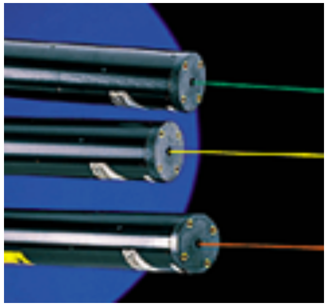
Quantum Electronics
Laser Physics

Chapter 3

The Optical Resonator



3.1 The Plane Mirror Resonator

3.2 The Spherical Mirror Resonator

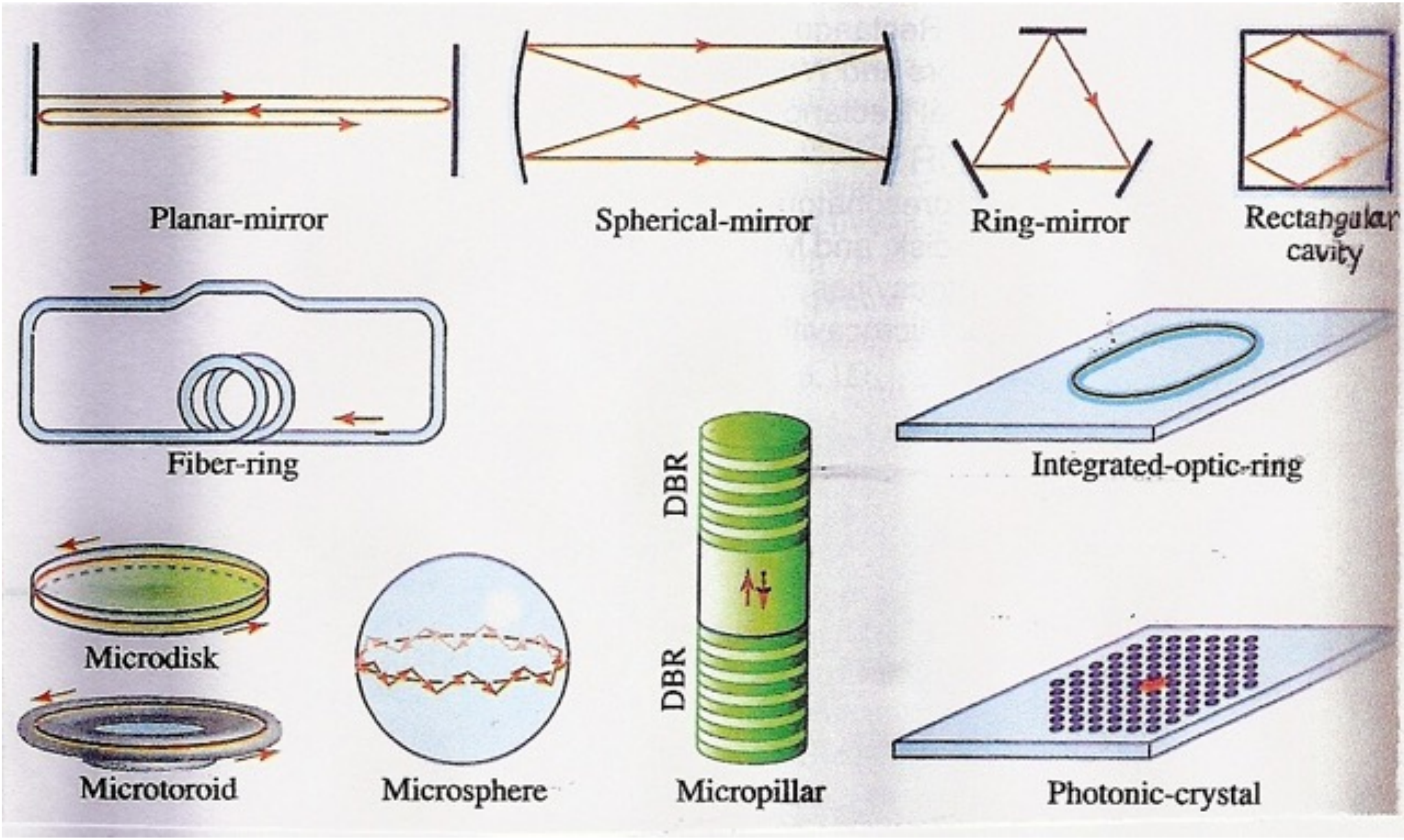
3.3 Gaussian modes and resonance frequencies

3.4 The Unstable Resonator

Introduction

- The optical resonator (OR) is the optical counterpart of an electronic resonant circuit: it confines and stores light at certain frequencies.
- Most important application: OR as a container within which laser light is generated.
- LASER=OR containing a light amplifying medium
- OR determines frequency and spatial distribution of the laser beam

Introduction



From Fundamentals of Photonics, Saleh and Teich, Wiley, chap 10, p.366

Introduction

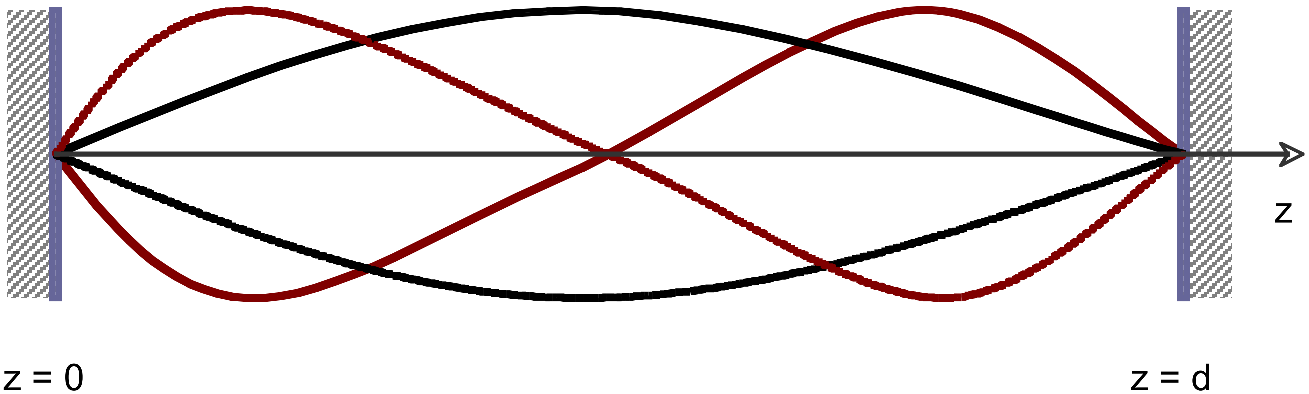
- Mirror resonators: 2 or 3 mirrors, 2D or 3D cavities
- Dielectric Resonators: use TIR instead of mirrors:
  - Fiber rings and integrated optic rings
  - Microdisks, microspheres, etc (Whispering Gallery modes)
  - Micropillars
  - Photonic Crystals
- Currently: Nanolasers with quantum confinement of carriers (e.g electrons) or photons

Introduction

- Two key parameters:
  - Modal volume V: volume occupied by confined optical mode
  - Quality factor Q: proportional to storage time in units of optical period
- V and Q represent the degrees of spatial and temporal confinements, respectively
- Large Q means low-loss resonator

3.1- Plane Mirror Resonator

- Fabry-Perot interferometer: pair of plane mirrors separated by distance d.



Monochromatic plane: $u(\vec{r}) = \mathrm{Re}\left[U(\vec{r})e^{2\pi i\nu t}\right]$

Satisfies Helmholtz equation:

$$\nabla^2 U + k^2 U = 0 \text{ with } k = 2\pi\frac{\nu}{c}$$

3.1- Plane Mirror Resonator

- Standing wave solution is obtained for the boundary conditions:

$$U(\vec{r}) = 0 \text{ at } z = 0 \text{ and } U(\vec{r}) = 0 \text{ at } z = d$$

$$U(\vec{r}) = A\, sin\, kz \text{ with } kd = q\pi \Rightarrow k = q\frac{\pi}{d},\ q = 1,2,3,\ldots$$

- q is the mode number, mode frequencies are:

$$\nu_q = q\frac{c}{2d}$$

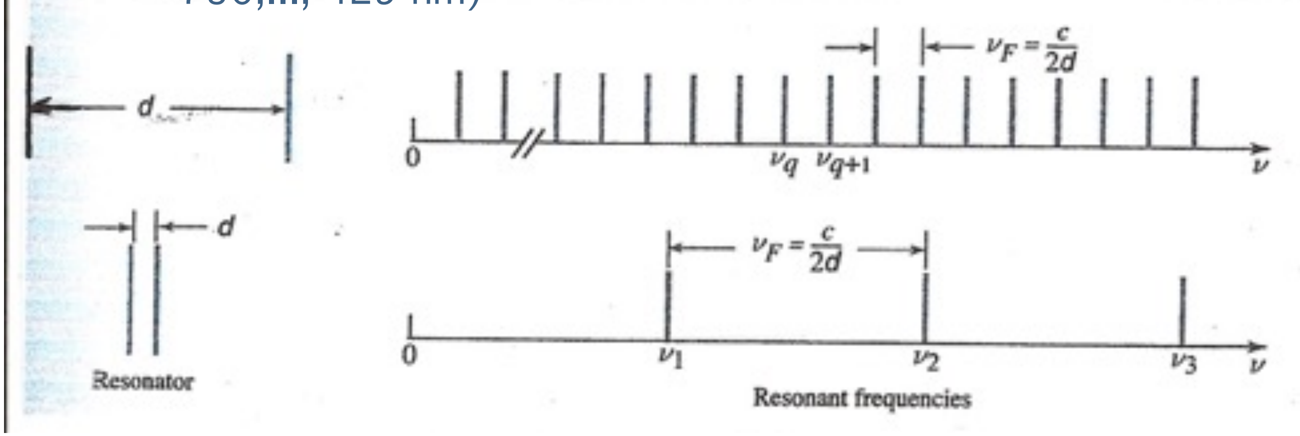
- Arbitrary wave = superposition of modes

$$U(r) = \sum_q A_q\, sin\, k_q z$$

- Constant frequency difference between adjacent modes (free spectral range): $\nu_F = \frac{c}{2d}$

3.1- Plane Mirror Resonator

- The resonance wavelengths in the optical medium are: $\lambda_q = \dfrac{c}{\nu_q} \Rightarrow 2d = q\lambda_q$
- Examples:
  - d = 30 cm, n =1 (air), free spectral range = 500 MHz
  - d = 3 microns, n = 1(air), 50 THz (7 modes in visible range: q = 8,...,14, λ_q = 750,..., 429 nm)



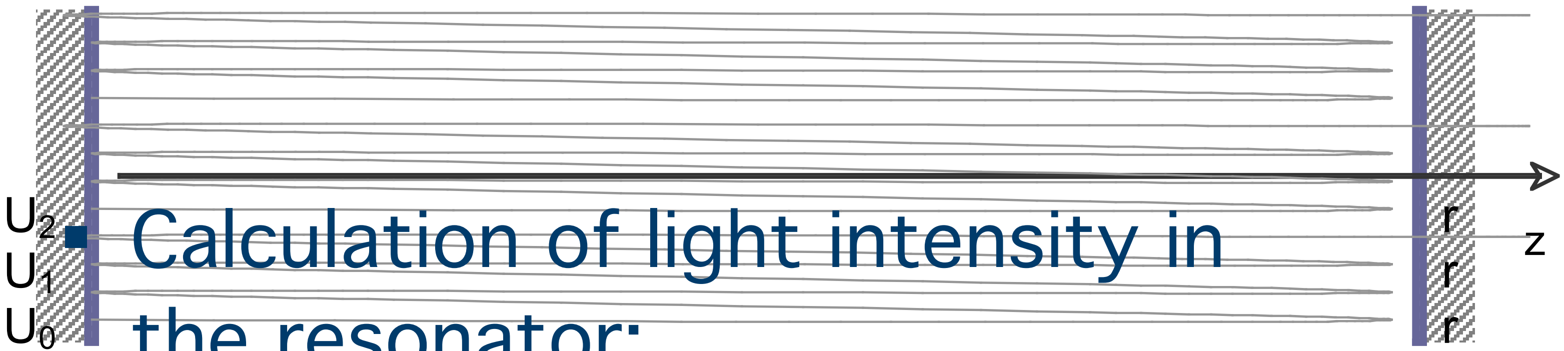
  - Free spectral range can be adjusted by placing resonators in series

3.1- Plane Mirror Resonator

- Calculation of light intensity in the resonator:
  - Summation of multiply reflected amplitudes
  - Phase shift after one round trip of propagation (2d) is $\varphi = \frac{2\pi}{\lambda} 2d = 2kd$
  - Wave reproduces itself after a round trip, thus: $\frac{2\pi}{\lambda} 2d = 2kd = 2q\pi,\ q = 1,\ 2,3\ldots$

$$U(r,t) = U_0 e^{i(k.r - 2\pi\nu t)}, k.r = \frac{2\pi}{\lambda} 2d \; for \; r = 2d$$

3.1- Plane Mirror Resonator



- Calculation of light intensity in the resonator:
  - Summation of multiply reflected amplitudes for perfect “non lossy” resonator:

$$U_1 = U_0 e^{-i\frac{2\pi}{\lambda}2d}, U_2 = U_1 e^{-i\frac{2\pi}{\lambda}2d} = U_0 e^{-i\frac{2\pi}{\lambda}4d}, etc...$$

$$\sum_{j=0}^{N} U_0\, e^{j(i2^n kd)}$$

Total amplitude:

$$U = U_0 + U_1 + U_2 + \ldots$$

$$U_{TOT} = \sum_{j=0}^{N} U_0\, e^{j\left(i\frac{2\pi}{\lambda}2^n d\right)}$$

$$\sum_{q=0}^{N} U_0\, e^{j(i2^n q\pi)}$$

3.1-Plane Mirror Resonator

- If resonator has losses, amplitude reduction upon reflection is taken into account (r reflection coefficient):

Total amplitude: $U = U_0 + rU_1 + r^2U_2 + \ldots$

$r =$ complex reflection coefficient (overall amplitude attenuation)

Intensity : $I = |U|^2$

$$U_{TOT} = \sum_{j=0}^{N} U_0 \, r^n e^{j\left(i\frac{2\pi}{\lambda}2^n d\right)}$$

$$I = \frac{I_0}{1 + \left(\frac{2F}{\pi}\right)^2 \sin^2\left(\frac{\varphi}{2}\right)} \quad = \text{ transmitted intensity}$$

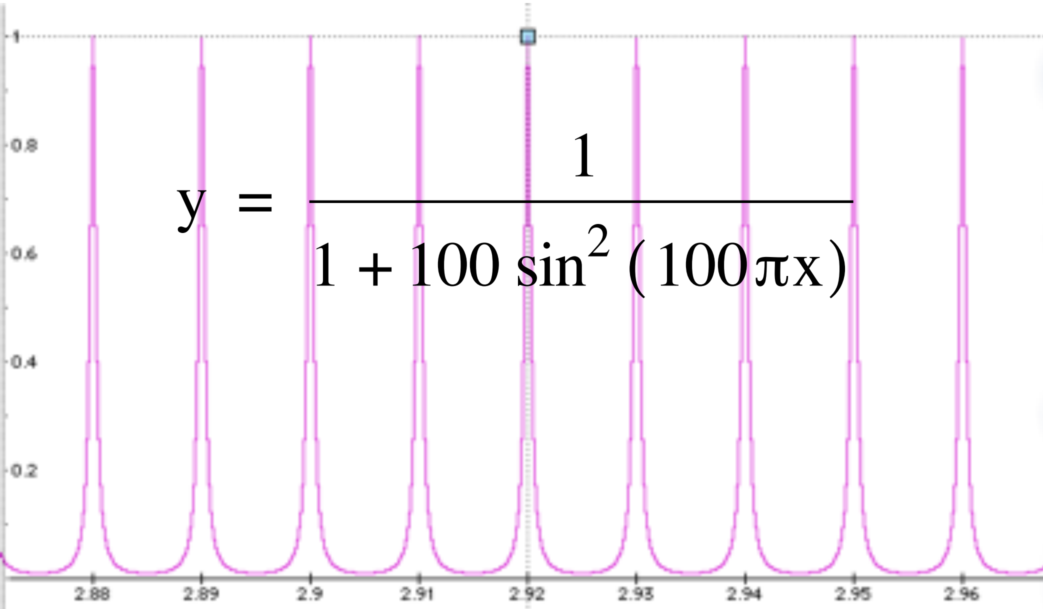
$$\frac{\phi}{2} = kd = \frac{2\pi}{\lambda} d = \frac{2\pi \nu d}{c}$$

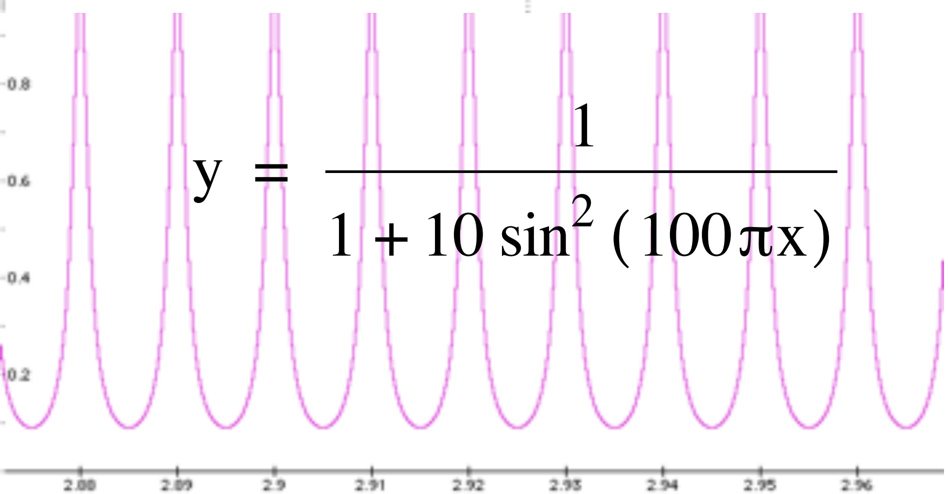
$I_0 = |U_0|^2$ = incident intensity

$R = |r|^2$, reflectivity of lossy mirror (or overall losses over round trip)

$$F = \frac{\pi\sqrt{R}}{1-R}, \text{ Finesse of resonator} = \frac{\text{Intermode spacing}}{\text{Width of a mode}} = \frac{\nu_F}{\delta\nu}$$

3.1- Plane Mirror Resonator





Spectral Response of the lossy resonator:

$$\frac{I}{I_0} = \frac{1}{1 + \left(\frac{2F}{\pi}\right)^2 \sin^2\left(\pi \frac{\nu}{\nu_F}\right)}$$

$$\nu_F = \frac{c}{2d}, I_0 \equiv \text{ Incident intensity}$$

$$\frac{1}{\nu_F} = \text{round-trip time}$$

Spectral width of a mode $\approx \frac{\nu_F}{F}$

Spectral width depends strongly on finesse (F). Cf. F=100 vs 10…

3.1- Plane Mirror Resonator

- The two principal sources of loss in the optical resonator are
  - Absorption and scattering in the medium between the mirror (see laser amplifier):

Round trip power attenuation: $exp(-2\alpha_S d)$

α_S : linear absorption coefficient of the medium

  - Losses arising from imperfect reflection at the mirrors (necessary transmission + finite size effects):

Mirrors of reflectance: R_1 and R_2

Overall round trip loss of intensity:

$$R_1 R_2 \exp(-2\alpha_S d) \equiv r^2$$

Overall distributed-loss written as:

$$\exp(-2\alpha_r d) = R_1 R_2 \exp(-2\alpha_S d)$$

$$\alpha_r = \alpha_S + \frac{1}{2d} Log \frac{1}{R_1 R_2}$$

Ultimately (after maths): $F \approx \frac{\pi}{\alpha_r d}$

if $\alpha_r d << 1$ (small loses)

3.1 Plane Mirror Resonator

- The resonance linewidth is inversely proportional to the loss factor ($\alpha_r d$)

Finesse is by definition $F = \dfrac{\nu_F}{\delta\nu} \rightarrow \delta\nu \approx \dfrac{\frac{c}{2d}}{\frac{\pi}{\alpha_r d}} = \dfrac{c\alpha_r}{2\pi}$

α_r is the loss per unit length, $c\alpha_r$ is the loss per unit time

The resonator lifetime or photon lifetime in cavity is:

$\tau_p = \dfrac{1}{c\alpha_r}$, thus $\delta\nu = \dfrac{1}{2\pi\tau_p}$

3.1 Plane Mirror Resonator

- The Quality factor Q can be used to characterize the losses:

$$Q = 2\pi \frac{\text{Stored energy}}{\text{Energy loss per cycle}}$$

In the case of an optical resonator (laser), one can show that :

$$\mathrm{Q} = 2\pi\nu_0\tau_p$$

$$Q = \frac{\nu_0}{\nu_F} F,\ \nu_0 = \text{ frequency of one of the modes}$$

- Since the resonator frequencies are much larger than the mode spacing, then $Q >> F$

3.1-Plane Mirror Resonator

- What are the requirements for a laser:
  - Assume 3-D resonator (3 pairs of parallel mirrors, closed resonator), equivalent to black-body cavity.
  - Number and frequency of modes is given by the particle in the box model (photons):

$$\frac{dN}{V} = \frac{8\pi\nu^2}{c^3} d\nu$$

$V = 1cm^3, \nu = 3\times 10^{14}$ Hz, $d\nu = 3\times 10^{10}$ Hz

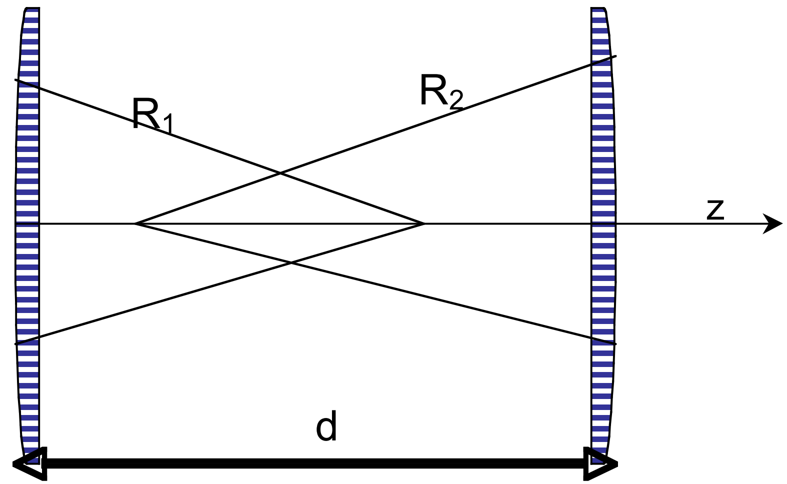
$dN = 2\times 10^{9}$ modes

3.1-Plane Mirror Resonator

- All the modes would have comparable Q in the 3D resonator
  - To be avoided in a laser as it would cause all the atoms to emit power into a large number of modes (would differ in their frequency and spatial characteristics)
- Large, open resonators consisting of opposite flat/curved reflectors must be used:
  - Energy of the vast majority of modes lost after a single pass
  - Surviving modes are near the axis

3.2 Spherical Mirror Resonator

- Ray confinement:
  - Concave R<0,
  - Convex R>0,
  - Only meridional (lie in a plane passing through the optical axis) and paraxial rays are considered
- Geometric optics is sufficient to find the condition for the existence of the confined modes



$$g_1 = 1 + \frac{d}{R_1}, \; g_2 = 1 + \frac{d}{R_2}$$

P378-381. Chapter 10.

3.2 Spherical Mirror Resonator

- Condition for the existence of confined modes:
  - Outside this domain the resonator is said to be unstable

$$0 \le g_1 g_2 \le 1$$

- For same radii, stability condition becomes:

$$R_1 = R_2 \Rightarrow g_1 = g_2 = g$$

$$-1 \le g \le +1 \Rightarrow 0 \le \frac{d}{(-R)} \le 2$$

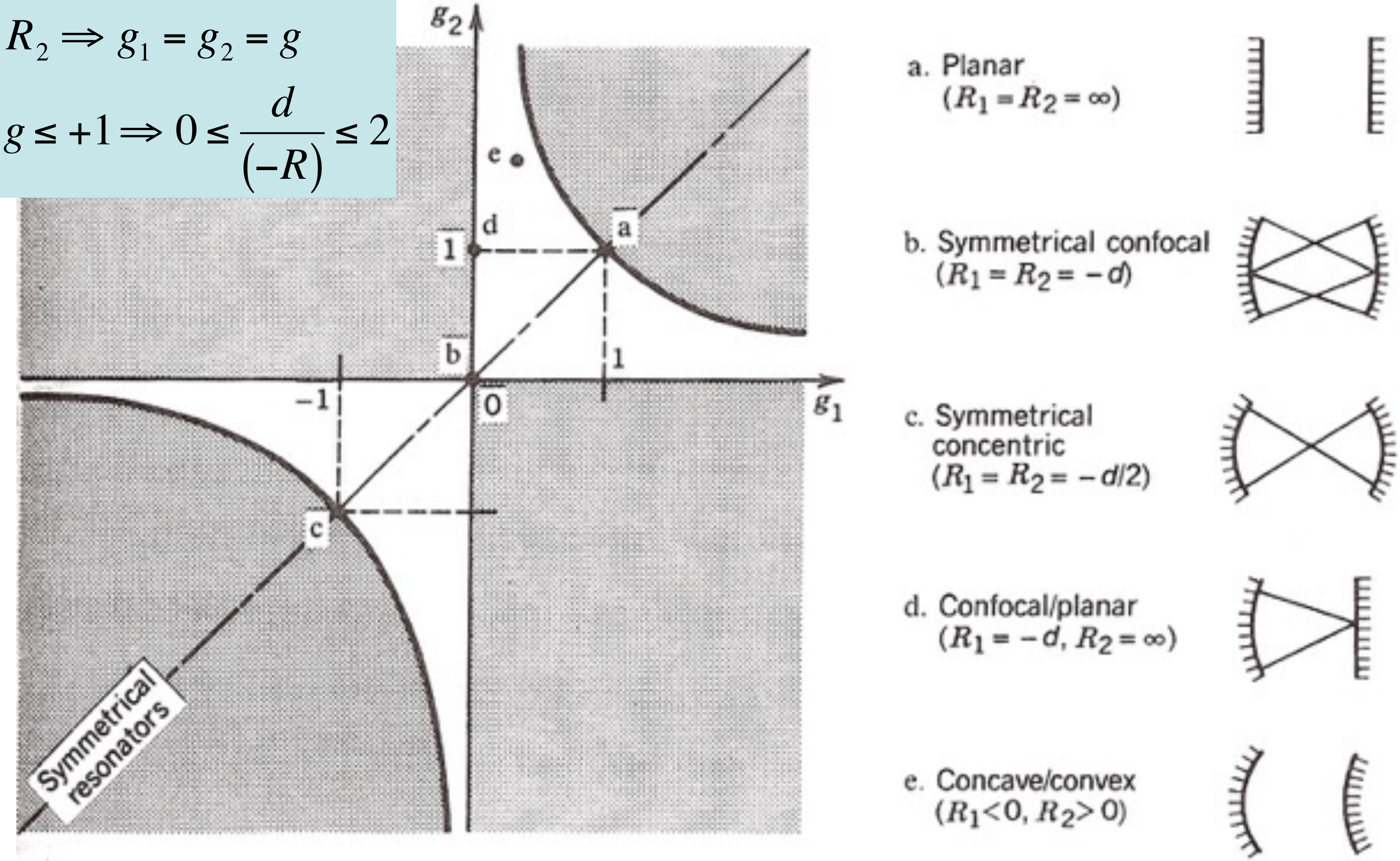
- Three resonators of practical interest: confocal concentric, confocal/planar

3.2-Spherical Mirror Resonator

Stability condition:

$$R_1 = R_2 \Rightarrow g_1 = g_2 = g$$

$$-1 \leq g \leq +1 \Rightarrow 0 \leq \frac{d}{(-R)} \leq 2$$



3.3 Gaussian Modes and resonance frequencies

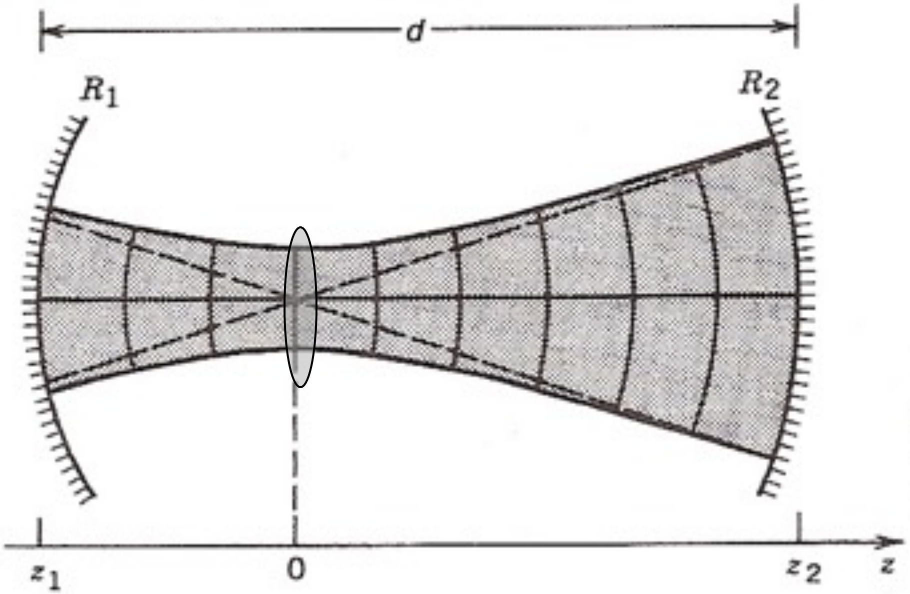
- Gaussian beams are stable modes of the spherical mirror resonator: wavefronts and phase match exactly the boundary conditions imposed by spherical mirror resonator (Helmholtz paraxial equation).
- Gaussian beam retraces incident beam if the radius of the wavefronts is exactly the same as the mirror radius.
- Phase of Gaussian beam:

$$\varphi(R,z) = kz - \zeta(z) + \frac{k\rho^2}{2R(z)} \text{ with } \rho^2 = x^2 + y^2$$

$$\text{On-axis: } \varphi(0,z) = kz - \zeta(z)$$

$$\zeta(z) = \arctan\left(\frac{z}{z_0}\right) \text{: phase retardation with respect to plane wave}$$

3.3 Gaussian Modes and resonance frequencies



$$z_0 = \frac{d}{2}\left(2\frac{|R|}{d} - 1\right)^{\frac{1}{2}}$$

$$W_0^2 = \frac{\lambda d}{2\pi}\left(2\frac{|R|}{d} - 1\right)^{\frac{1}{2}}$$

$$W_1^2 = W_2^2 = \frac{\lambda d / \pi}{\left\{\left(d/|R|\right)\left[2 - \left(d/|R|\right)\right]\right\}^{\frac{1}{2}}}$$

- See chapter 10 for details (symmetrical resonator):

$$R_1 = R_2 = -|R|$$

$$z_1 = -d/2,\ z_2 = d/2$$

$$R(z) = z\left[1 + \left(\frac{z_0}{z}\right)^2\right]$$

$$W(z) = W_0\left[1 + \left(\frac{z}{z_0}\right)^2\right]^{\frac{1}{2}}$$

3.3 Gaussian Modes and resonance frequencies

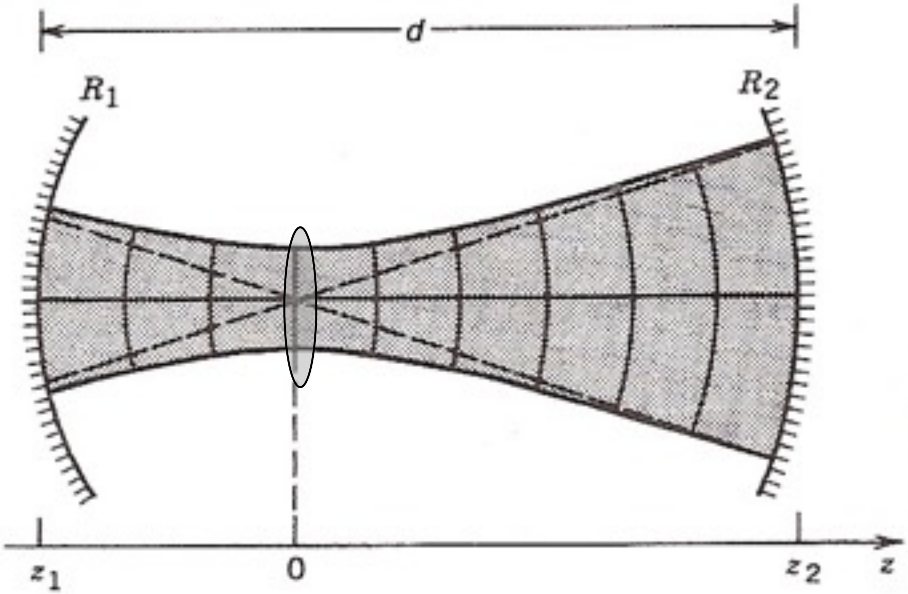
$$z_2 = z_1 + d \qquad\qquad R(z) = z + \frac{z_0^2}{z}$$

$$R_1 = z_1 + \frac{z_0^2}{z_1}, -R_2 = z_2 + \frac{z_0^2}{z_2}$$

$$z_1 = \frac{-d(R_2 + d)}{R_2 + R_1 + 2d}$$

$$z_0^2 = \frac{-d(R_1 + d)(R_2 + d)(R_1 + R_2 + d)}{(R_2 + R_1 + 2d)^2}$$

3.3 Gaussian Modes and resonance frequencies



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3.3 Gaussian modes and resonance frequencies

- Resonance frequencies can be calculated from the resonance condition (round trip phase change is exactly 2π):

At mirrors location:

$\varphi(0,z_1) = kz_1 - \zeta(z_1)$ and $\varphi(0,z_2) = kz_2 - \zeta(z_2)$

Phase change from z1 to z2:

$$\Delta\varphi = \varphi(0,z_2) - \varphi(0,z_1) = k(z_2 - z_1) - [\zeta(z_2) - \zeta(z_1)] = kd - \Delta\zeta$$

For one round trip + phase matching condition:

$$\Delta\varphi = 2(kd - \Delta\zeta) = 2q\pi \ (q = \pm 1, \pm 2, \ldots)$$

$\nu_q = q\nu_F + \dfrac{\Delta\zeta}{\pi}\nu_F,$ frequency spacing: $\left(\nu_F = \dfrac{c}{2d}\right)$

3.3 Gaussian modes and resonance frequencies

- All the Hermite-Gaussian beams of order (l,m) are also good solutions.
- All (l,m) modes have same wavefronts as $(0,0)$ but different amplitudes: Conditions for wavefront matching are identical.
- The entire family of $A_{l,m}\, G_l\, G_m$ are also modes of the spherical mirror resonator
- The resonance frequencies depend on (l,m)

3.3 Gaussian modes and resonance frequencies

- Phase matching conditions provide resonance frequencies:

Phase of the axial modes:

$$\varphi(0,z) = kz - (l+m+1)\zeta(z)$$

After a round trip + phase matching condition

$$2kd - 2(l+m+1)\Delta\zeta = 2q\pi \quad (q = \pm 1, \pm 2, \ldots)$$

Resonance frequencies:

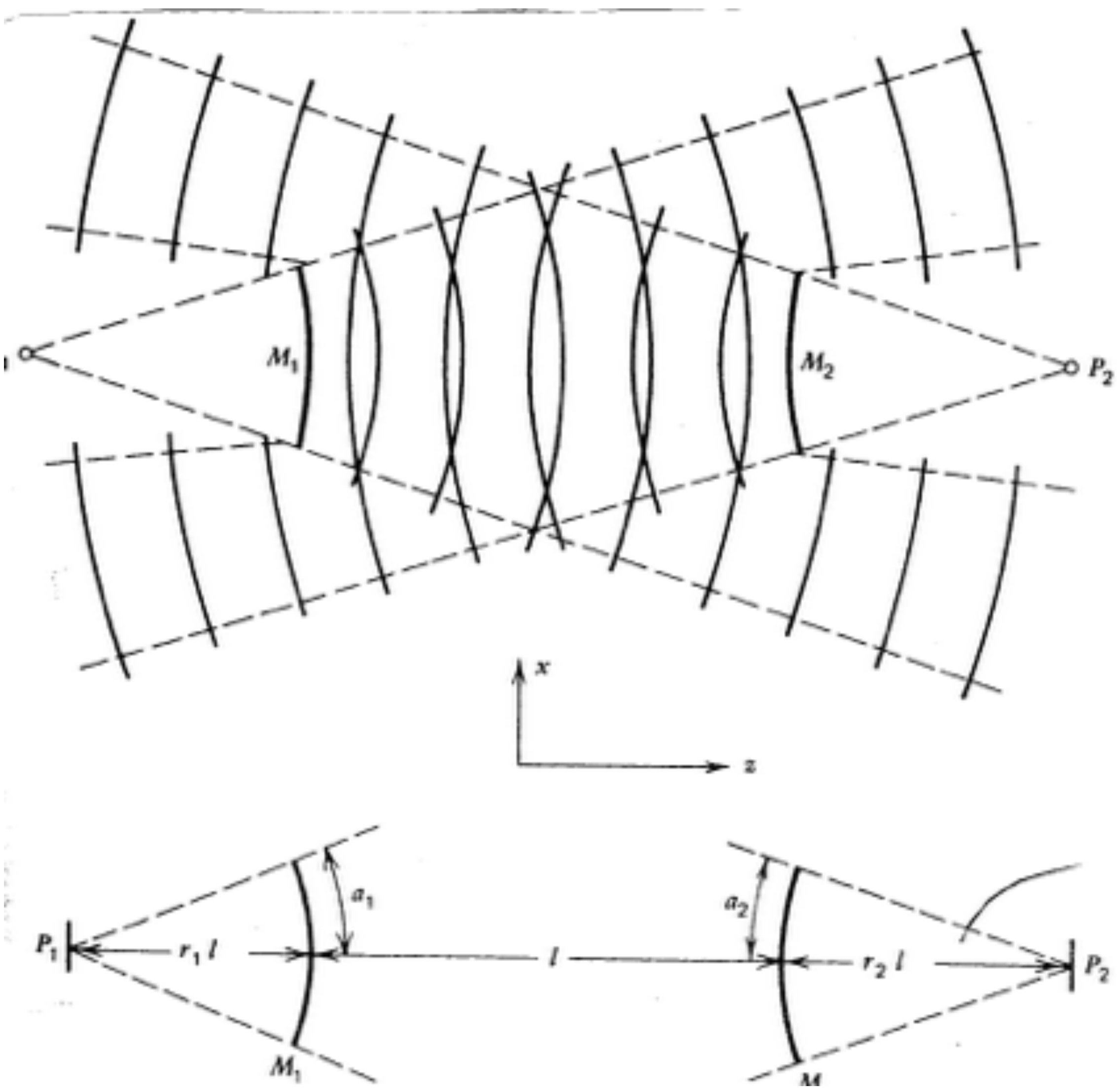
$$\nu_q = q\nu_F + (l+m+1)\frac{\Delta\zeta}{\pi}\nu_F$$

- Modes of different q but same (l,m) are called <u>longitudinal (axial)</u> modes
- Modes with different (l,m) represent different <u>transverse</u> modes

3.4 Unstable Resonator

- Close to regions of ‘unconfinement’, beam size increases
- Light losses due to missing the mirror become important (diffraction losses).
- For high power applications, large volume modes and diffraction losses are desirable
- High diffraction losses are good for a high gain situation (see later).
- Output beam has large aperture: optics are simplified
- Losses depend only on mirrors radii of curvature and separation distance.

3.4 Unstable Resonator



- Spherical wave picture of the mode in an unstable resonator.
- Points P_1 and P_2 are the virtual centres of the spherical waves.