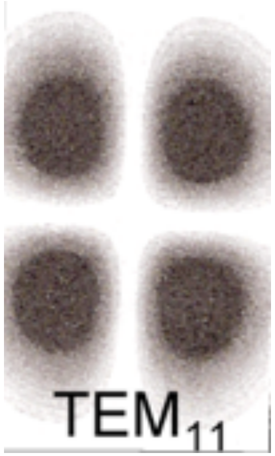
$$\nu_{q,l,m} = q\nu_F + \left(l + m + 1\right)\frac{\Delta\zeta}{\pi}\nu_F$$

V. Laser output: Spatial distribution

- Due to different spatial distributions, (l,m) modes undergo different gain and losses.



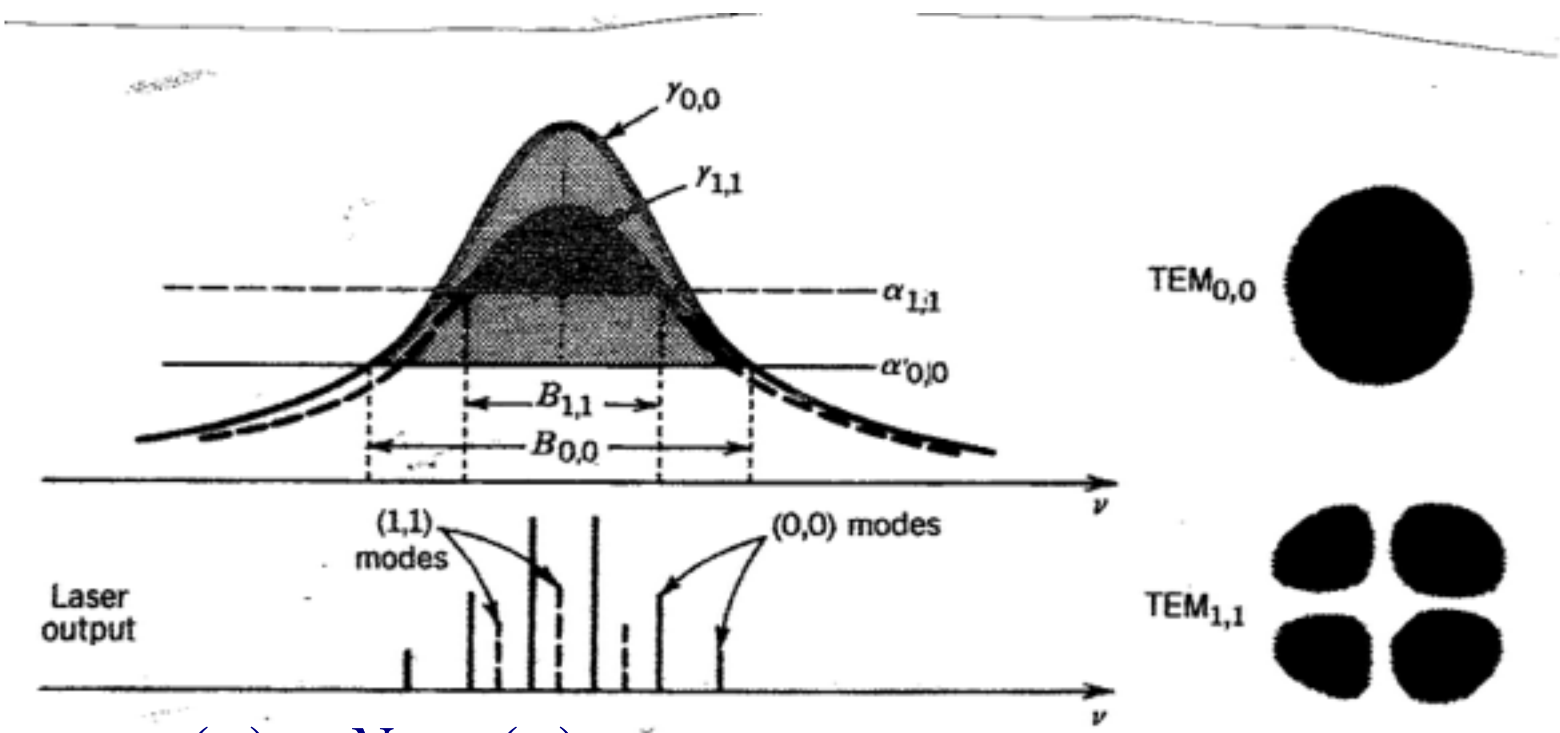
Mostly confined about the axis + least diffraction losses at mirrors



Larger gain due to larger volume, higher diffraction losses

- Therefore, each mode has a different gain curve and loss coefficient.
- This determines the competitive nature of the different transverse modes in their contribution to the laser oscillation.

V. Laser output: Spatial distribution



$$\gamma_{0,0}(\nu) = N_{0,0}\sigma(\nu)$$

$$\gamma_{1,1}(\nu) = N_{1,1}\sigma(\nu)$$

Power in different modes

drawn from atoms at different locations

V. Laser output: Spatial distribution

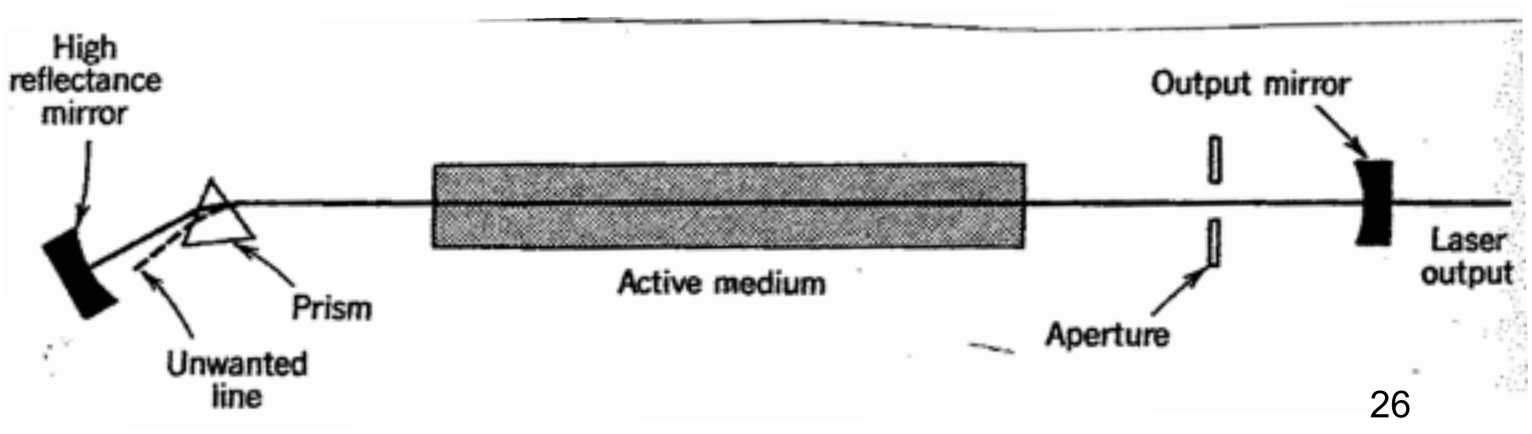
- In a homogeneously broadened laser: strongest mode tends to suppress gain for the other modes
- Spatial hole burning allows transverse modes with substantially different spatial distributions to oscillate simultaneously (draw energy from atoms at different locations).
- Partial spatial overlap and atomic migrations (as in gas lasers) allow for mode competition.
- Lasers designed to operate on (either):
  - Single transverse mode, typically (0,0) as it has the smallest diameter and can be focused to the smallest spot size (single mode laser)
  - Several higher-order modes for generating large optical power (multimode laser)

V. Laser output: Polarisation

- Each (q,l,m) mode has 2 degrees of freedom corresponding to 2 independent orthogonal polarizations (degeneracy due to spin of photon = 2).
- Two polarizations are regarded as independent modes.
- As the two “polarization” modes have the same spatial distribution and equal gains and losses are provided by the resonator: laser oscillates on these 2 modes independently.
- Typically, laser output is **unpolarized**.

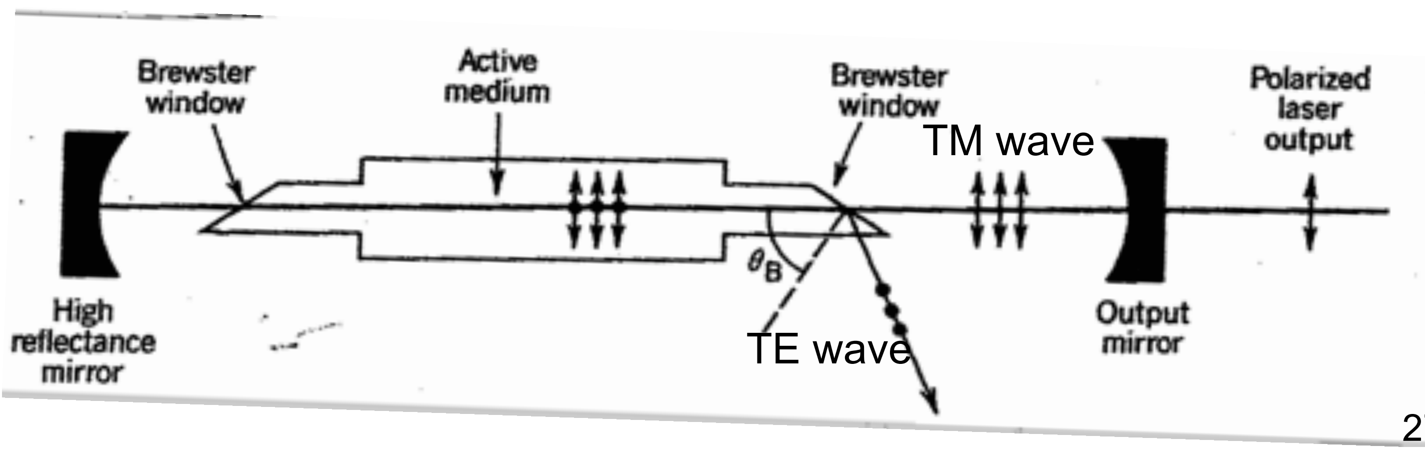
VI Mode selection: laser line

- Laser amplifier with multiple transitions will produce multiple laser lines (see chapter 5).
  - e.g. He-Ne laser: 0.633, 1.15 and 3.39 μm
  - Argon-ion laser 6 lines in the blue to blue-green
- Line selected by placing a prism/grating inside the resonator. Rotation of the dispersive element allows tuning.
- Not suitable for selection of axial modes (too close).



VI. Mode selection: TE mode and polarisation

- Aperture of controllable shape can be used for **TEM mode selection** + mirror design can help favour a particular transverse mode.
- Polariser is placed inside cavity to select a polarisation. All the power goes into the selected polarisation as the other one suffers high losses.
  - Usually implemented with Brewster's windows
  - Output is 100 % TM polarised



VI. Mode selection: axial mode

- To select a single axial mode:
  - Increase the losses sufficiently so that only the mode with the largest gain oscillates: leads to low power output.
  - Increase the axial mode spacing by reducing the resonator length. Again, leads to diminished laser power as the active volume is also reduced.
- Use of intracavity tilted (frequency-selective) étalon to alter the frequency spacing of the resonator modes.
  - Etalon of width d_1 has modes with spacing > B: only one etalon mode fits within the laser bandwidth.
  - The etalon mode is made to coincide with the resonator mode exhibiting largest gain.
  - Etalon can be fine-tuned by means of rotation, change of temperature, or change of width with piezoelectric transducer
  - Etalon slightly tilted to prevent reflections from reaching the mirrors and thereby creating unwanted additional resonances.

VI. Mode selection: axial mode