Quantum Electronics
Laser Physics

Chapter 5.

The Laser Amplifier

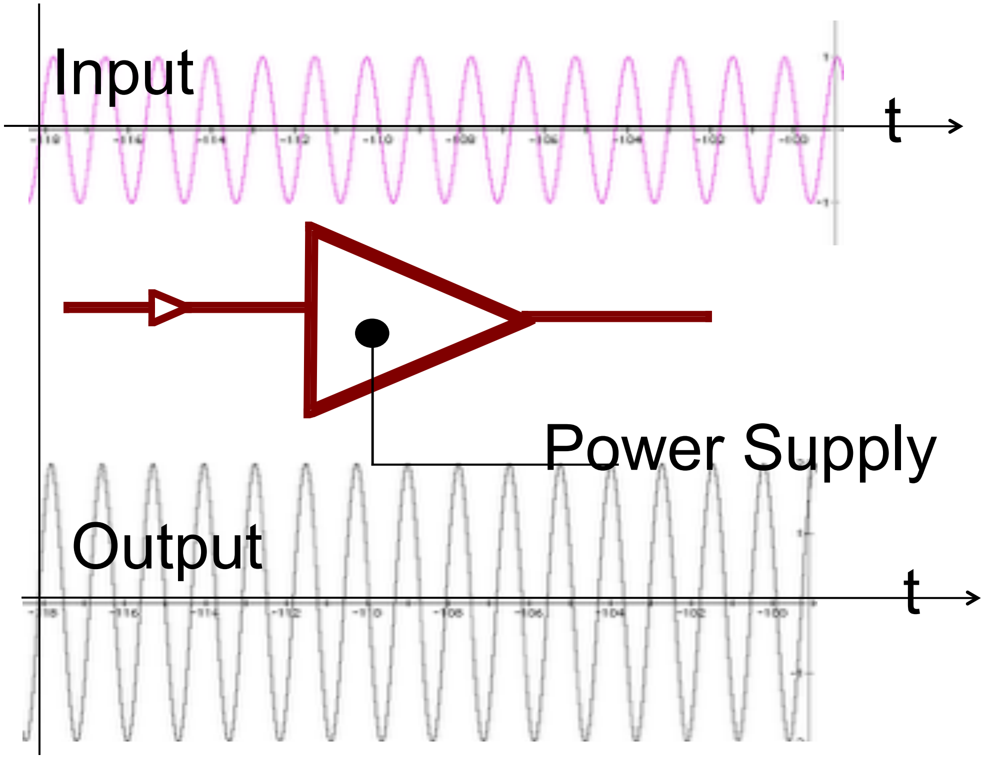
The laser amplifier

- 5.1 Amplifier Gain
- 5.2 Amplifier Bandwidth
- 5.3 Amplifier Phase-Shift
- 5.4 Amplifier Power source and rate equations
- 5.5 Amplifier non-linearity and gain saturation
- 5.6 Saturable absorbers

Laser: Light Amplification by Stimulated Emission of Radiation

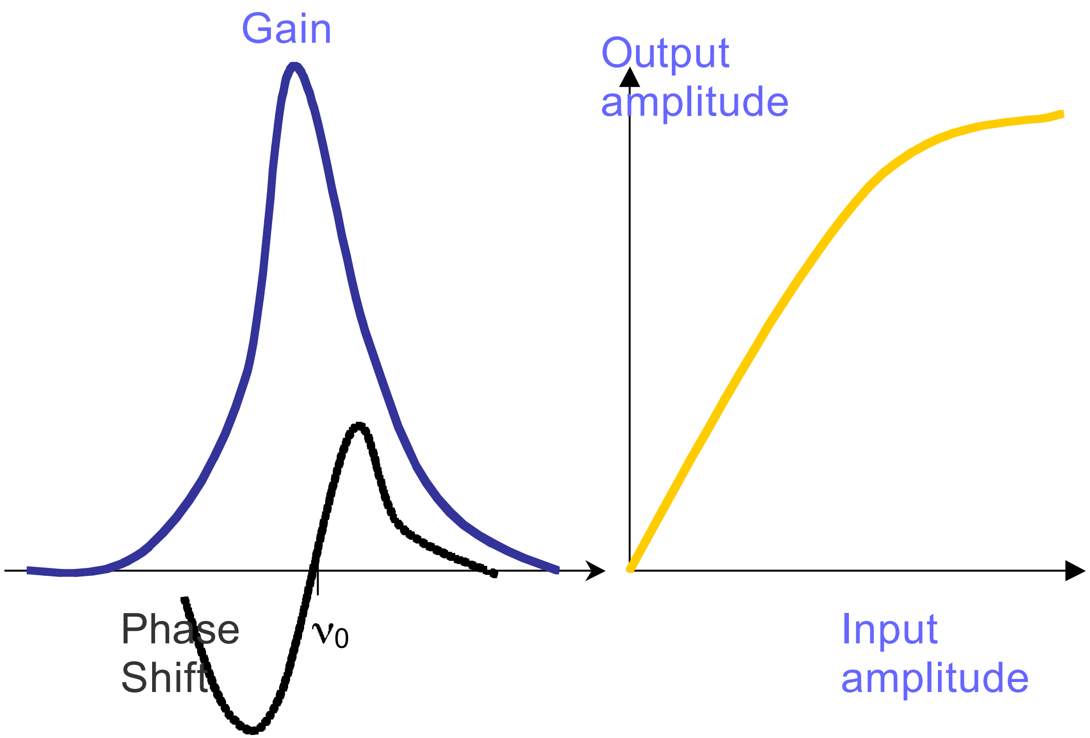
- Laser = coherent optical amplifier: increases the amplitude of an optical wave + maintains the phase
- Underlying physical process: stimulated emission of radiation (see Einstein theory in chapter 4)

Laser: Light Amplification by Stimulated Emission of Radiation



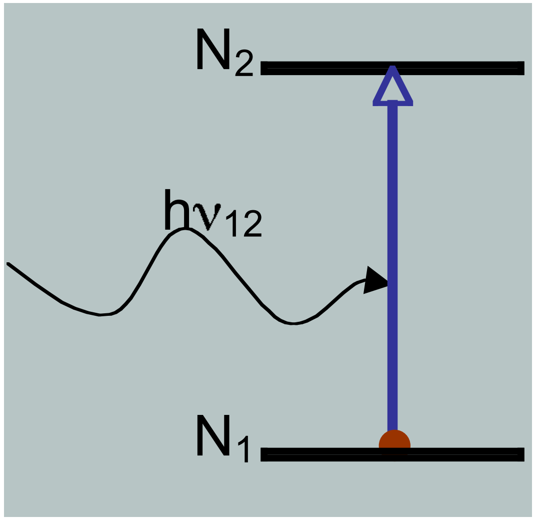
- Ideal amplifier:
  - Output scales linearly with input
  - Gain is constant in a certain bandwidth
  - Linear phase shift introduced by amplifier

Laser: Light Amplification by Stimulated Emission of Radiation

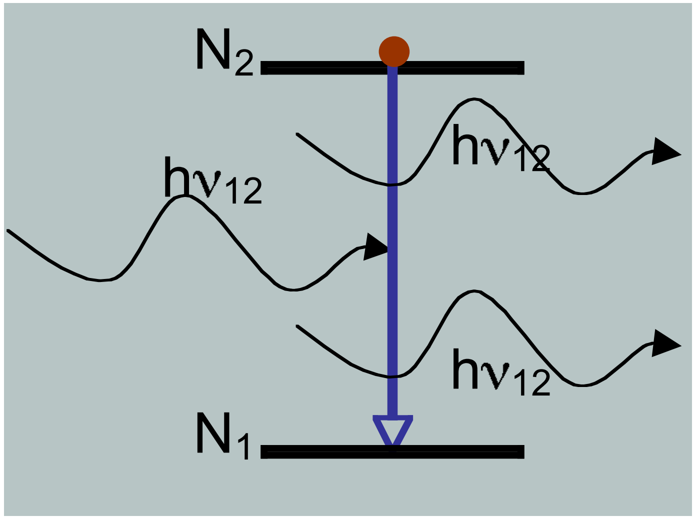


- Real amplifier:
  - Gain and phase shift (amplifier transfer function) are frequency dependent
  - For large input values, the output saturates (nonlinear behaviour)

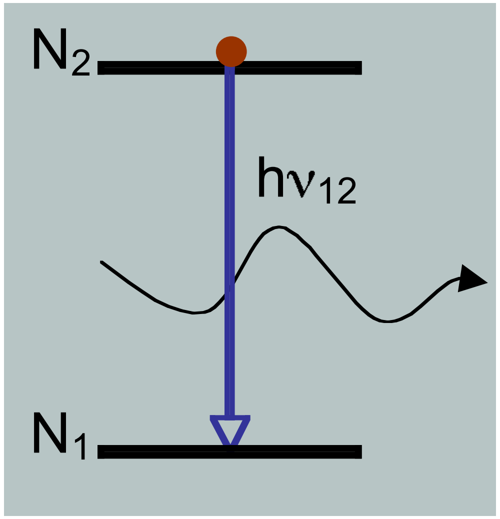
Elementary Interaction Processes



- Stimulated absorption: one photon lost
- Attenuation



- Stimulated emission: one photon gained
- Amplification
- Stimulated photon: same direction and phase



- Spontaneous emission: one photon emitted at random
- Noise

The Gain coefficient $\gamma(\nu)$

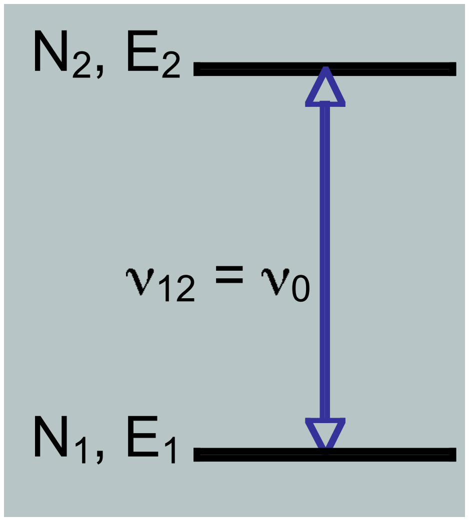
- Monochromatic plane wave travels in $+z$ direction
- Through a medium of optical impedance η with resonance between levels 1 and 2
- Intensity depends on z: $I(z)$
- Photon flux density Φ: number of photons per second per unit surface area of the medium

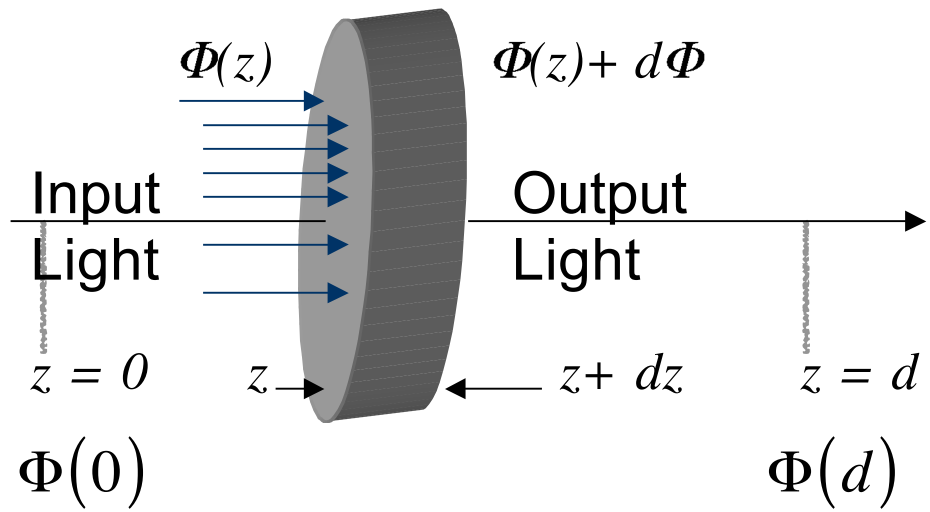
$$E(z) = Re\left[A_0 e^{i(2\pi\nu t - kz)}\right]$$

$$I(z) = \frac{|E(z)|^2}{2\eta}$$

$$\Phi(z) = \frac{I(z)}{h\nu}$$

The Gain coefficient $\gamma(\nu)$





- If the medium is amplifying:
  - Gain coefficient per unit length $\gamma(\nu)$
  - Phase shift per unit length $\varphi(\nu)$

The Gain coefficient $\gamma(\nu)$

- W is the probability per unit time that a photon is absorbed or emitted by the stimulated processes:

$$d\Phi = \left(N_2 - N_1\right) \times W \times dz$$

$$d\Phi = N \times W \times dz$$

- If $N > 0$, there exists a population inversion and the medium is amplifying
- If $N < 0$, the medium attenuates and the photon flux decreases

The Gain coefficient $\gamma(\nu)$

- W is given by Quantum Mechanics:

$$W = \Phi \times \sigma(\nu)$$

$\sigma(\nu)$ is atomic cross-section

$$d\Phi = N\Phi\sigma(\nu)dz = \gamma(\nu)\Phi dz$$

$\gamma(\nu) \equiv$ Gain coefficient $\quad \gamma(\upsilon) = N.\sigma(\nu)$

After integration : $\Phi(z) = \Phi_0 e^{\gamma(\nu)z} \Rightarrow I(z) = I_0 e^{\gamma(\nu)z}$

Overall for a length d of the amplifier :

$$\frac{\Phi(d)}{\Phi(0)} = G(\nu) = e^{\gamma(\nu)d}$$

$\varphi(\nu) = \textit{phase change per unit length}$

$\Phi(\nu) = \textit{photon flux density} = \frac{I(\nu)}{h\nu}$

The atomic cross-section $\sigma(\nu)$

- It is a fundamental atomic property characterising the "strength" of the emission or absorption processes in the E_2-E_1 resonance.

$$\sigma_{ij}(\nu) = \frac{1}{4\pi\varepsilon_0}\frac{\pi e^2}{mc} \times f_{ij} \times g(\nu)$$

$f_{ij} \equiv$ Oscillator strength of the $i \rightarrow j$

value found in Tables of data (e.g NIST web site)

$g(\nu) \equiv$ Normalized line shape function

$$\int_{-\infty}^{+\infty} g(\nu)\,d\nu = 1$$

Atomic cross-section can be related to Einstein coefficients:

$$\sigma(\nu) = \frac{c^2}{8\pi\nu^2} A_{ij} g(\nu)$$

$$\sigma(\nu) = \frac{c^2}{8\pi\nu^2}\frac{1}{\tau_{ij}} g(\nu)$$

Units: m^2; commonly: barns (10^{-28} m^2 = 10^{-22} cm^2); megabarns (10^{-22} m^2 = 10^{-18} cm^2) (Mb)

Amplifier bandwidth

- A band of frequencies will get amplified due to the lineshape function $g(\nu)$.
- Lineshape is usually a Lorentzian curve $L(\nu)$ (see chap.4). Thus, the gain curve $\gamma(\nu)$ is also a Lorentzian with same width.

$$\gamma(\nu) = N\sigma_{\mathrm{ij}}(\nu) = N\frac{c^2}{8\pi\nu^2}\frac{1}{\tau_{ij}}g(\nu)$$

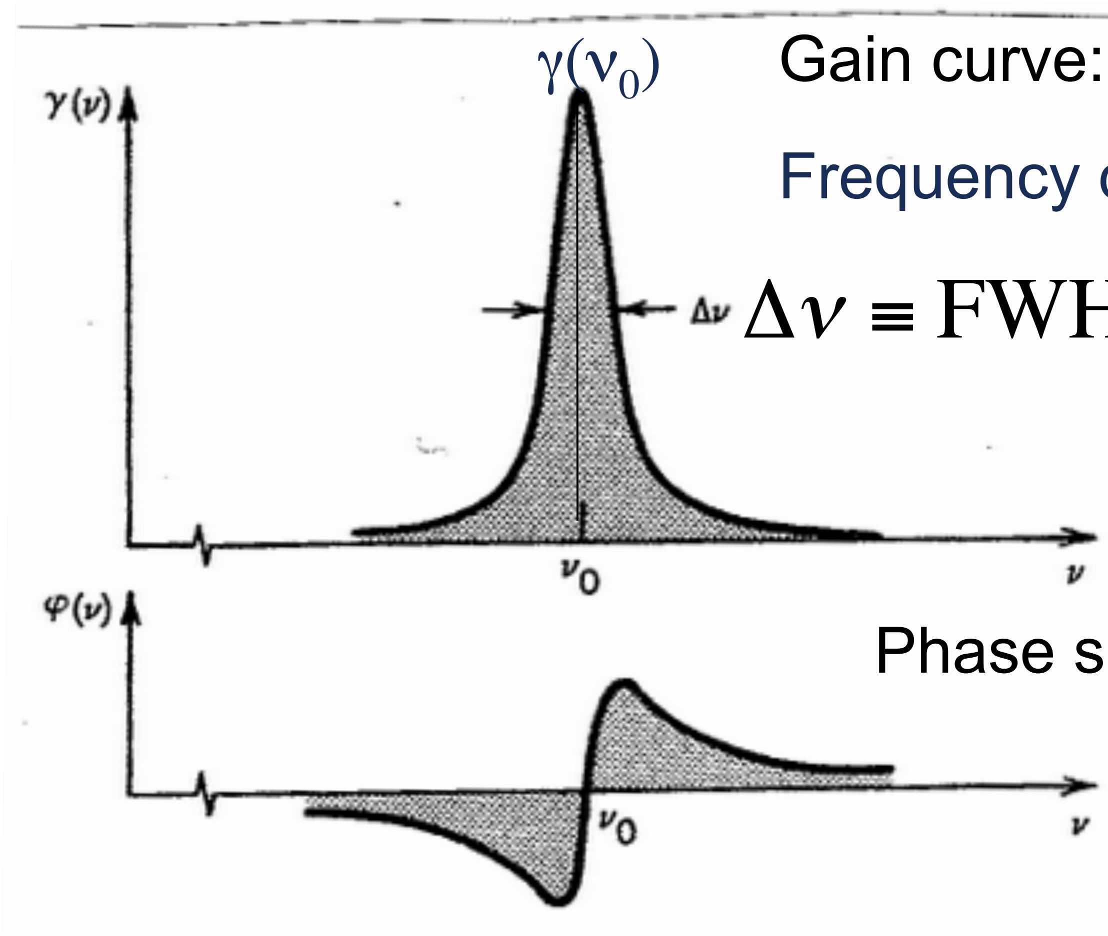
$$g(\nu) = \frac{\Delta\nu/2\pi}{(\nu-\nu_0)^2 + (\Delta\nu/2)^2}$$

$$\gamma(\nu) = \gamma(\nu_0)\frac{(\Delta\nu/2)^2}{(\nu-\nu_0)^2 + (\Delta\nu/2)^2}$$

$$\gamma(\nu_0) = N\frac{c^2}{4\pi^2\nu^2\tau_{ij}\Delta\nu}$$

$\equiv$ gain coefficient at centre of line

Amplifier bandwidth



Gain curve: $\gamma(\nu)$

Frequency dependent

$$\Delta\nu \equiv \mathrm{FWHM}$$

$\varphi(\nu) = \textit{phase change per unit length}$

$\Phi(\nu) = \textit{photon flux density} = \dfrac{I(\nu)}{h\nu}$

Phase shift: $\varphi(\nu)$

$$\varphi(\nu) = \left(\frac{\nu - \nu_0}{\Delta\nu}\right)\gamma(\nu)$$

Amplifier Phase shift

At z, the optical field is

$$E(z) = E_0 e^{\frac{\gamma(\nu)z}{2}} e^{-i\varphi(\nu)z} \quad \varphi(\nu) \equiv \text{phase shift per unit length}$$

Some Δz further, the field has increased by :

$$E(z+\Delta z) = E(z) e^{\frac{\gamma(\nu)\Delta z}{2}} e^{-i\varphi(\nu)\Delta z} \approx E(z)\left[1 + \frac{\gamma(\nu)\Delta z}{2} - i\varphi(\nu)\Delta z\right]$$

(Using Taylor series expansion)

$$\Delta E(z) = E(z+\Delta z) - E(z)$$

$$\Rightarrow \boxed{E(z)} = E(z)\left[\frac{\gamma(\nu)\Delta z}{2} - i\varphi(\nu)\Delta z\right]$$

$$\Rightarrow \text{Output} = \text{Input}\left[\text{Transfer function}\right]$$

Amplifier phase shift

$$-\varphi(\nu) = Hilbert\ Transform\ of\ \gamma(\nu)/2$$

$$H'(\nu) = \frac{1}{\pi}\int_{-\infty}^{\infty} \frac{H''(s)}{s-\nu} ds$$

$$H''(\nu) = \frac{1}{\pi}\int_{-\infty}^{\infty} \frac{H'(s)}{\nu-s} ds$$

The amplifier phase shift is determined by its gain coefficient. If it is Lorentzian:

$$\varphi(\nu) = \left(\frac{\nu-\nu_0}{\Delta\nu}\right)\gamma(\nu)$$

$\varphi(\nu)$ is zero at ν_0 and small near ν_0

Amplifier Power source

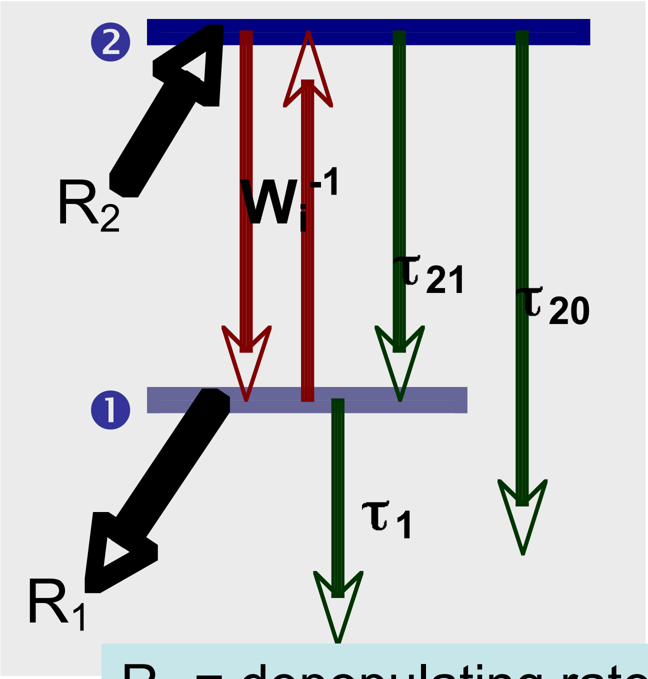
- External power source is required to add to the input signal.
- To achieve amplification: pump must provide a population inversion on the transition of interest. Thermal equilibrium conditions do not exist: situation equivalent to "negative temperature"

$$N = \left(N_2 - N_1\right) > 0$$

- Pumping may be achieved
  - Optically (flashlamp, other laser)
  - Electrically (gas discharge, electron beam, injected charge carriers)
  - Chemically (excimer laser)
- For CW (continuous wave) operation, a steady-state population inversion must be maintained.

Amplifier Power source

- Rate equations showing the balance between decay and excitation of the various levels involved must be considered:
  - Without amplifying radiation (green arrows)
  - With amplifying radiation (red



$$\frac{1}{\tau_2} = \frac{1}{\tau_{21}} + \frac{1}{\tau_{20}}$$

$$\frac{1}{\tau_2} \equiv \text{Decay rate of 2}$$

$$\frac{1}{\tau_1} \equiv \text{Decay rate of 1}$$

R_1 = depopulating rate of level 1 due to pump ($m^{-3}s^{-1}$).

R_2 = populating rate of level 2 due to pump ($m^{-3}s^{-1}$).

W_i^{-1} = time constant of stimulated processes

Amplifier Power source

- Rate equations in the absence of stimulated processes:

$$\frac{dN_2}{dt} = R_2 - \frac{N_2}{\tau_2}$$

$$\frac{dN_1}{dt} = -R_1 - \frac{N_1}{\tau_1} + \frac{N_2}{\tau_{21}}$$

- Steady-state solution

$$\frac{dN_2}{dt} = \frac{dN_1}{dt} = 0$$

$$\Rightarrow \left(N_2 - N_1\right) = N = N_0$$

$$N_0 = R_2\tau_2\left(1 - \frac{\tau_1}{\tau_{21}}\right) + R_1\tau_1$$

For large gains: large pump rates,

long τ_2,

short τ_1

Amplifier Power source

$$\frac{dN_1}{dt} = 0 => R_1 = \left(-\frac{N_1}{\tau_1} + \frac{N_2}{\tau_{21}}\right)$$

$$\frac{dN_2}{dt} = 0 => R_2 = \frac{N_2}{\tau_2}$$

$$N_2 = R_2\tau_2$$

$$R_1\tau_1 = -N_1 + \frac{N_2\tau_1}{\tau_{21}}$$

$$N_1 = -R_1\tau_1 + R_2\frac{\tau_2\tau_1}{\tau_{21}}$$

$$N_0 = N_2 - N_1 = R_2\tau_2 + R_1\tau_1 - R_2\frac{\tau_2\tau_1}{\tau_{21}}$$

$$N_0 = R_2\tau_2\left(1 - \frac{\tau_1}{\tau_{21}}\right) + R_1\tau_1$$

Amplifier Power source

- The upper level should be pumped strongly and decay slowly to retain its population.
- The lower level should decay/empty ("depump") fast.

Ideally :

$\tau_{21} << \tau_{20} \Rightarrow \tau_2 \approx \tau_{21}$ (spontaneous rate) and $\tau_{21} >> \tau_1$

Simplified result obtains for N_0 :

$N_0 \approx R_2 \tau_{21} + R_1 \tau_1$

If further $R_1 = 0$ or $\mathrm{R}_1 << \dfrac{\tau_{21}}{\tau_{21}} R_2$

$\Rightarrow N_0 \approx R_2 \tau_{21}$ or ($R_2 \tau_{21}^{sp}$) or ($R_2 t^{sp}$)

Amplifier Power source: effect of stimulated processes

- N_0 controlled by: decay rates, pump rates and stimulated processes at rate W_i

$$\frac{dN_2}{dt} = R_2 - \frac{N_2}{\tau_2} - \underline{N_2 W_i + N_1 W_i}$$

$$\frac{dN_1}{dt} = -R_1 - \frac{N_1}{\tau_1} + \frac{N_2}{\tau_{21}} + \underline{N_2 W_i - N_1 W_i}$$

- Steady-state solution:

$$\frac{dN_2}{dt} = \frac{dN_1}{dt} = 0$$

$$N = \frac{N_0}{1 + \tau_S W_i}$$

$$\tau_S = \tau_2 + \tau_1 \left(1 - \frac{\tau_2}{\tau_{21}}\right)$$

$\equiv$ Saturation time constant ($\tau_S > 0$)

$N = N_2 - N_1$ (*in the presence of amplifier radiation*)

$N_0 = N_2 - N_1$ (*in the absence of amplifier radiation*)

Amplifier Power source: effect of stimulated processes

- Steady-state population difference is smaller in presence of stimulated processes.

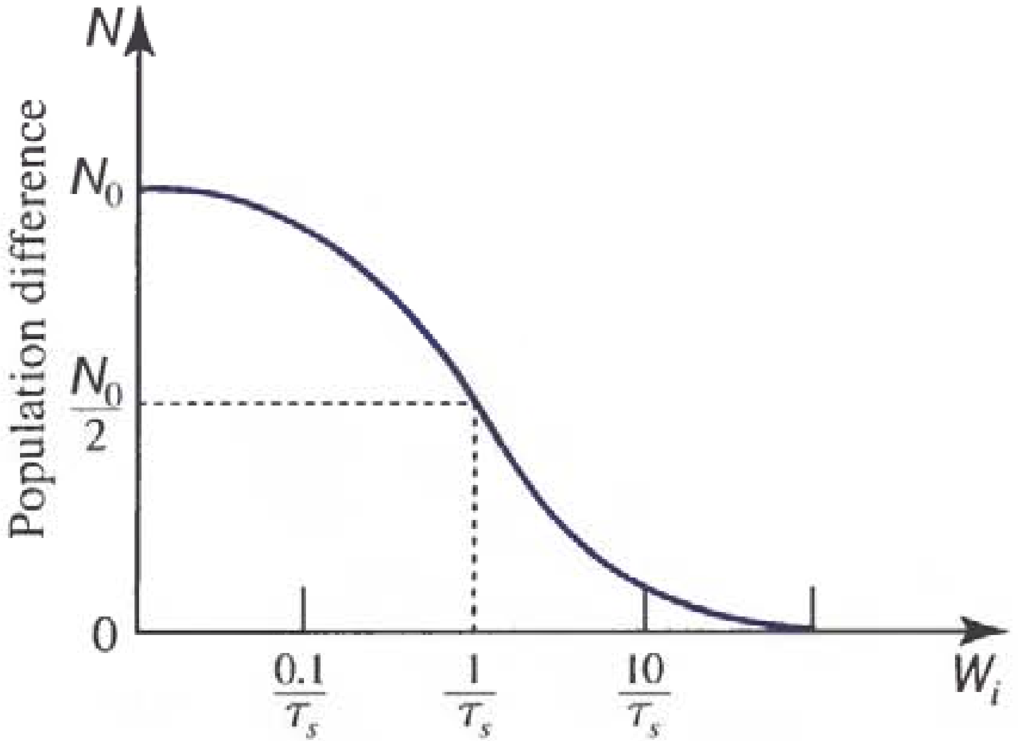
If $\tau_S W_i << 1 \Rightarrow N \approx N_0$ (small signal approximation).

- When large fluxes, stimulated processes dominate and population inversion becomes zero.

Recall $W_i = \phi \times \sigma(\nu) \Rightarrow$ if $\tau_S W_i >> 1 \Rightarrow N \approx 0$

τ_S plays the role of saturation time constant.

Amplifier Power source: effect of stimulated processes



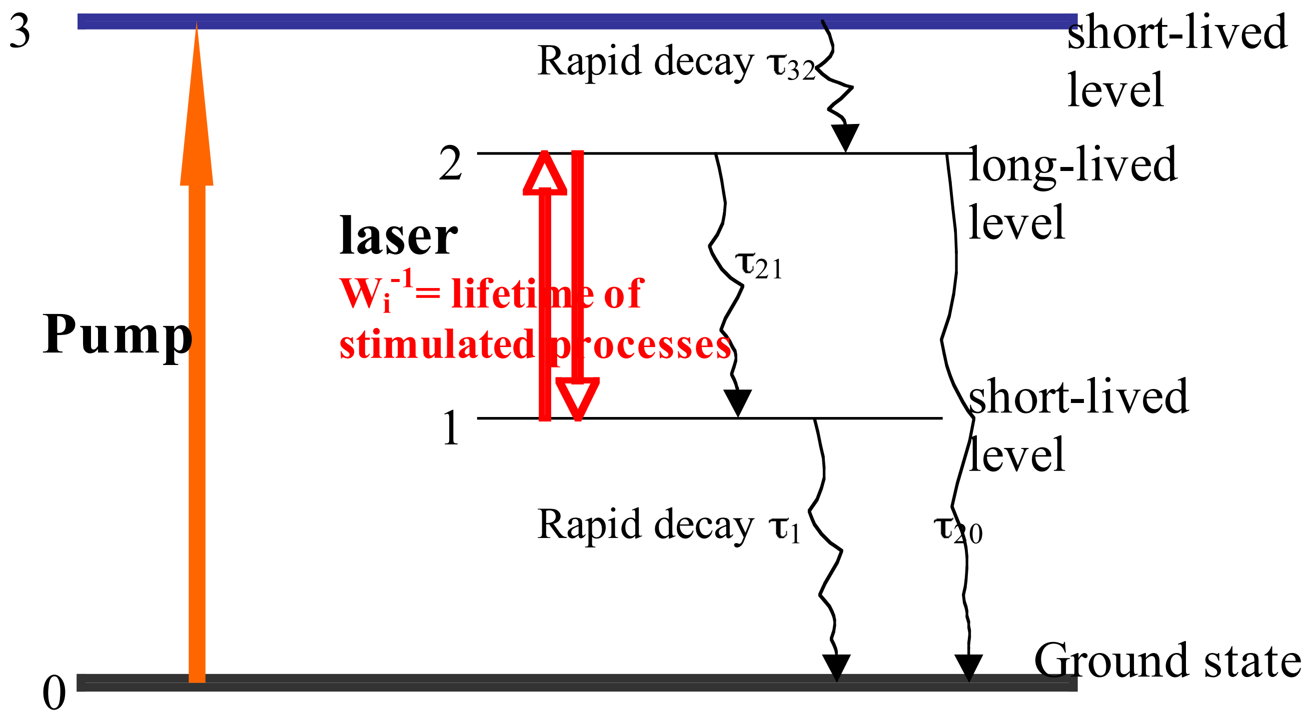
Adapted from Fundamental of Photonics, Saleh and Teich, 2nd ed. Wiley

- Depletion of steady-state population difference as the rate of stimulated processes increases.
- Time constant = time to halve the initial inversion.

Amplifier Power source: effect of stimulated processes

- Practical lasing schemes (population inversion between E_1 and E_2) involve 3 or 4 levels:
  - 4-level pumping schemes
  - 3-level pumping schemes
  - (Direct optical pumping of 2-level system does not achieve steady-state N).

Amplifier Power source: 4-level pumping scheme

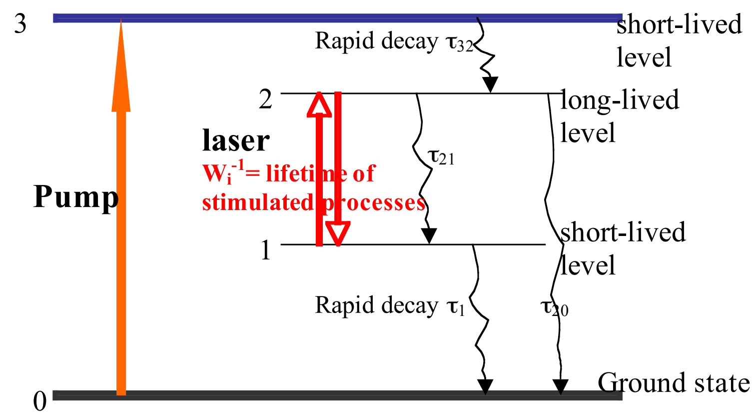


4-level pumping scheme

•In normal conditions, level 1 -the lower laser level- will be virtually unpopulated (highly desirable situation for the operation of the laser).

•There is little accumulation of population in level 3 as it decays non-radiatively to level 2 with a large transition probability.

•Level 2 is pumped via relaxation of level 3 and accumulates population.



•R is the pumping rate: as decay of level 3 is very fast, pumping of level at a rate R is equivalent to pumping of level 2 at rate R_2 = R.

•In this configuration, there is no pumping of atoms neither into nor out of level 1: R_1 = 0

4-level pumping scheme

In the absence of amplifier radiation $(W_i = \Phi = 0)$, the steady-state population difference can be calculated:

$$N_0 = R\tau_2\left(1 - \frac{\tau_1}{\tau_{21}}\right)$$

The non-radiative component of τ_{21} is negligible: $\tau_{21} \approx t_{sp}$

and $\tau_{20} >> t_{sp} >> \tau_1$.

$N_0 = Rt_{sp}$ and $\tau_S \approx t_{sp}$

$$N \approx \frac{Rt_{sp}}{\left(1 + t_{sp}W_i\right)}$$

4-level pumping scheme

The pumping rate is however explicitely dependent on the population inversion :

$N = N_2 - N_1$ (due to $N_g + N_1 + N_2 + N_3 = N_a$)

N_a = Total atomic density = constant

Pumping at rate R depopulates N_g and populates N_3 with a transition probability W : $R = \left(N_g - N_3\right)W$

R~NgW

Levels 1 and 3 are short - lived : $N_1 = N_3 = 0$

Then $N_g + N_2 = N_a \Rightarrow N_g \approx N_a - N_2 \approx N_a - N$

The pumping rate $R \approx \left(N_a - N\right)W$

4-level pumping scheme

$$N \approx \frac{t_{sp} N_a W}{1 + t_{sp} W_i + t_{sp} W}$$

Rewritting in the form of $N = \dfrac{N_0}{1 + \tau_S W_i}$

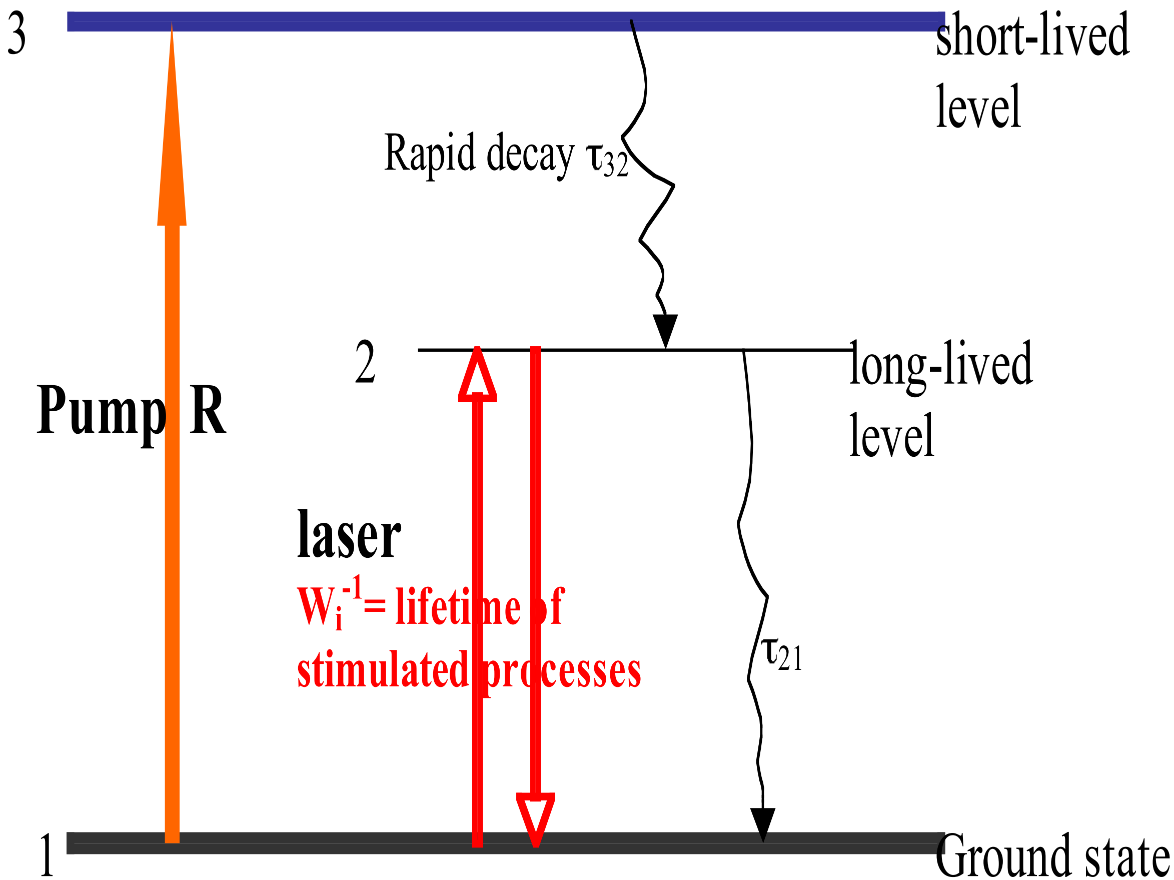
$$N_0 \approx \frac{t_{sp} N_a W}{1 + t_{sp} W} \text{ and } \tau_S \approx \frac{t_{sp}}{1 + t_{sp} W}$$

If weak pumping conditions $\left(W << 1/t_{sp} \right)$

$N_0 = t_{sp} N_a W$ and $\tau_S \approx t_{sp}$

Retrieving the previous results

3-level pumping scheme



- Rapid 3-2 decay: no build up of population in 3
- 3-1 decay is slow: pumping populates upper laser level
- Population accumulates in 2 as it's long-lived

3-level pumping scheme

Using the original set of rate equations:

$$\frac{dN_1}{dt} = \frac{dN_2}{dt} = 0 = R - \frac{N_2}{\tau_{21}} - N_2 W_i + N_1 W_i$$

$N_1 + N_2 = N_a$ (as N_3 is negligible).

Solving for N_1, N_2, obtains the saturation time constant τ_S and the population inversion N_2-N_1.

Using: $N = \dfrac{N_0}{1 + \tau_S W_i}$

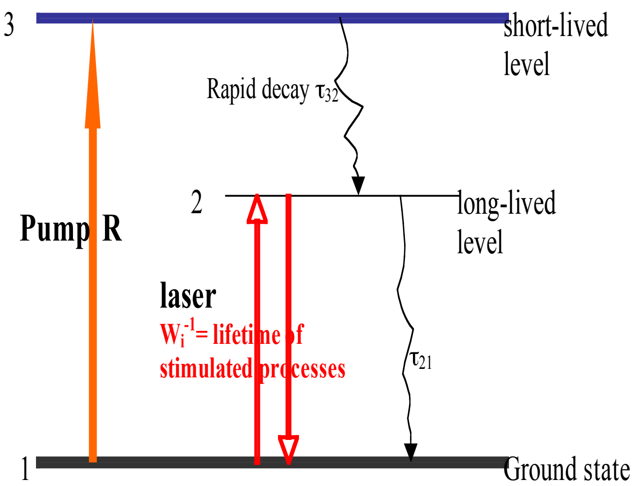
$N_0 = 2R\tau_{21} - N_a$

$\tau_S = 2\tau_{21} \approx 2t_{sp}$ (non-radiative processes are negligible).

To attain a population inversion ($N > 0$ and $N_0 > 0$) in 3-level system requires a pumping rate $R > N_a/2t_{sp}$. The corresponding pump power density is $E_3 N_a/2t_{sp}$. (E_3 : energy of level 3)

This is a large number as N_a is large (ground state). (compare with 4-level system where the lower laser level 1 is normally empty).

3-level pumping scheme



R depends on N: $R = \left(N_1 - N_3\right)W;\ N_3 \approx 0$

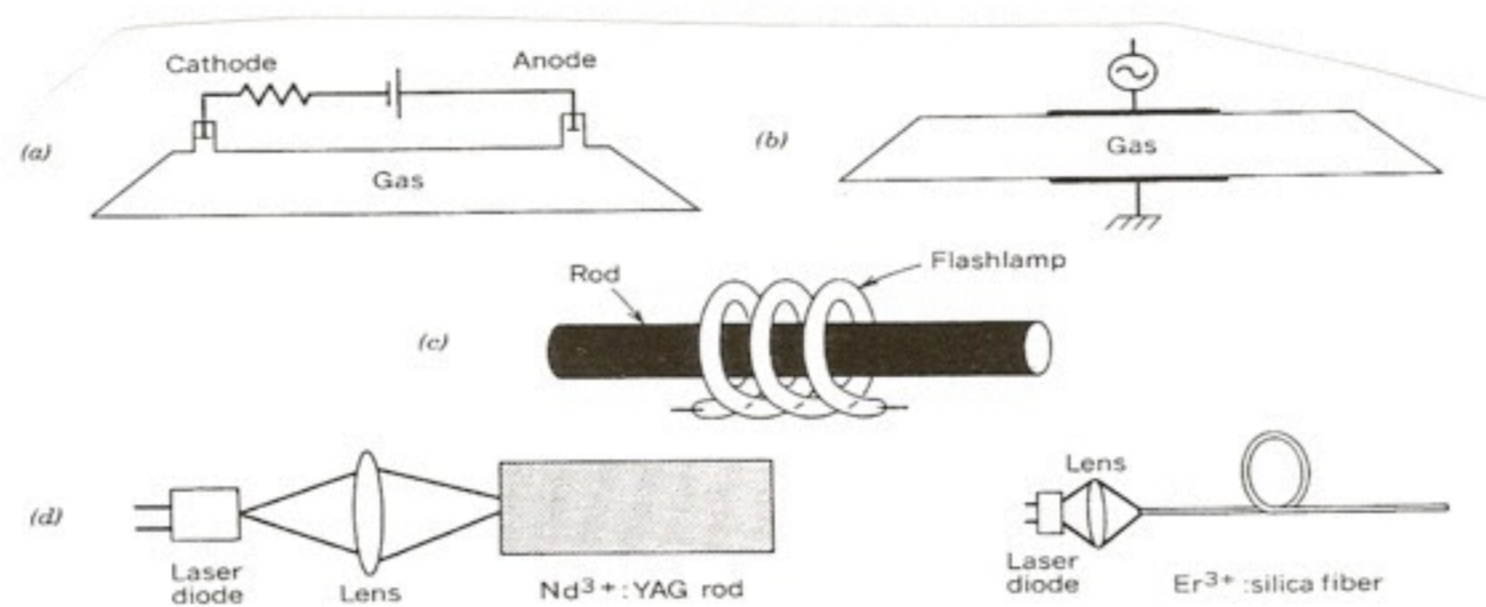
$N_1 + N_2 \approx N_a \Rightarrow N_1 = \frac{1}{2}\left(N_a - N\right),$

thus: $R = \frac{1}{2}\left(N_a - N\right)W$

Substituting into: $N = \dfrac{2R\tau_{21} - N_a}{1 + 2t_{sp}W_i}$, rearranging

$$N_0 = \frac{N_a\left(t_{sp}W - 1\right)}{1 + t_{sp}W} \quad \text{and } \tau_S \approx \frac{2t_{sp}}{1 + t_{sp}W}$$

Examples of laser amplifier

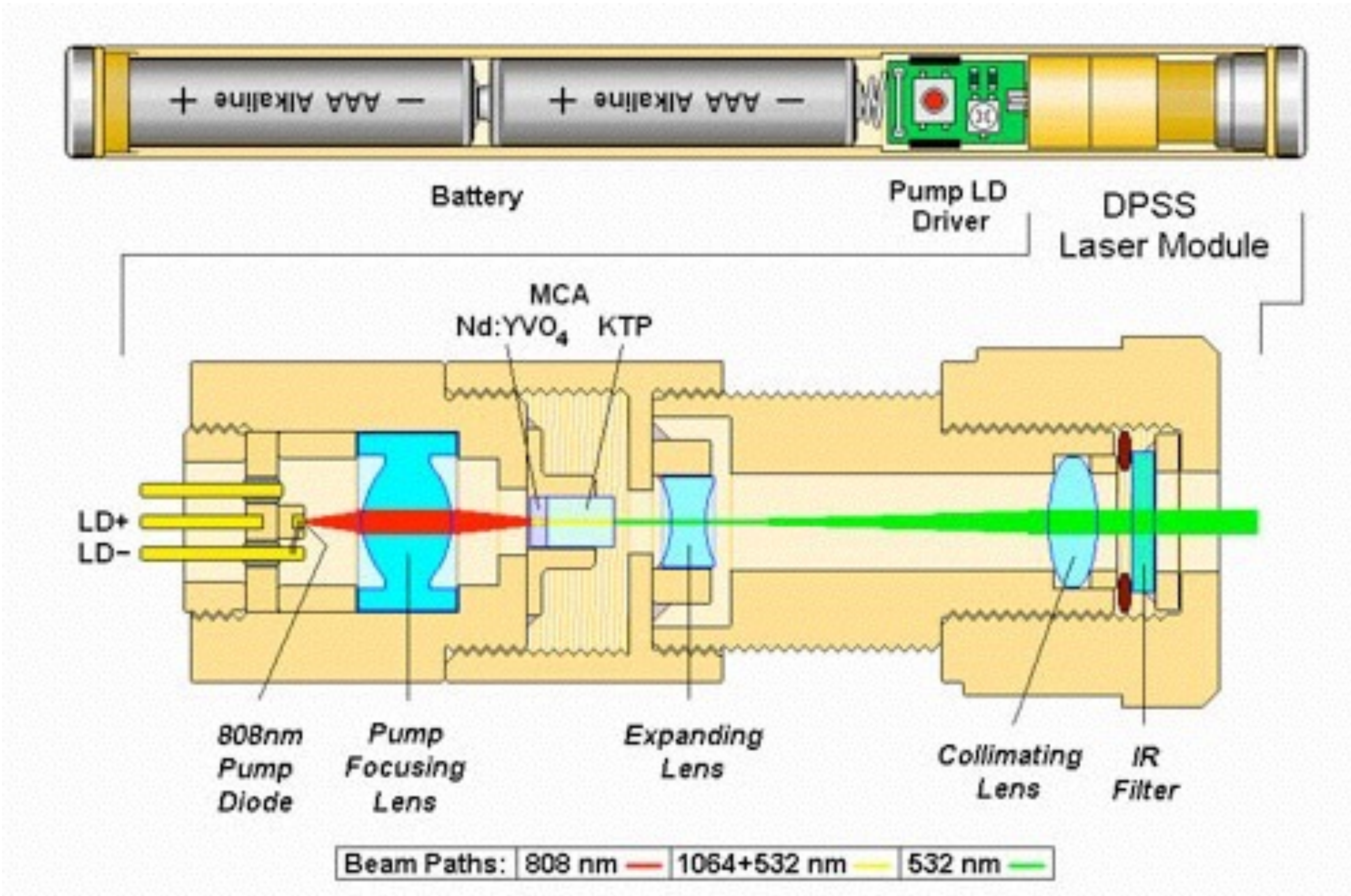


a) and b) DC or RF discharge

c) Flashlamp pump

d) Laser pumps: laser diode pumps Nd:YAG rod or fiber laser

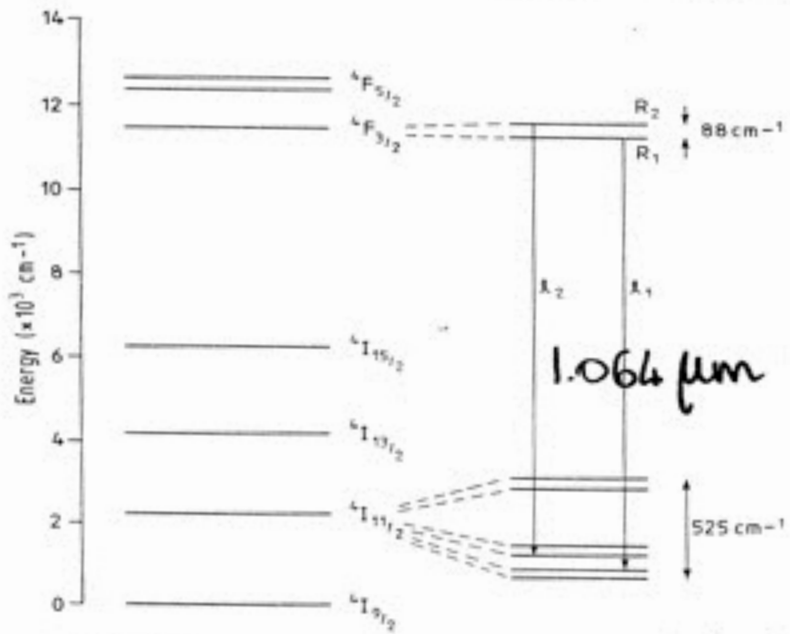
Diode pumped solid state frequency-doubled (DPSSFD) laser



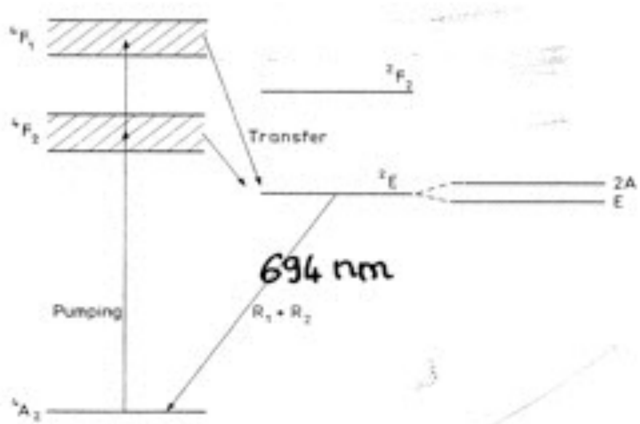
http://en.wikipedia.org/wiki/Laser_pointer#Green

Solid State laser amplifiers

Nd:YAG Laser

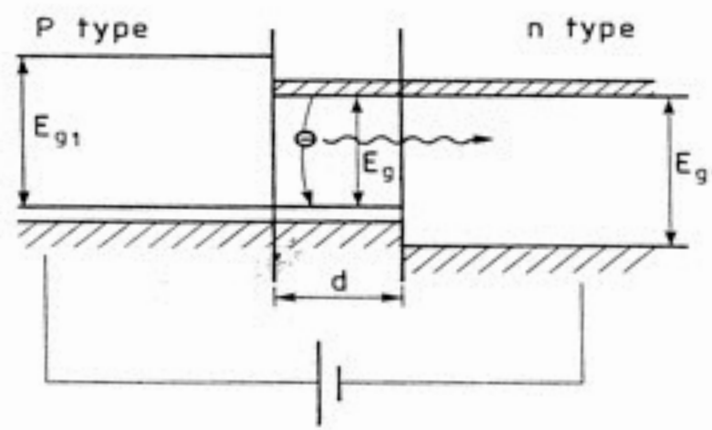


The important energy levels for Nd:YAG crystal, showing the origin of the tw components, l_1 and l_2, of the 1·064 μm line.



Energy levels for a ruby laser.

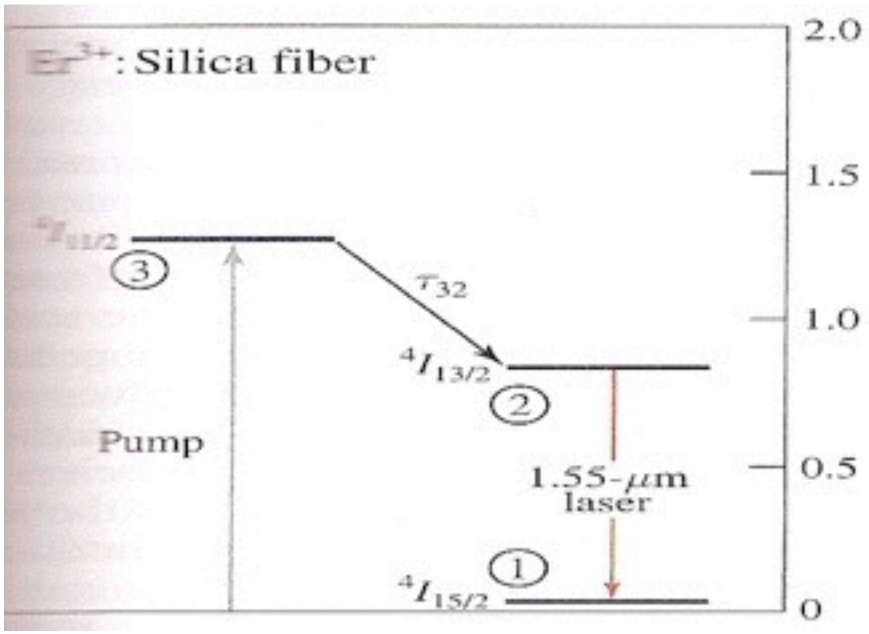
Ruby Laser



A heterojunction semiconductor laser.

Laser Diode

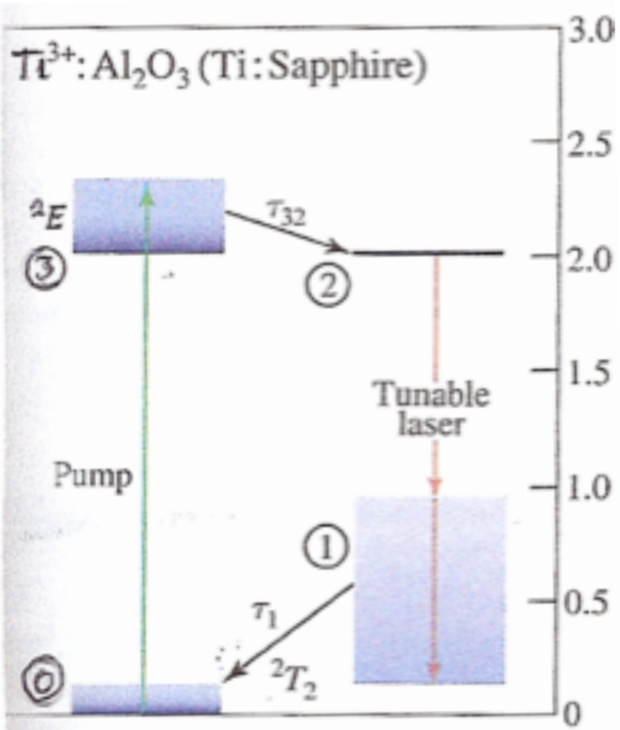
Erbium-doped silica fibre amplifier

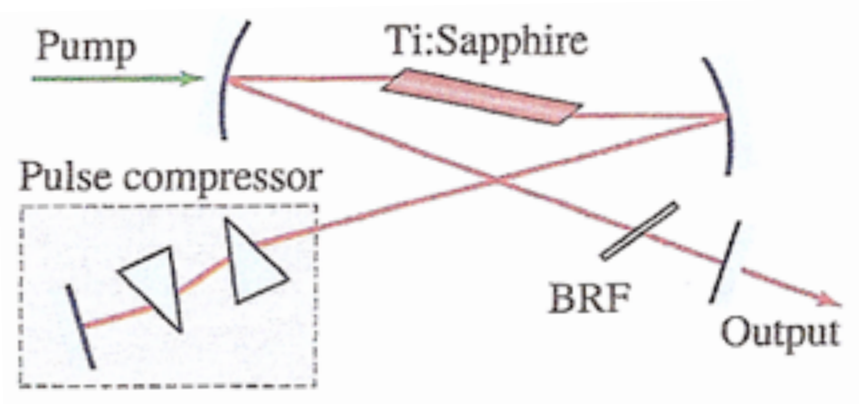


Adapted from Fundamental of Photonics, Saleh and Teich, 2nd ed. Wiley

- Longitudinal pumping of EDFA (erbium-doped fibre amplifier) using 980 nm InGaAs QW laser diode
- 3-level pumping scheme at 300 K
- EDFA has high gain and high output power: 30 dB gain achieved with 5 mW of pump through 50 m of fibre (300 ppm Er_2O_3).
- High powers can be obtained (100W).

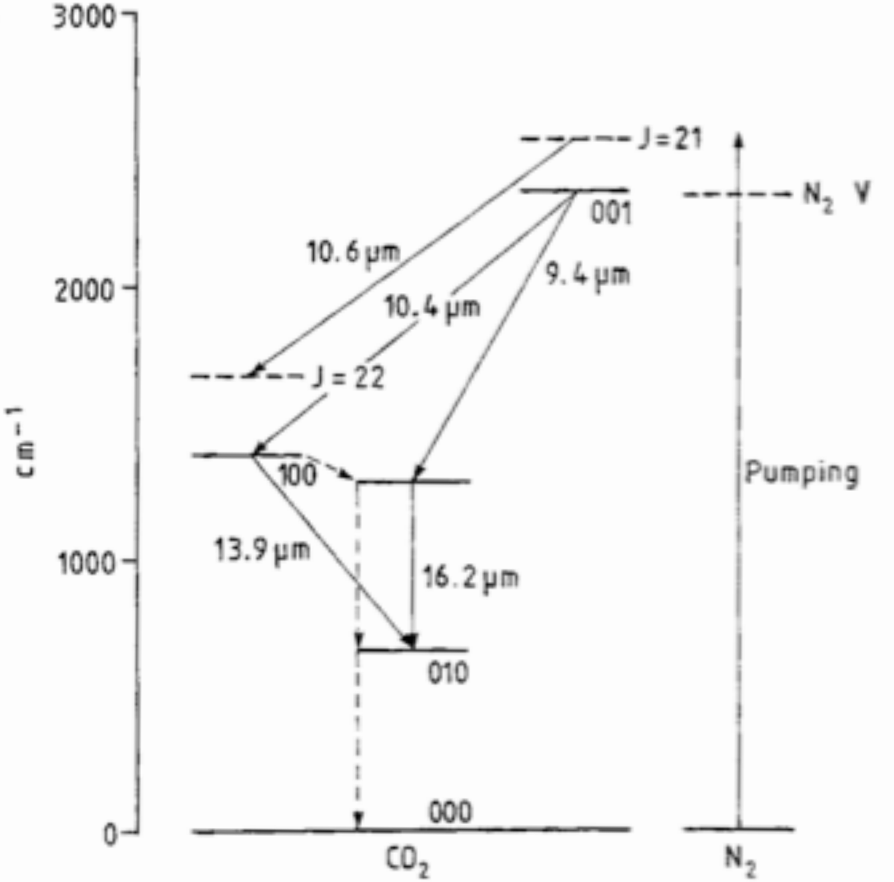
Titanium-sapphire laser





- Titanium Ti^{3+} ion dopants of sapphire matrix.
- Strong coupling of electronic levels to lattice vibrations: broadband vibronic levels (stimulated emission + phonons emission)
- Green pump is frequency-doubled Nd:YAG or green laser diode.
- 4-level pumping scheme
- Tunability is achieved by means of intracavity band pass filters (BRF: birefrengent rotatable filter at Brewster's angle): 700 nm - 1050 nm
- 5W when CW operation
- If mode-locked: 10 fs, 50 nJ, 80 MHz, 1 MW peak power

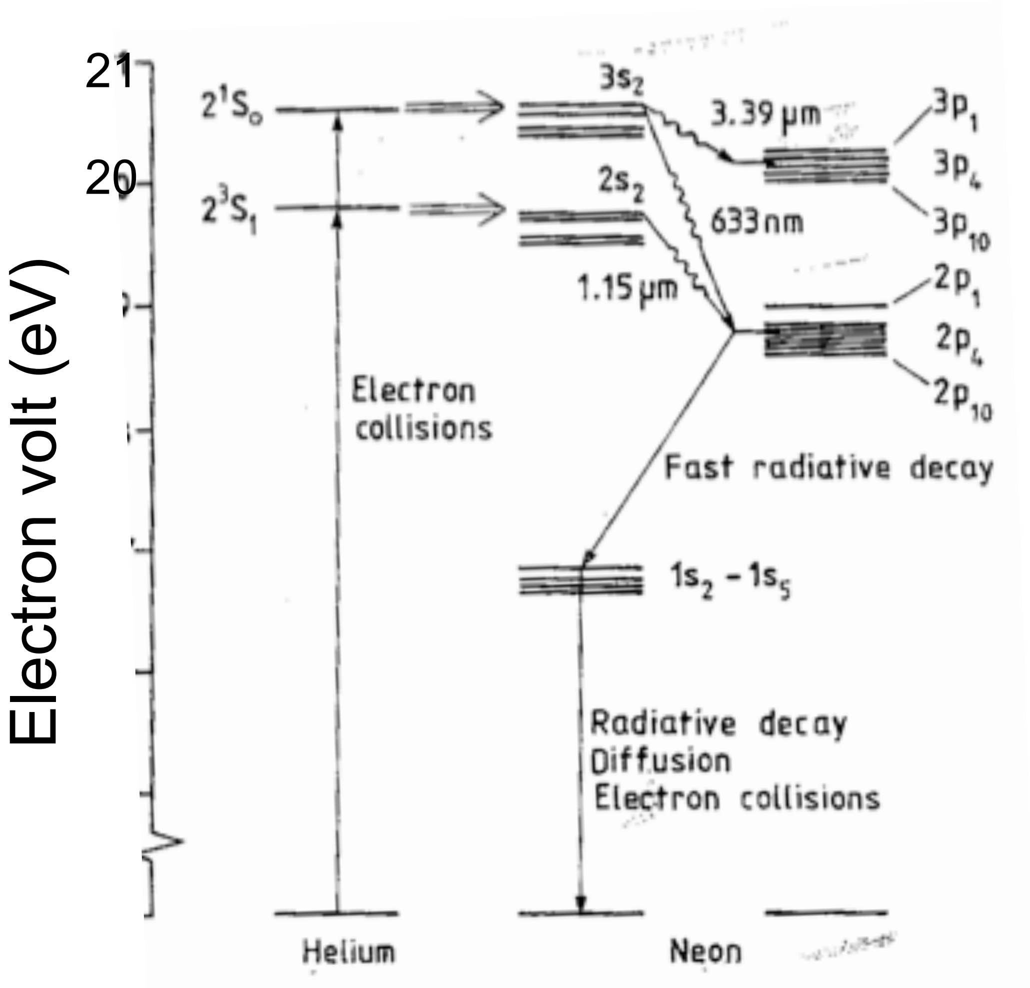
Gas laser amplifiers



Lowest vibrational states in CO_2 showing the origin of the 9·4 μm and 10·4 μm bands. Strong relaxation paths are shown dotted. The position of the energy levels for the strongest transition in the 10·6 μm *P* branch is also shown.

CO_2 Laser

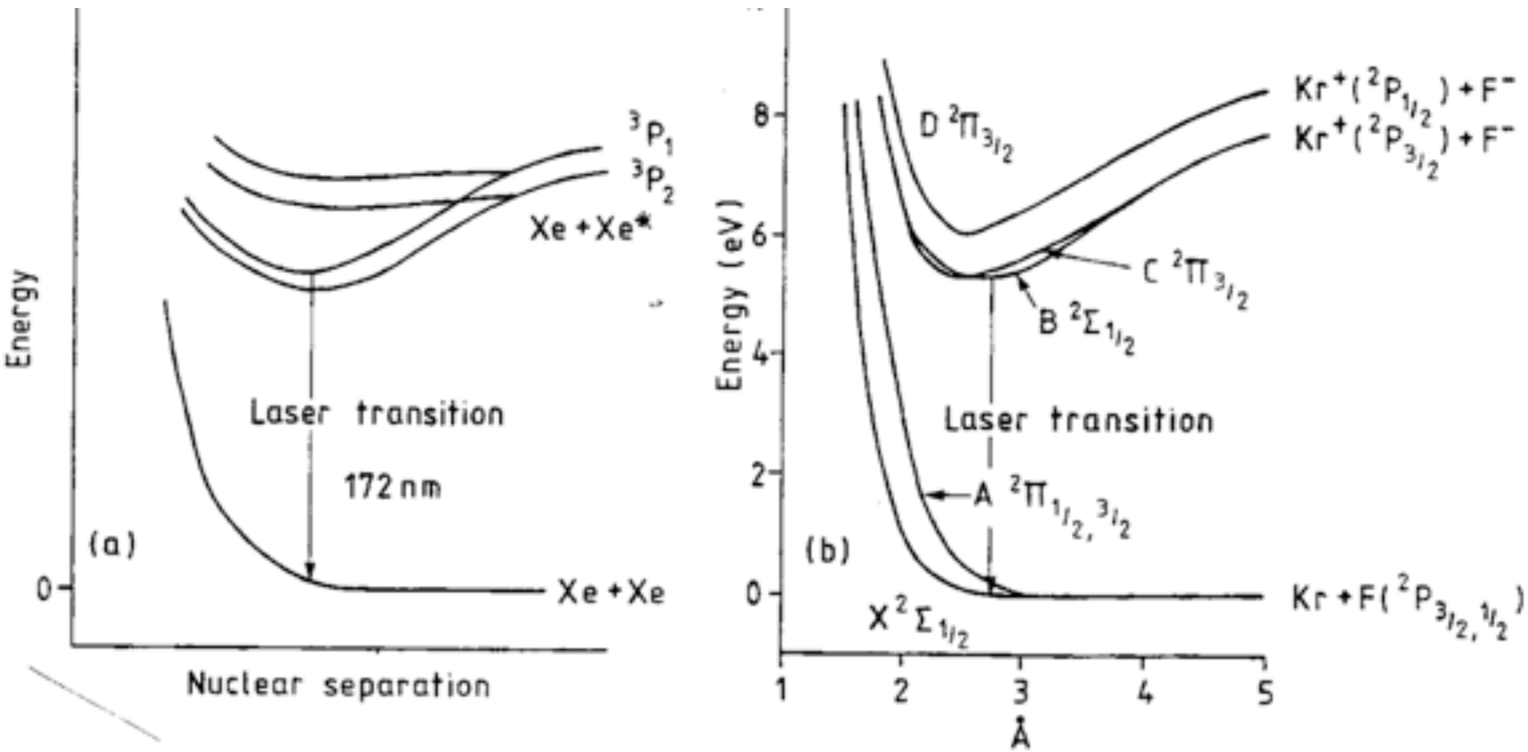
Gas laser amplifiers



Energy level diagram for a He–Ne laser.

- He-Ne (helium-neon)

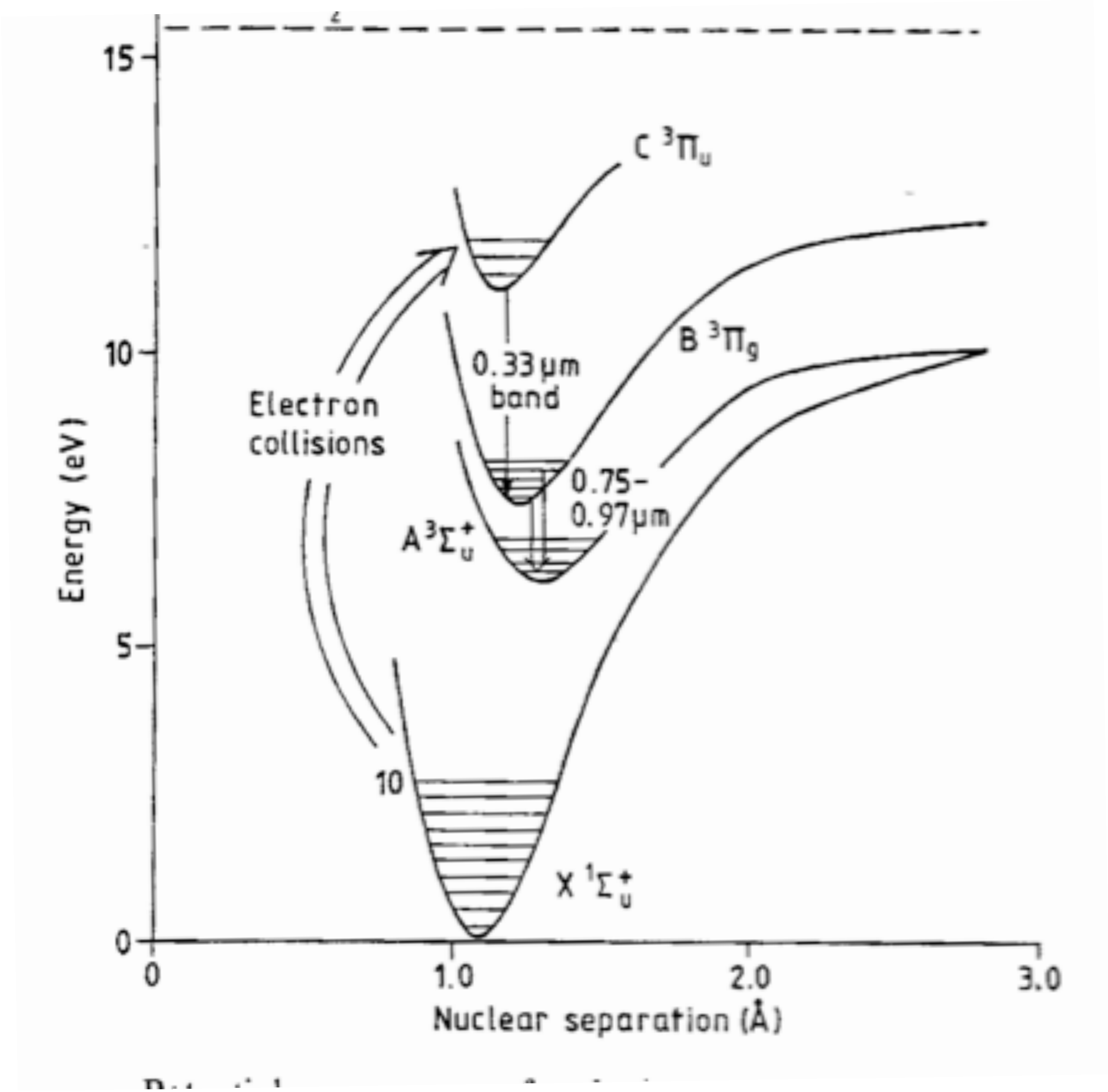
Gas laser amplifiers



Simplified potential energy curve for (a) the lowest states of Xe_2 and (b) the lowest levels in KrF.

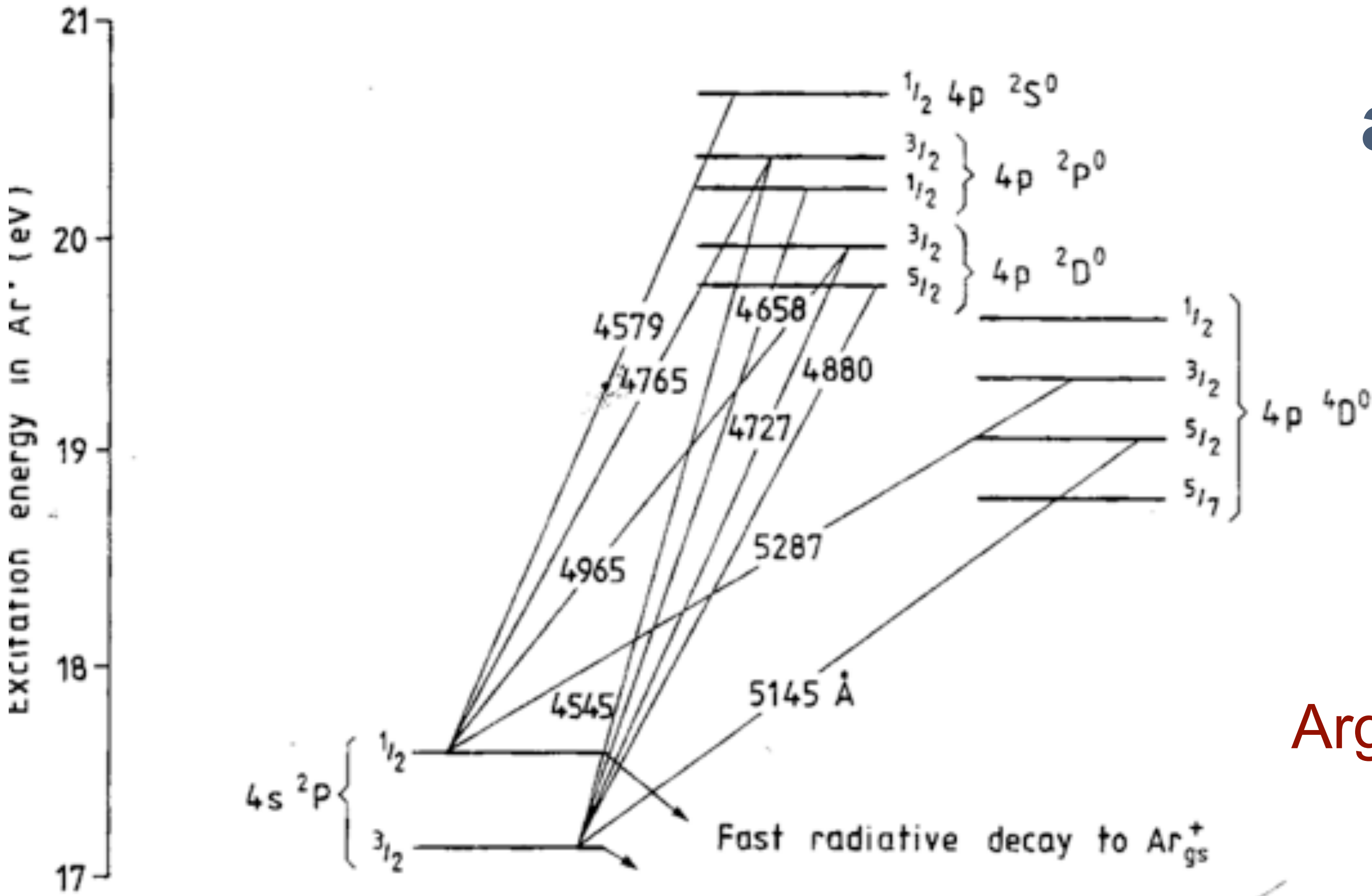
Excimer Laser

Gas laser amplifiers



Nitrogen Laser

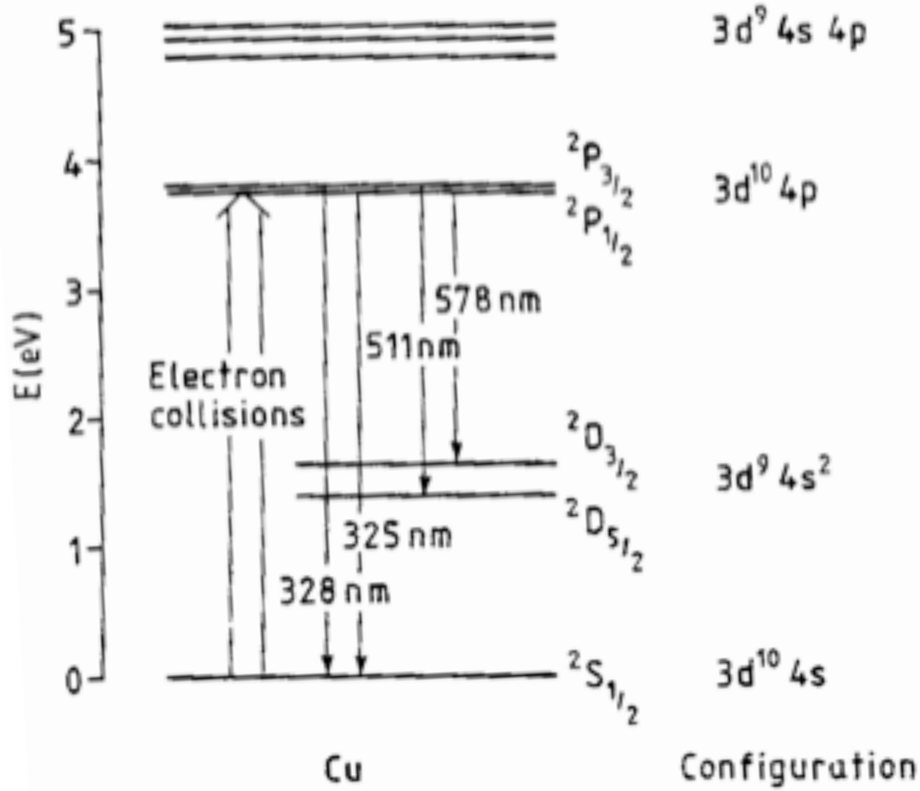
Gas laser amplifiers



$4p \rightarrow 4s$ laser transitions in Ar^+.

Argon-ion Laser

Gas laser amplifiers



Energy diagram for Cu.

Copper vapour Laser

Amplifier nonlinearity and gain saturation

- Nonlinearity and gain saturation arise due to dependence of the gain coefficient on photon flux.

By definition: $W_i = \phi\sigma(\nu)$ and $\mathrm{N} = \dfrac{\mathrm{N}_0}{1+\tau_S W_i}$

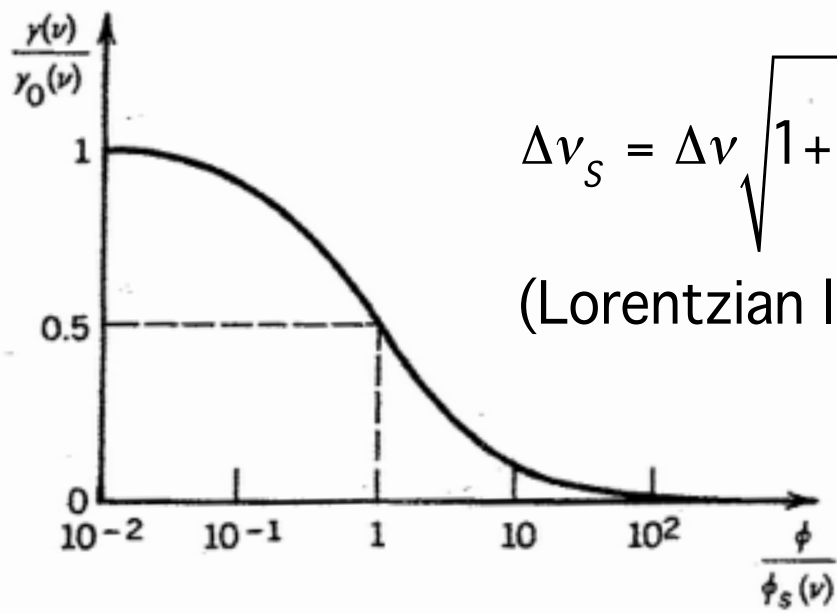
$$\Rightarrow \mathrm{N} = \frac{\mathrm{N}_0}{1+\phi/\phi_S(\nu)} \text{ with } \frac{1}{\phi_S(\nu)} = \tau_S\sigma(\nu)$$

Since $\gamma(\nu) = N \times \sigma(\nu) \Rightarrow$

$$\gamma(\nu) = \frac{\gamma_0(\nu)}{1+\phi/\phi_S(\nu)} \text{ with } \gamma_0(\nu) = N_0 \times \sigma(\nu)$$

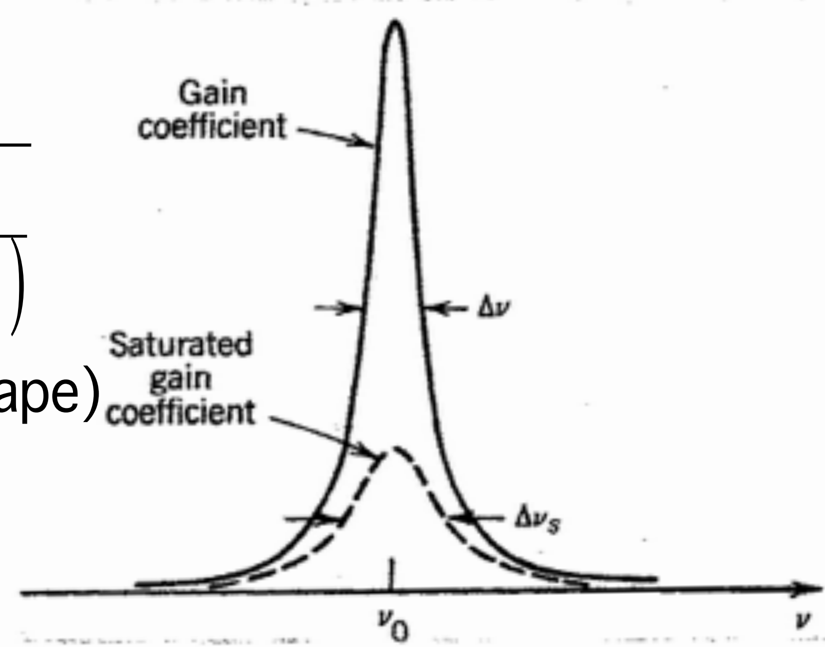
Amplifier nonlinearity and gain saturation

- Gain coefficient is a decreasing function of the photon flux density ϕ.
- Gain reduction is accompanied by increase in bandwidth:



$$\Delta\nu_S = \Delta\nu\sqrt{1+\frac{\phi}{\phi_S(\nu_0)}}$$

(Lorentzian lineshape)



Amplifier nonlinearity and gain saturation

- Behaviour of saturated gain coefficient in a homogeneously broadened laser amplifier of length d.
- Drop the frequency dependence out of notation for simplicity: $\gamma \equiv \gamma(\nu)$ and $\phi_S \equiv \phi_S(\nu)$

Basic gain: $d\phi = \gamma\phi dz$ (γ is now satured gain) $\Rightarrow$

$$\frac{d\phi}{dz} = \frac{\gamma_0 \phi}{1+\phi/\phi_S} \Rightarrow \int_0^z \gamma_0 \, dz = \int_0^z \frac{1}{\phi}\left(1+\phi/\phi_S\right) d\phi$$

$$\gamma_0 z = \left[\phi/\phi_S + Ln\phi\right]_0^z$$

$$Ln\frac{\phi(z)}{\phi_0} + \frac{\phi(z)-\phi_0}{\phi_0} = \gamma_0 z$$

Should be Phi(s) and not Phi (0)..

Amplifier nonlinearity and gain saturation

Solving for $\phi(z)$ and using the variables:

Normalised input photon-flux density: $X = \dfrac{\phi(0)}{\phi_S}$

Normalised output photon-flux density: $Y = \dfrac{\phi(d)}{\phi_S}$

Overall gain: $G = \dfrac{Y}{X} = \dfrac{\phi(d)}{\phi(0)}$

$$[LnY + Y] = [LnX + X] + \gamma_0 d$$

Two limiting cases:

(1) X and $Y << 1$: Photon flux densities small (below saturation)

(2) $X >> 1$ (Large initial photon flux)

Amplifier nonlinearity and gain saturation

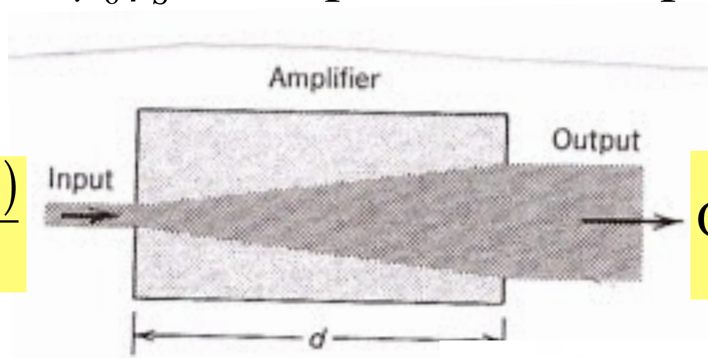
(1) $LnY \approx LnX + \gamma_0 d \Rightarrow G = e^{\gamma_0 d}$

Small signal case: gain is independent of photon flux

(2) $Y \approx X + \gamma_0 d \Rightarrow \phi(d) \approx \phi(0) + \gamma_0 \phi_S d \approx \phi(0) + \dfrac{N_0 d}{\tau_S}$

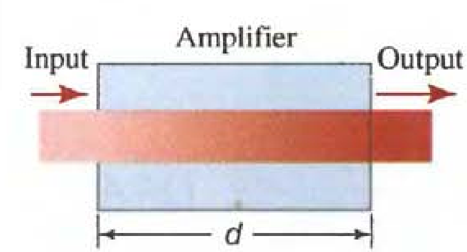
Heavily saturated conditions: initial flux augmented by constant flux $\gamma_0 \phi_S d$ independent of amplifier input.

Input $X = \dfrac{\phi(0)}{\phi_S}$

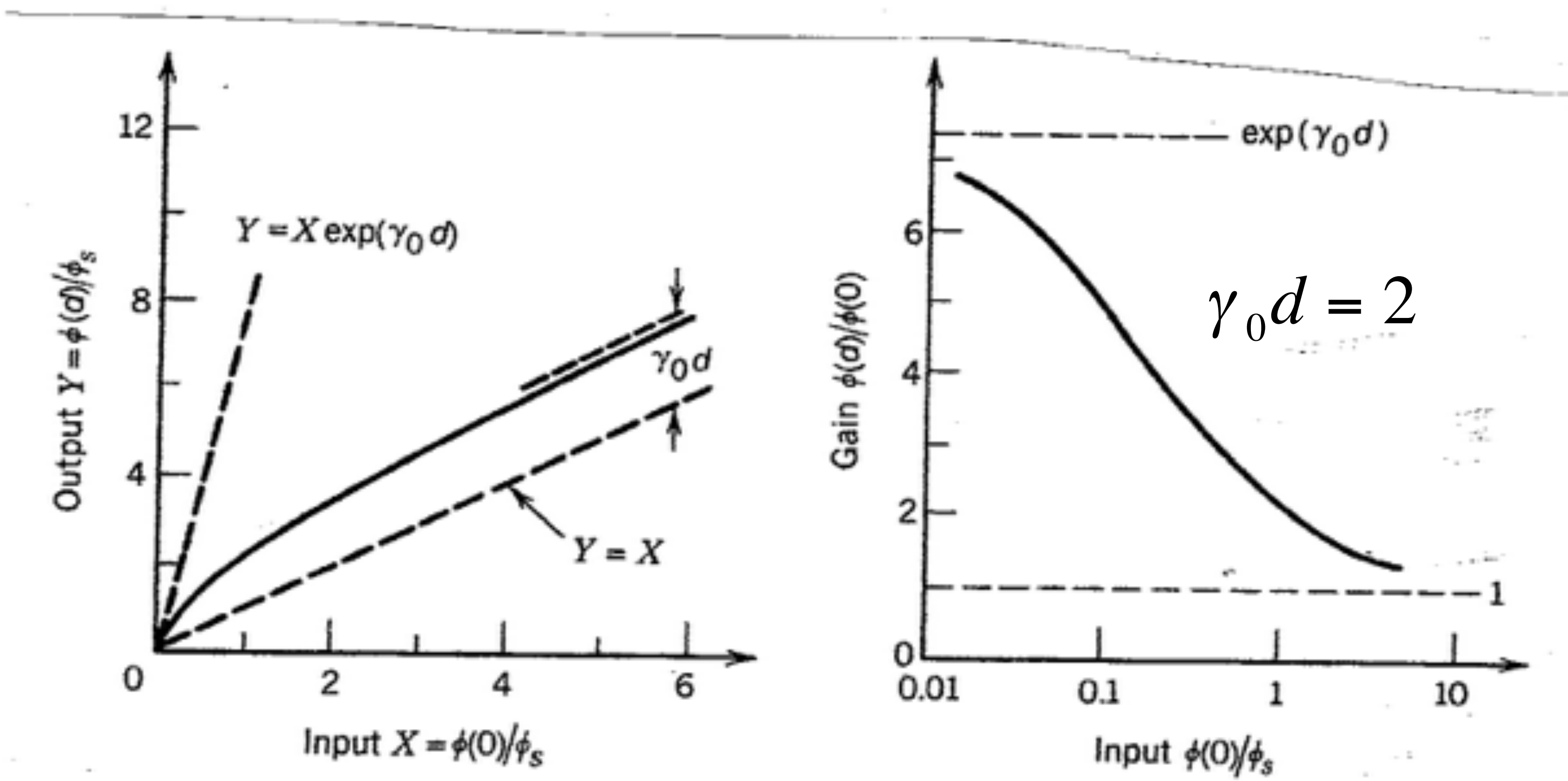


Output: $Y = \dfrac{\phi(d)}{\phi_S}$

Overall gain: $G = \dfrac{Y}{X} = \dfrac{\phi(d)}{\phi(0)}$



Amplifier nonlinearity and gain saturation



Saturable absorbers

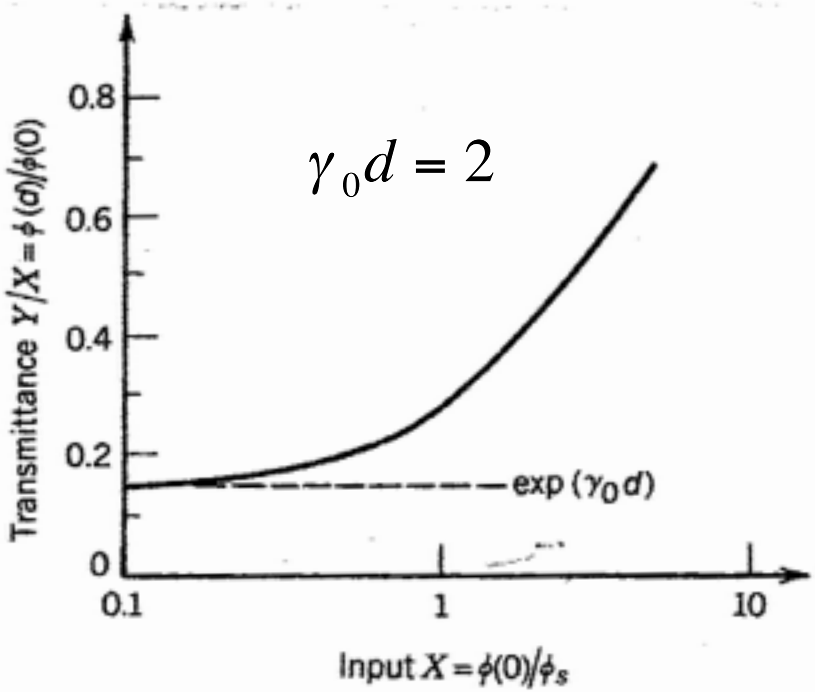
- If the gain coefficient is <0, the medium attenuates, there is no population inversion ($N_0<0$).
- The attenuation (absorption) coefficient is:

$$\alpha(\nu) = -\gamma(\nu)$$

- It also suffers from saturation:

$$\boxed{\alpha(\nu) = \frac{\alpha_0(\nu)}{1+\phi/\phi_S(\nu)}}$$

Saturable absorbers



Saturable absorbers are widely used to pulse lasers (passive Q-switching or mode-locking).

- Same flux equation with $\gamma_0 < 0$

$$\left[LnY + Y\right] = \left[LnX + X\right] + \gamma_0 d$$

- Transmittance increases for increasing input photon-flux density.
- The medium transmits more due to the effect of stimulated emissions