Chapter 1

I Wave Optics

II Electromagnetic Optics

III Polarization

IV Crystal Optics

V Polarization devices

I-1 Wave Optics: definitions

- From experimental evidence: light propagates in the form of waves
- Light wave (vibration) = scalar wave = wavefunction
- This description accounts for a large number of optical phenomena
- Nature of light remains unspecified.

I-1 Wave Optics: definitions

- Wave travels in a homogeneous, non-absorbing medium with phase velocity c (wave speed)
- In vacuum, subscript 0 (zero) is used,e.g.,

$$c_0, \mu_0, \varepsilon_0$$

- The index of refraction defined as:

$$n = \frac{c_0}{c}$$

I-1 Wave Optics: definitions

- Wavefunction is a real function of position - defined by position vector $\vec{r}$ and time t: $u(\vec{r},t)$
- Wavefunction satisfies the wave equation:

$$\nabla^2 u(r,t) - \frac{1}{c^2}\frac{\partial^2 u(r,t)}{\partial t^2} = 0$$

- Principle of superposition applies:

$$u(\vec{r},t) = u_1(\vec{r},t) + u_2(\vec{r},t)$$

I-1 Wave Optics: definitions

- **OPTICAL INTENSITY** is the optical power per unit surface area ($W.cm^{-2}$). It is the measurable quantity
- It is proportional to the time average of $u^2(\vec{r},t)$

$$I(\vec{r},t) = 2\left\langle u^2(\vec{r},t)\right\rangle_{\Delta t}$$

- Δt is taken over many light cycles……

I-1 Wave Optics: definitions

- OPTICAL POWER P = power (W) flowing into an area A normal to the direction of propagation:

$$P(t) = \int_A I(\vec{r},t)\,dA$$

- OPTICAL ENERGY: time integral of optical power over the time interval

$$P = \int_{\Delta t} P(t)\,dt$$

I-1 Wave Optics: definitions

- FLUENCE = Optical energy per unit surface area ($J.cm^{-2}$). Commonly specified for laser light at the focus of a converging lens.
- Photodetectors:
  - Photoelectric detectors: photon releases an electron (photocurrent). Photodiode (p-i-n), Schottky diodes (metal-semiconductors), Photomultiplier tubes. Sensitive to intensity of incident light
  - Conversion of photon energy into heat: Power meters. Temperature rise is measured with a thermopile. Sensitive to total power absorbed

I-2 Wave Optics: monochromatic waves

- Monochromatic waves have a harmonic (sine, cosine) time dependence:

$$u(\vec{r},t) = a(\vec{r})\cos\left[2\pi\nu t + \varphi(\vec{r})\right]$$

$a(\vec{r})$: Amplitude (in metres)

$\varphi(\vec{r})$: Phase (in radians) (determined by initial conditions)

ν (nu) : Frequency (Hz)

ω (omega) : Angular frequency (in rads^{-1}) = $2\pi\nu$

I-2 Wave Optics: monochromatic waves

- It is convenient to use a complex wavefunction function instead:

$$U(\vec{r},t) = a(\vec{r})e^{i\left[2\pi\nu t + \varphi(\vec{r})\right]}$$

- From above definition:

$$u(\vec{r},t) = Re\left[U(\vec{r},t)\right] \text{ (Re = real part)}$$

$$u(\vec{r},t) = \frac{1}{2}\left[U(\vec{r},t) + U^*(\vec{r},t)\right]$$

I-2 Wave Optics: monochromatic waves

- Can be rewritten in the form:

$$U(\vec{r},t) = U(\vec{r})e^{2\pi i\nu t}$$

- The amplitude is now a complex function:

$$U(\vec{r}) = a(\vec{r})e^{i\varphi(\vec{r})}$$

- Helmholtz equation obtains (after substitution into wave equation):

$$\left(\nabla^2 + k^2\right)U(\vec{r}) = 0$$

$$k = \frac{2\pi}{\lambda} = \frac{2\pi\nu}{c} \quad = \text{ wavenumber } (\mathrm{m}^{-1})$$

I-2 Wave Optics: monochromatic waves

- Notes:
  - The choice $cos\left[2\pi\nu t+\varphi(\vec{r}\,)\right]$ is arbitrary; depends on the initial conditions
  - $sin\left[\varphi(\vec{r}\,)-2\pi\nu t\right]$ Would be also an acceptable function
- Most optical phenomena are steady-state (no time dependence): it is therefore often customary to drop the time factor: $e^{2\pi i\nu t}$

I-2 Wave Optics: monochromatic waves

- The optical intensity: $I(\vec{r}) = \left|U(\vec{r})\right|^2$
- The intensity does not vary with time
- Surfaces of equal phase are called wavefronts:

$$\varphi(\vec{r}) = constant$$

Typically: $\varphi(\vec{r}) = 2\pi q$ (q is an integer)

I-3 Wave Optics: Elementary waves

- Various possible solutions of the Helmholtz equation in homogeneous medium:
  - PLANE WAVE
  - SPHERICAL WAVE
  - PARAXIAL WAVES (➡GAUSSIAN BEAM➡OPTICAL RESONATOR)

I-3 Wave Optics: Plane wave

- The Plane Wave with complex amplitude:

$$U(\vec{r}\,) = Ae^{-i\vec{k}\cdot\vec{r}}, \;\; \varphi(\vec{r}\,) = \vec{k}\cdot\vec{r}$$

- A is the complex envelope and ***k*** is the wave vector, with $\vec{k}\cdot\vec{r} = k_x x + k_y y + k_z z = cons\,tan\,t$
- Equation describing parallel planes separated by a distance of one wavelength:

$$\lambda = \frac{2\pi}{k}$$

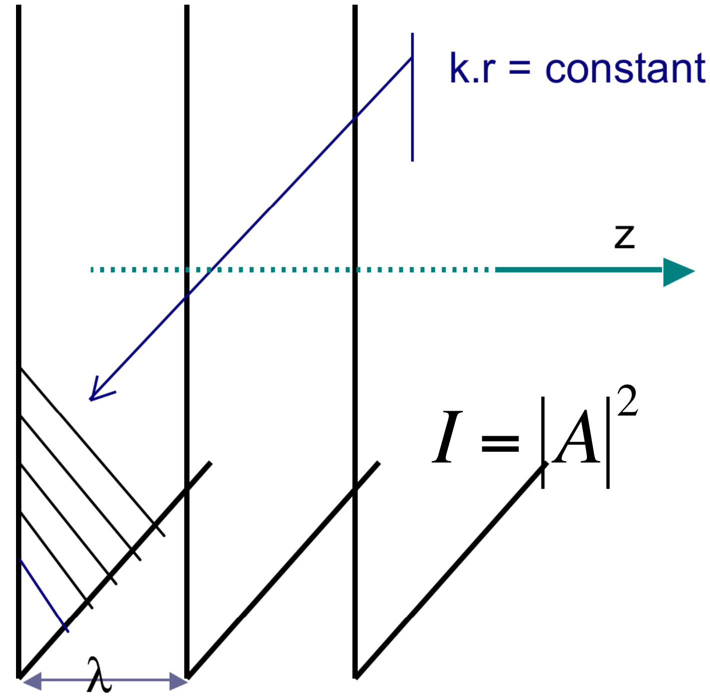
I-3 Wave Optics: Plane wave

- Can choose z axis in the direction of k:

$$U = Ae^{-ikz}$$

$$u(\vec{r},t) = |A| cos\left[2\pi\nu t - kz + arg\{A\}\right]$$

$$u(\vec{r},t) = |A| cos\left[2\pi\nu (t - \frac{z}{c}) + arg\{A\}\right]$$



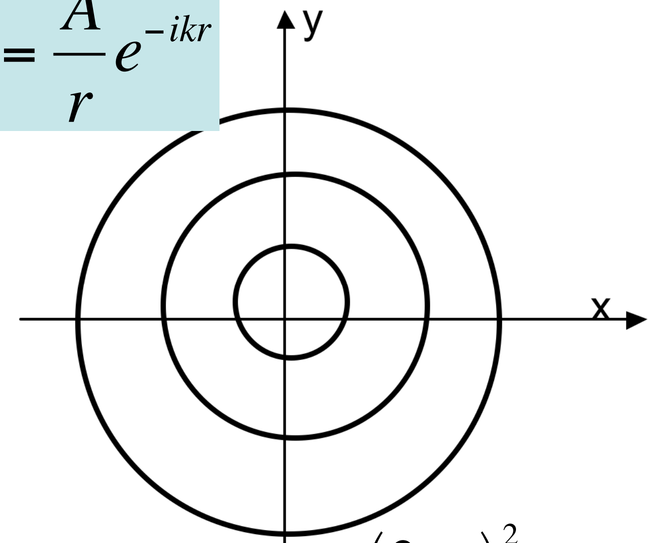
$$I = |A|^2$$

- c and λ are the phase velocity and wavelength in the medium:

$$c = \frac{c_0}{n} \text{ and } \lambda = \frac{\lambda_0}{n}$$

I-3 Wave Optics: Spherical wave

- The complex amplitude is: $U(r) = \frac{A}{r} e^{-ikr}$
- r is the radial distance from origin
- Optical Intensity: $I(r) = \frac{|A|^2}{r^2}$
- If A is real, ie arg{A} = 0, the surfaces of equal phase: $kr = 2\pi n$ or $r_x^2 + r_y^2 + r_z^2 = \left(\frac{2\pi n}{k}\right)^2$ define concentric spheres, separated by a distance of $\frac{2\pi}{k}$
- Large r $\longrightarrow$ becomes plane

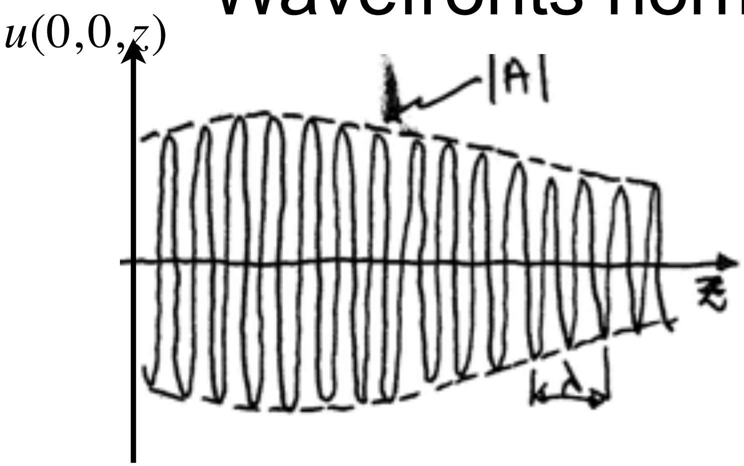


I-3 Wave Optics: Spherical wave

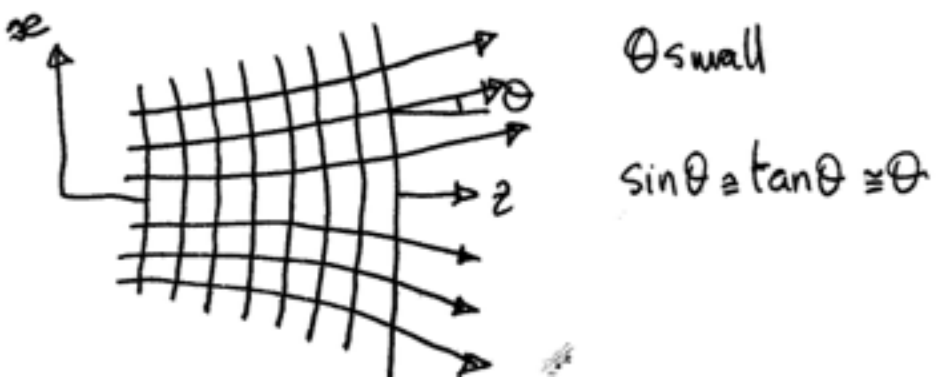
- At points close to z axis and far from origin:
  - Paraboloidal wave: approximation for behaviour between spherical and planar.
  - At large z, behaviour is almost planar
- This is typically the behaviour of <u>paraxial waves </u>(eg. the Gaussian beam)

I-3 Wave Optics: Paraxial waves

- Wavefronts normal are paraxial rays:



Wavefunction of paraxial wave at point on z axis



Wavefronts and wavefront normals

- Basic approximation of Geometric Optics

I-3 Wave Optics: Paraxial waves

- To construct a paraxial wave: start with a plane wave Ae^{-ikz} and modulate the complex envelope A making it a slowly varying function of r:

$$U(\vec{r}\,) = A(\vec{r}\,)e^{-ikz}$$

$A(\vec{r}\,)$ variation with position is very small over a distance of one λ.

It is still approximately planar.

I-3 Wave Optics: Paraxial waves

- Paraxial waves satisfy the paraxial Helmholtz equation:

$$\nabla_T^2 A - i2k\frac{\partial A}{\partial z} = 0$$

$$\nabla_T^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \quad \equiv \text{ Transverse Laplace operator}$$

- Most useful is the Gaussian beam (mode of the spherical-mirror resonator)

II -Electromagnetic Optics

- Light is an electromagnetic phenomenon: carries electric $\vec{E}(\vec{r},t)$ and magnetic fields $\vec{H}(\vec{r},t)$
- These are **vector** waves: scalar wave equation fails to explain electric and magnetic effects induced by light
- **Problem**: how can we describe the electromagnetic state of matter in the presence of light?

II -Electromagnetic Optics: Definitions

- New set of vectors is required to describe the response of matter:

Electric current density $\vec{j}$

Electric displacement (electric flux density) $\vec{D}$

Magnetic displacement (magnetic induction) $\vec{B}$

ρ density of free charges

- ***E***, **H, B, D, j** and ρ are related by Maxwell's equations (set of 4 coupled pde's)

II -Electromagnetic Optics: Definitions

- General solution of Maxwell's eqs. is complicated (would provide electromagnetic response of matter -***D*** and ***B***- in the presence of ***E*** and ***H*** fields)
- For **harmonic** fields and **isotropic** media, relation between applied fields and response is simple

II -Electromagnetic Optics: In Vacuo

- ε = Electric permittivity or dielectric constant

$$\vec{D} = \varepsilon \vec{E}$$

- μ = magnetic permeability
  - μ ~ 1 non-magnetic (most substances)
  - μ > 1 paramagnetic
  - μ< 1 diamagnetic

$$\vec{B} = \mu \vec{H}$$

- σ = specific conductivity
  - σ negligibly small: insulators (dielectrics)
  - σ not negligibly small: conductors

$$\vec{j} = \sigma \vec{E}$$

II -Electromagnetic Optics: Definitions

- Previous set of equations describes the response of matter in the presence of weak fields.
- Linear response: 1st power of fields
- For strong fields (strength of the order of valence electrons binding energies):
  - Response is non linear
  - Must include higher-order components of the fields

II -Electromagnetic Optics: Definitions

$$\vec{D} = \varepsilon\vec{E} + (\varepsilon)_2\vec{E}\vec{E} + (\varepsilon)_3\vec{E}\vec{E}\vec{E} + \ldots$$

$$\vec{D} = \varepsilon\vec{E} + (\varepsilon)_2\vec{E}^2 + (\varepsilon)_3\vec{E}^3 + \ldots$$

- The laws of Optics must be modified (Non-linear Optics, Bloembergen, 1965)

II -Electromagnetic Optics: In Medium

- Effects of the fields can be described using “additive” relations:

$$\vec{D} = \varepsilon_0 \vec{E} + \vec{P}$$

$\vec{P}$ = Polarization = Dipole moment/m^3

$$\vec{B} = \mu_0 \vec{H} + \mu_0 \vec{M}$$

$\vec{M}$ = Magnetization = Magnetic moment/m^3

II -Electromagnetic Optics: Definitions

- For **weak** fields, polarization and magnetization are assumed to be linearly proportional to the applied fields:

$$\vec{P} = \chi \varepsilon_0 \vec{E}$$

χ = electric susceptibility

$$\vec{D} = \varepsilon_0 \vec{E} + \chi \varepsilon_0 \vec{E} = \varepsilon_0 (1 + \chi) \vec{E}$$

$$\varepsilon = \varepsilon_0 (1 + \chi)$$

$$\varepsilon_r = \frac{\varepsilon}{\varepsilon_0} = (1 + \chi)$$

relative permittivity

$$\mu_0 \vec{M} = \mu_0 \chi_m \vec{H}$$

χ_m = magnetic susceptibility

$$\vec{B} = \mu_0 \vec{H} + \chi_m \mu_0 \vec{H} = \mu_0 (1 + \chi_m) \vec{H}$$

$$\mu = \mu_0 (1 + \chi_m)$$

$$\mu_r = \frac{\mu}{\mu_0} = (1 + \chi_m)$$

relative permeability

II -Electromagnetic Optics: Maxwell's Equations

$$\nabla \times \vec{E} = -\mu \frac{\partial \vec{H}}{\partial t}$$

$$\nabla \cdot \vec{D} = \rho$$

$$\nabla \times \vec{H} = \varepsilon \frac{\partial \vec{E}}{\partial t} + \vec{j}$$

$$\nabla \cdot \vec{B} = 0$$

- In optics, generally non-magnetic media and no currents ($\vec{M} = \vec{0}$ and $\vec{j} = \vec{0}$)

- The flow of electromagnetic energy is given by the Poynting vector: $\vec{P} = \vec{E} \times \vec{H}$

II -Electromagnetic Optics: Maxwell's Equations

- Most optical materials are dielectrics:
  - L = linear : if **P** is linearly related to **E**
  - ND = non-dispersive: instantaneous response: **P** at t is determined by **E** at t.
  - H = homogeneous: relation between **P** and **E** is independent of **r**
  - I = isotropic: relation between **P** and **E** is independent of the direction of **E**. Medium is identical from all directions of space.

II -Electromagnetic Optics: Maxwell's Equations

- Medium is L, ND, H and I:

$$\vec{P} = \chi\varepsilon_0\vec{E};\vec{D} = \varepsilon\vec{E};\varepsilon = \varepsilon_0(1+\chi)$$

- Each component of E, H satisfy separately the wave equation (same as wave optics):

$$\nabla^2 u(r,t) - \frac{1}{c^2}\frac{\partial^2 u(r,t)}{\partial t^2} = 0 \text{ with } c = \frac{1}{(\varepsilon\mu_0)^2} = \frac{c_0}{n}$$

$$n = \left(\frac{\varepsilon}{\varepsilon_0}\right)^{\frac{1}{2}} = (1+\chi)^{\frac{1}{2}}$$

II -Electromagnetic Optics: Maxwell's Equations -inhomogeneous medium

- Medium is L, ND, I, inhomogeneous
- (e.g. a graded-index optical fibre)
- The spatial variations of $n = n(\vec{r})$ are small over distances of a few wavelengths

$$\vec{P} = \chi(\vec{r})\varepsilon_0 \vec{E}; \vec{D} = \varepsilon(\vec{r})\vec{E}$$

$$\nabla^2 \vec{E} - \frac{1}{c(\vec{r})^2}\frac{\partial^2 \vec{E}}{\partial t^2} = 0$$

II -Electromagnetic Optics: Maxwell's Equations

- Medium is L, ND, H but anisotropic: relation between **P** and **E** depends on the direction of **E**
- **P** and **E** are not necessarily parallel:
  - Dielectric properties described by an array of (3x3) constants called the susceptibility tensor

II -Electromagnetic Optics: Maxwell's Equations-Anisotropic medium

- Each component of **P** (or **D**) is given by:

$$P_i = \sum_j \varepsilon_0 \chi_{ij} E_j$$

$i, j = 1,2,3$ denotes x, y, z components

$$D_i = \sum_j \varepsilon_{ij} E_j$$

ε_{ij} components of electric permittivity tensor

- Typically crystals with non cubic symmetries are anisotropic media

II -Electromagnetic Optics: Maxwell's Equations nonlinear medium

- The relation between P and E is non linear: $\vec{P} = \Psi(\vec{E})$, e.g. $\vec{P} = a_1\vec{E} + a_2\vec{E}^2 + a_3\vec{E}^3$
- Maxwell's equations must be used to derive a non-linear partial differential eqn

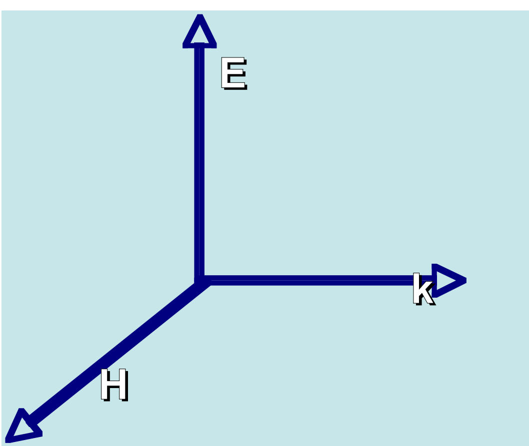
$$\nabla^2\vec{E} - \frac{1}{c^2}\frac{\partial^2\vec{E}}{\partial t^2} = \mu_0\frac{\partial^2\vec{P}}{\partial t^2} = \mu_0\frac{\partial^2\Psi(\vec{E})}{\partial t^2}$$

Basic equation of non linear optics

II -Electromagnetic Optics: Elementary EM waves

- The Transverse Electromagnetic (TEM) Plane Wave (medium L,H,I):

$$\vec{E}(\vec{r}) = \vec{E}_0 e^{-i\vec{k}\cdot\vec{r}} \quad \overrightarrow{H}(\vec{r}) = \vec{H}_0 e^{-i\vec{k}\cdot\vec{r}}$$



(1) From Maxwell: $\left(\frac{E_0}{H_0}\right) = \left(\frac{\omega\mu_0}{k}\right) = \left(\frac{c_0\mu_0}{n}\right) = \frac{\left(\frac{\mu_0}{\varepsilon_0}\right)^{\frac{1}{2}}}{n} = \eta$

Eta = (optical) impedance of medium

(2) From Poynting: $I = \frac{|E_o|^2}{\eta}$

III-Polarisation of Light

- Polarisation = time course of the direction of the electric field vector **E**(r,t)
- In paraxial optics, EM waves are approximately TEM: ***E***(r,t) lies in transverse plane
- If medium is isotropic: wave is elliptically polarized

III-Polarisation of Light

- Polarisation plays an important role in optics:
  - Amount of reflected light depends on polarisation state at the boundary (interface)
  - Amount of light absorbed depends on state of polarisation (dichroism)
  - Refractive index of anisotropic materials depends on polarisation state (see optical devices - birefringent materials)
  - Rotation of plane of polarisation of linearly polarised light in presence of external electric or magnetic field

III-Polarisation of Light: polarisation ellipse

- $$\vec{E}(z,t) = Re\left[\vec{A}e^{-i2\pi\nu(t-\frac{z}{c})}\right]$$

Monochromatic plane wave travelling in Oz direction with velocity c

- Complex envelope (amplitude): $\vec{A} = A_x\hat{x} + A_y\hat{y}$

$$A_x = a_x e^{-i\varphi_x};\ A_y = a_y e^{-i\varphi_y}$$

III-Polarisation of Light: polarisation ellipse

- Polarisation = End point of **E**(z,t) = location of points whose coordinates are (E_x, E_y): $\vec{E}(z,t) = E_x\hat{x} + E_y\hat{y}$

Defining $\tau = 2\pi\nu(t - \frac{z}{c})$

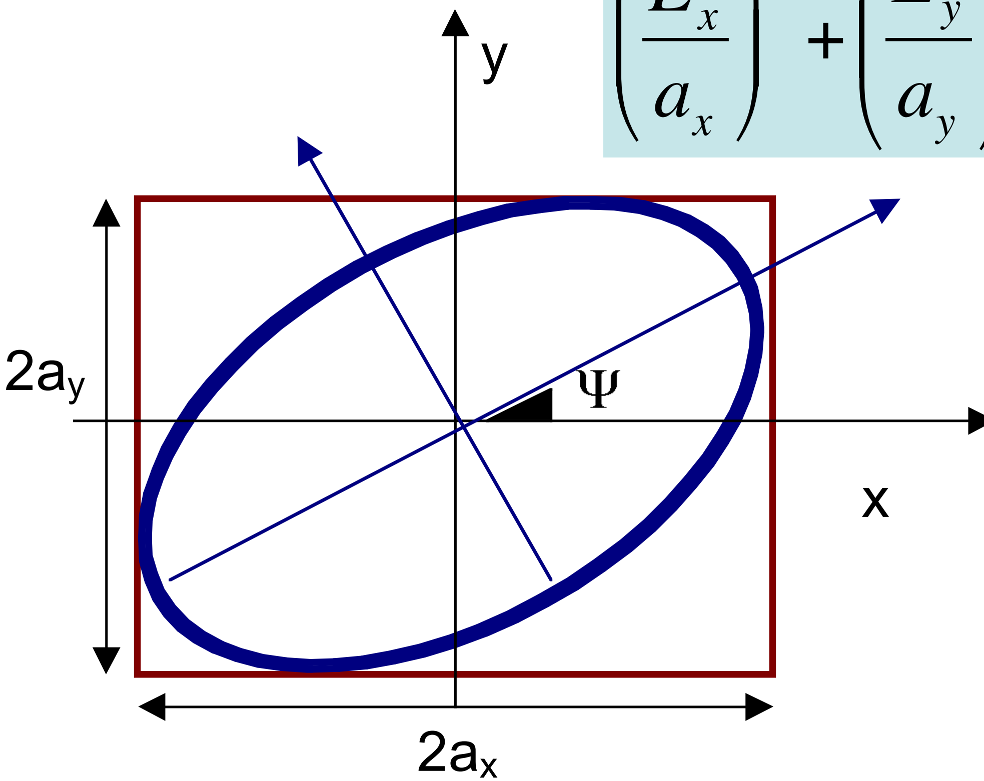
$$E_x = a_x cos(\tau + \varphi_x),\ E_y = a_y cos(\tau + \varphi_y),\ E_z = 0$$

$\frac{E_x}{a_x} = cos\tau\, cos\varphi_x - sin\tau\, sin\varphi_x$ etc...and $\varphi = \varphi_y - \varphi_x$

III-Polarisation of Light: polarisation ellipse

- Equation of ellipse (conic):

$$\left(\frac{E_x}{a_x}\right)^2 + \left(\frac{E_y}{a_y}\right)^2 - 2\left(\frac{E_x}{a_x}\frac{E_y}{a_y}\right)cos\varphi = sin^2\varphi$$



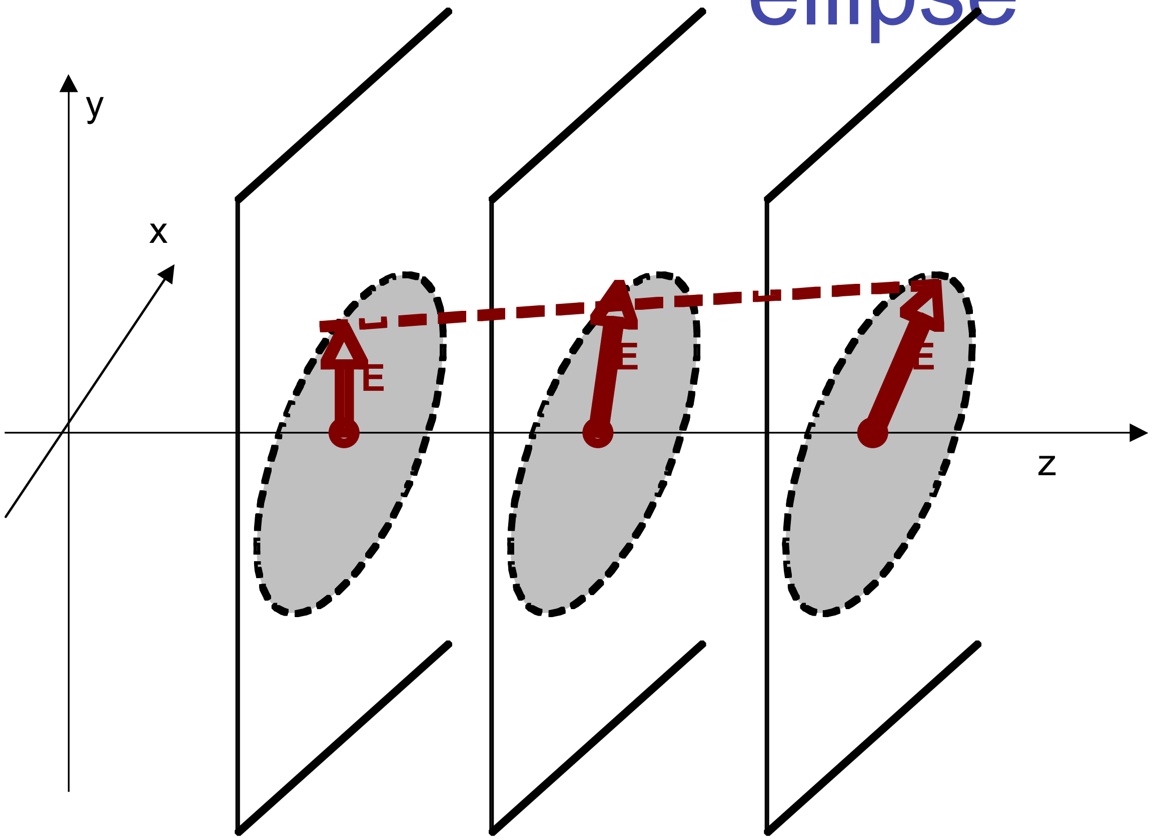
The "tilt" Ψ is obtained from:

$$tan\,2\Psi = \frac{2a_x a_y\,cos\varphi}{(a_x^2 - a_y^2)}$$

III-Polarisation of Light: polarisation ellipse

- The magnetic vector is also elliptically polarised
- At fixed value of z, **E** rotates at frequency (ν) nu in (x-y) plane tracing out an ellipse
- At fixed t (snap shot): the location of the tip follows a helical trajectory
- State of polarisation determined by tilt (value of psi) and ratio of major to minor axes

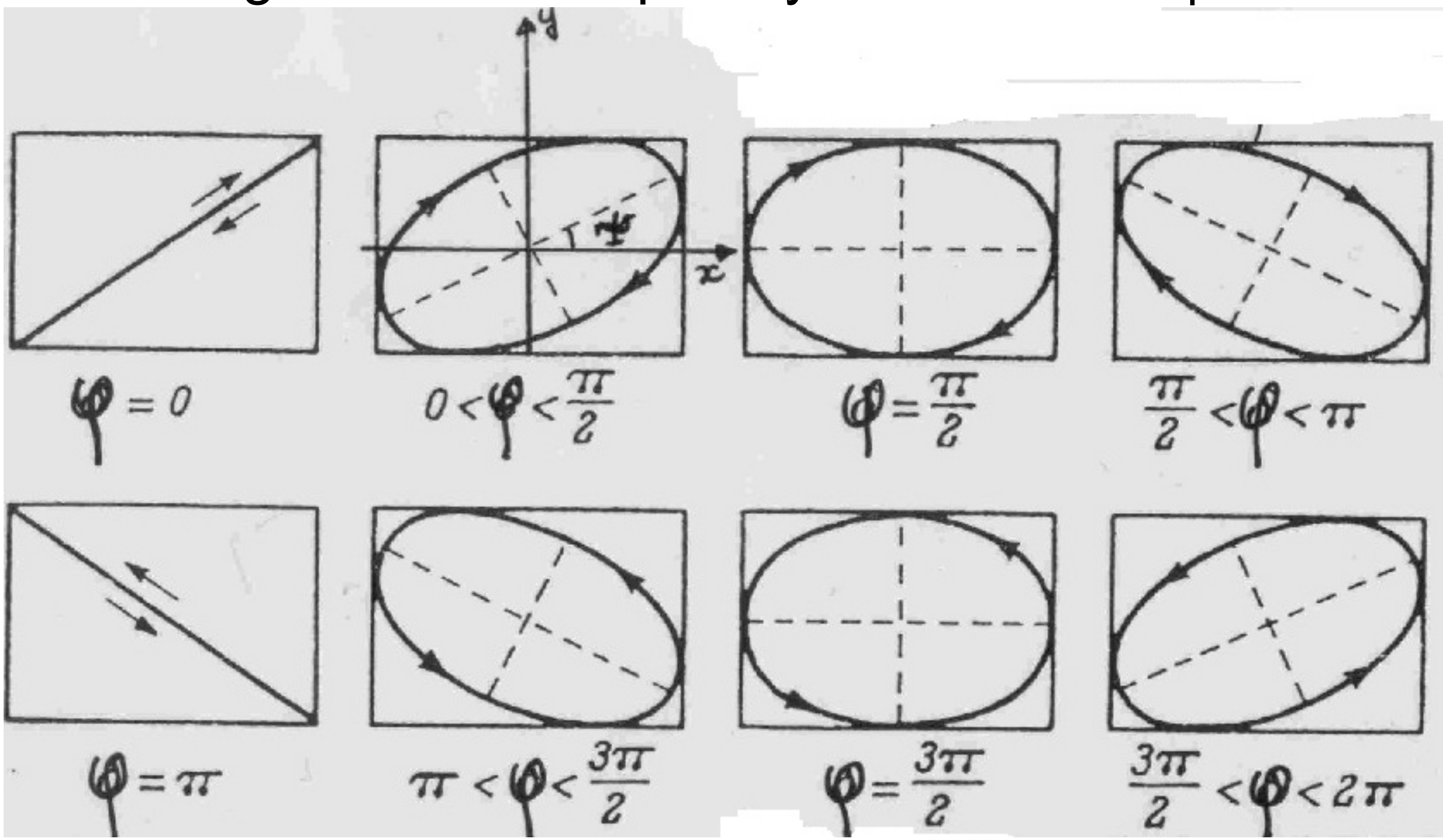
III-Polarisation of Light: Polarisation ellipse



Timecourse of tip of **E** is an elliptical helix:

III-Polarisation of Light: polarisation ellipse

Right-Handed Elliptically Polarised: sinφ>0



Left-Handed Elliptically Polarised: sinφ<0

III-Polarisation of Light: polarisation ellipse

- The nature of the polarisation can be determined from:

$$\frac{E_y}{E_x} = \frac{a_y}{a_x} e^{i(\varphi_x - \varphi_y)} = \frac{a_y}{a_x} e^{-i\varphi}$$

  - Linear Polarisation:

$$\frac{E_y}{E_x} = (-1)^m \frac{a_y}{a_x}, \text{ as ellipse reduces to a straight line}$$

when $\varphi = m\pi \; (m = 0, \pm 1, \pm 2, \ldots)$

Linear polarisation also for a_x or $a_y \; = 0$

III-Polarisation of Light: polarisation ellipse

- Circular Polarisation: the ellipse degenerates into a circle if a_x=a_y=a_0 and φ=mπ/2 (m =±1, ±3, ±5,...)

$$E_x^{\ 2} + E_y^{\ 2} = a_0^{\ 2}$$

- Using complex form:

$$\text{Right-handed circularly polarized: } a_x = a_y, \varphi = \pi/2$$

$$\frac{E_y}{E_x} = e^{-i\frac{\pi}{2}} = -i$$

$$\text{Left-handed circularly polarized: } a_x = a_y, \varphi = -\pi/2$$

$$\frac{E_y}{E_x} = e^{i\frac{\pi}{2}} = i$$

III-Polarisation of Light: Matrix Representation; Jones Vector

- A monochromatic plane wave is completely determined by the knowledge of the complex envelope A_x and A_y
- Can be represented in the form of a 2-component column matrix -the Jones vector:

$$\vec{J} = \begin{bmatrix} A_x \\ A_y \end{bmatrix}$$

III-Polarisation of Light: Jones Vector

- From J, one can calculate the total light intensity: $$I = \left(\left| A_x \right|^2 + \left| A_y \right|^2 \right) / 2\eta$$

- The orientation and shape of the polarisation ellipse can be obtained from:

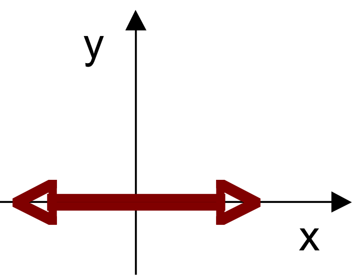
$$\frac{a_y}{a_x} = \frac{\left| A_y \right|}{\left| A_x \right|}; \varphi = \varphi_y - \varphi_x = \arg\left\{ A_y \right\} - \arg\left\{ A_x \right\}$$

III-Polarisation of Light: Jones Vector

- Jones vectors for typical polarisations:
  intensity is normalised so that:

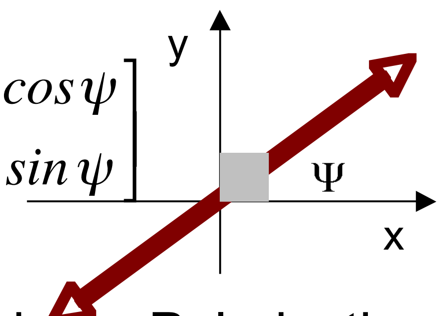
$$\left(\left|A_x\right|^2+\left|A_y\right|^2\right)=1 \text{ and } \varphi_x=0$$

$$\begin{bmatrix}1\\0\end{bmatrix}$$



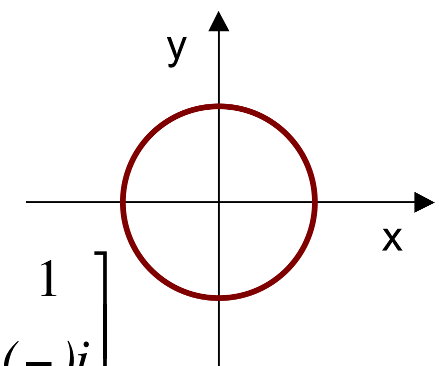
Linear Polarisation in x direction

$$\begin{bmatrix}\cos\psi\\ \sin\psi\end{bmatrix}$$



Linear Polarisation making angle ψ with x axis

$$\frac{1}{\sqrt{2}}\begin{bmatrix}1\\(-)i\end{bmatrix}$$



Right(left)-handed circular polarisation

III-Polarisation of Light: Jones Matrix

- Jones vectors $\mathbf{J_1}$ and $\mathbf{J_2}$ are orthogonal if (inner product is 0):

$$\vec{J}_1 \cdot \vec{J}_2^{\;*} = (A_{1x} A_{2x}^{*} + A_{1y} A_{2y}^{*}) = 0$$

- Any arbitrary Jones vector $\mathbf{J}$, can be analysed as a weighted superposition of two orthogonal polarisations:

$$\vec{J} = \alpha_1 \vec{J}_1 + \alpha_2 \vec{J}_2$$

$$\alpha_1 = \vec{J} \cdot \vec{J}_1^{\;*} ; \alpha_2 = \vec{J} \cdot \vec{J}_2^{\;*}$$

$\vec{J}_1, \vec{J}_2$ normalised to unity

$$\vec{J}_1 \bullet \vec{J}_1^{*} = \vec{J}_2 \bullet \vec{J}_2^{*} = 1$$

III-Polarisation of Light: Jones Matrix

- A linear optical system that maintains the plane wave nature of light but alters its polarisation can be represented by a (2×2) Jones matrix **T**:

$$\vec{J}_1 = \begin{bmatrix} A_{1x} \\ A_{1y} \end{bmatrix}$$

Input Wave

$$T = \begin{pmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{pmatrix}$$

Optical System

$$\vec{J}_2 = \begin{bmatrix} A_{2x} \\ A_{2y} \end{bmatrix}$$

Output Wave

$$\vec{J}_2 = T\vec{J}_1$$

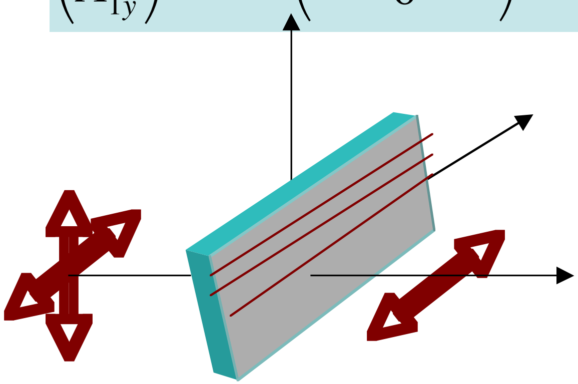
III-Polarisation of Light: Jones Matrix

- Examples of Jones matrices:

1. The Linear Polariser:

$$T = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$$

$$\begin{pmatrix} A_{1x} \\ A_{1y} \end{pmatrix} \xrightarrow{T} \begin{pmatrix} A_{2x} = A_{1x} \\ 0 \end{pmatrix}$$



2. The Wave Retarder:

$$T = \begin{pmatrix} 1 & 0 \\ 0 & e^{-i\Delta} \end{pmatrix}$$

$\Delta = \dfrac{\pi}{2}$ = quarter-wave retarder

$\Delta = \pi$ = half-wave retarder

$$\begin{pmatrix} A_{1x} \\ A_{1y} \end{pmatrix} \xrightarrow{T} \begin{pmatrix} A_{2x} = A_{1x} \\ A_{2y} = A_{1y} e^{-i\Delta} \end{pmatrix}$$

3. The Polarisation Rotator:

$$T = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$$

$$\begin{pmatrix} \cos\theta_1 \\ \sin\theta_1 \end{pmatrix} \xrightarrow{T} \begin{pmatrix} \cos\theta_2 \\ \sin\theta_2 \end{pmatrix}$$

$$\theta_2 = \theta + \theta_1$$

III-Polarisation of Light: Normal modes

- Normal modes of a polarisation system are the states of polarisation that remain unchanged when transmitted through the system
- Normal modes = eigenvectors of T matrix (2 modes)

$$\mathbf{T}\vec{J} = \mu\vec{J}$$

III-Polarisation of Light: Normal modes

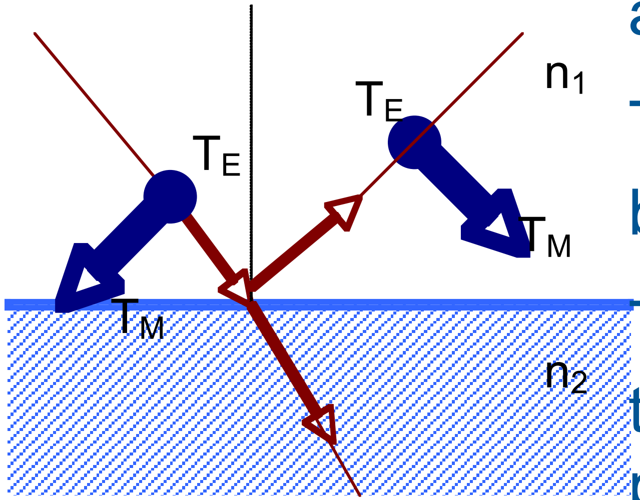
- Normal modes are orthogonal and form a basis set (**T** is hermitian)
- Any input wave **J** = superposition of normal modes: $\vec{\mathbf{J}} = \alpha_1 \vec{\mathbf{J}}_1 + \alpha_2 \vec{\mathbf{J}}_2$
- The response can be easily evaluated using:

$$\mathbf{T}\vec{\mathbf{J}} = \mathbf{T}(\alpha_1 \vec{\mathbf{J}}_1 + \alpha_2 \vec{\mathbf{J}}_2) = \alpha_1 \mathbf{T}\vec{\mathbf{J}}_1 + \alpha_2 \mathbf{T}\vec{\mathbf{J}}_2 = \alpha_1 \mu_1 \vec{\mathbf{J}}_1 + \alpha_2 \mu_2 \vec{\mathbf{J}}_2$$

- Problem: Find the Normal Modes

III-Polarisation of Light: Example of normal modes

- Reflection and refraction of monochromatic plane wave of arbitrary polarisation incident at dielectric boundary (n_1,n_2)



The normal modes (from Maxwell's) are the two linear polarisations:

T_E (transverse electric, parallel to the boundary): sigma or s polarisation

T_M (Transverse magnetic parallel to the plane of incidence): parallel or pi polarisation

IV-Crystal Optics

- Crystals are anisotropic media: electric displacement vector ***D*** depends (possibly) on all the components of applied ***E*** field.
- Each component of ***D*** can be written as:

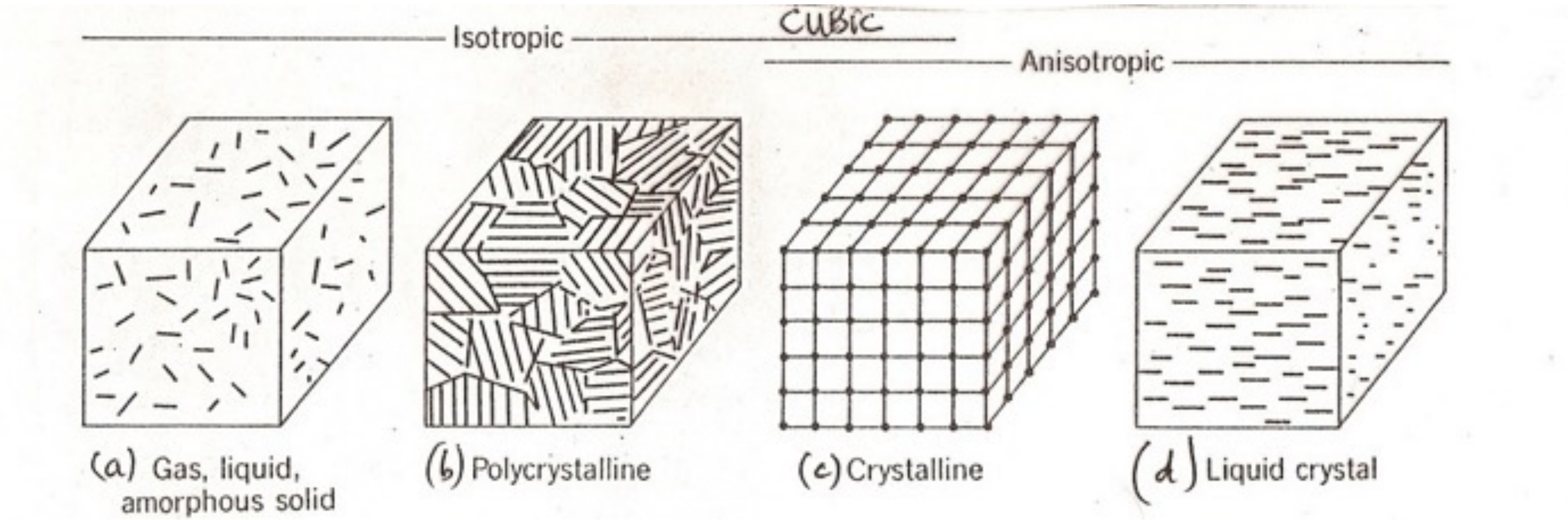
$$D_i = \sum_j \varepsilon_{ij} E_j \text{ with } i, j = 1,2,3 \equiv x, y, z$$

$\tilde{\varepsilon}$ is a second-rank tensor: the permittivity tensor

Electric displacement $\vec{D}$ is the contraction of a 2-tensor and a vector (tensor rank one): $\vec{D} = \tilde{\varepsilon}\vec{E}$

IV-Crystal Optics

- Examples of anisotropic media



(a) Completely isotropic: long and short-range disorder

(b) Short-range order, long-range disorder: average macroscopic behaviour is isotropic

(c) Positional and orientational orders: anisotropic (except fcc lattices)

(d) Short-range disorder, long-range order: average macroscopic behaviour is anisotropic

IV-Crystal Optics

- There always exists a system of coordinates in which ε has only diagonal elements: ε_{11}, ε_{22}, ε_{33}
- This system defines the **Principal Axes**: directions of space for which **E** and **D** are parallel.
- The principal refractive indices are:

$$n_1 = \left(\frac{\varepsilon_1}{\varepsilon_0}\right)^{\frac{1}{2}} \quad n_2 = \left(\frac{\varepsilon_2}{\varepsilon_0}\right)^{\frac{1}{2}} \quad n_3 = \left(\frac{\varepsilon_3}{\varepsilon_0}\right)^{\frac{1}{2}}$$

IV-Crystal Optics

- Anisotropy leads to birefringence: phase velocity of an optical beam clearly depends on the direction of polarisation of its ***E*** vector.
- Three types of crystals:
  - Uniaxial: $n_1 = n_2 = n_o$ (ordinary index), $n_3 = n_e$ (extraordinary index) calcite, quartz
  - Biaxial: n_1, n_2, n_3
  - Isotropic $n_1=n_2=n_3$

IV-Crystal Optics

- Geometrical construction completely describes the optical properties: it specifies the values of the principal refractive indices and the directions of the Principal axes.
- This is called the Index Ellipsoid (also Optical Indicatrix). It is the surface of equation:

$$\frac{x^2}{n_1^2}+\frac{y^2}{n_2^2}+\frac{z^2}{n_3^2}=1$$

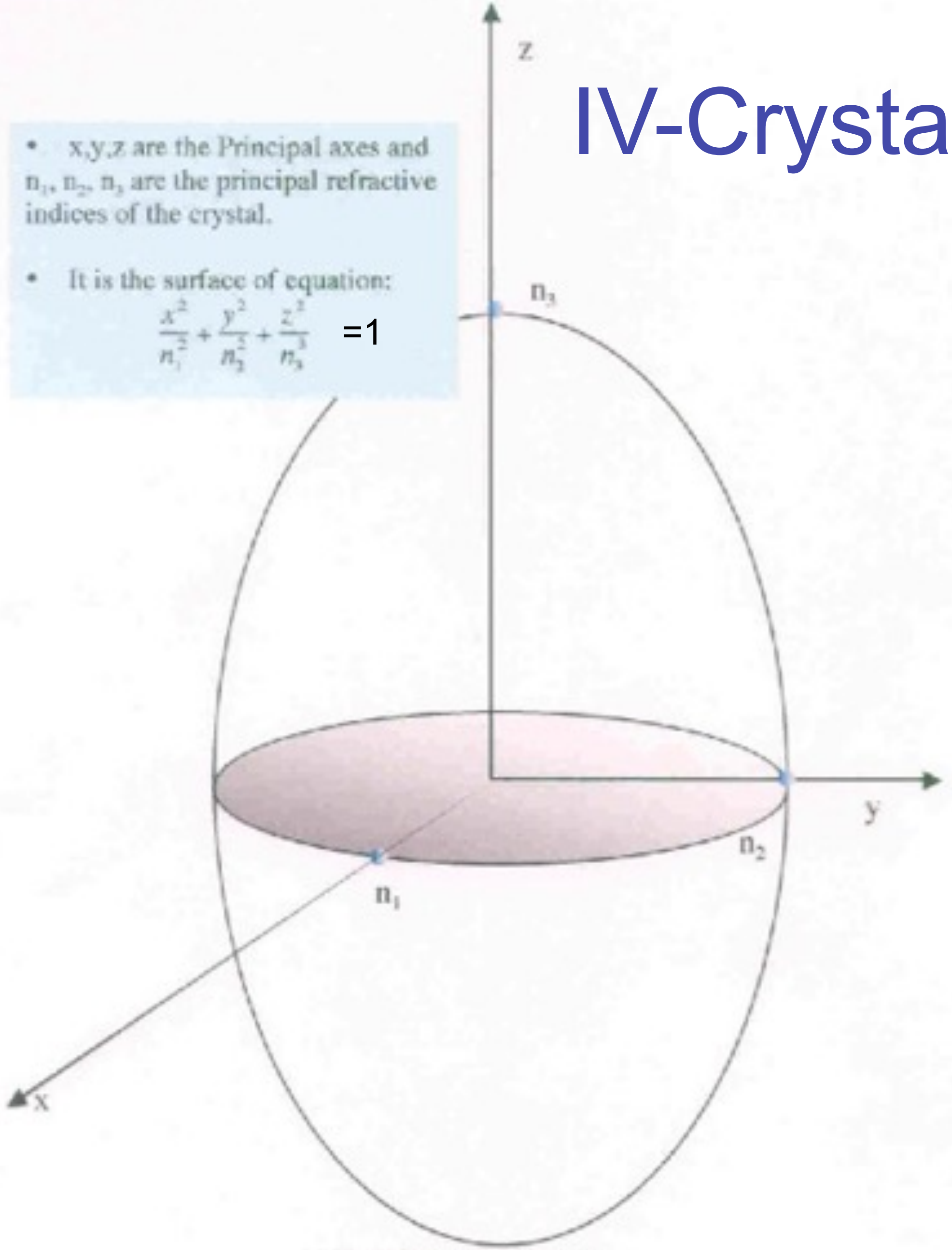
x, y, z : principal axes

n_1, n_2, n_3: principal indices

IV-Crystal Optics

- x,y,z are the Principal axes and n_1, n_2, n_3 are the principal refractive indices of the crystal.
- It is the surface of equation:

$$\frac{x^2}{n_1^2}+\frac{y^2}{n_2^2}+\frac{z^2}{n_3^2}=1$$

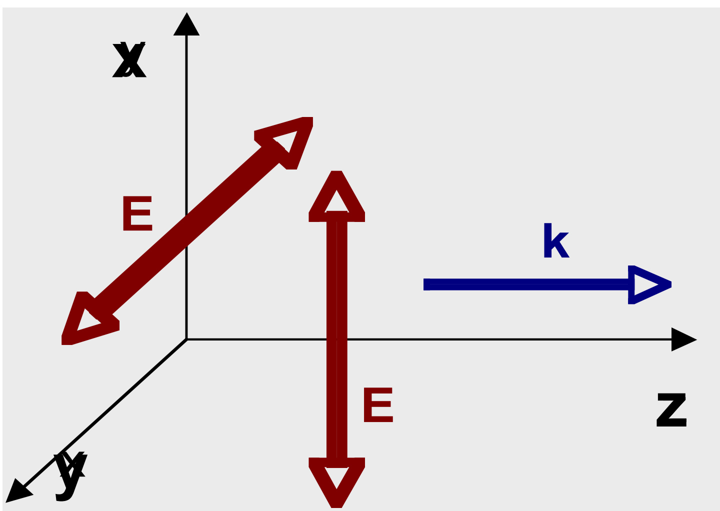


The Index Ellipsoid

1. Index ellipsoid is an ellipsoid of revolution for uniaxial crystals
2. Index ellipsoid is a sphere for cubic crystal
3. z is called optic axis for uniaxial crystals

IV-Crystal Optics

- Propagation of plane EM waves (linearly polarised) along one of Principal axes: what are the normal modes?



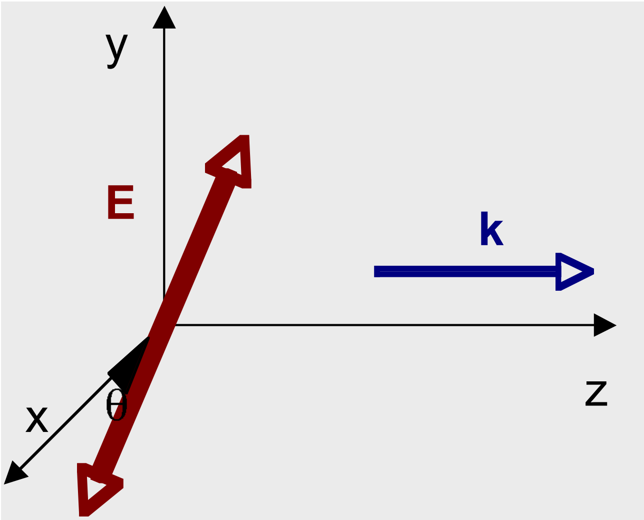
•Linear polarisation along x (y): wave travels at phase velocity c_0/n_1 (c_0/n_2) without change of polarisation.

•$D_1=\varepsilon_1 E_1$ ($D_2=\varepsilon_2 E_2$)

If **k** is along Oz, the Normal modes are the linearly polarised waves in the x and y directions respectively

IV-Crystal Optics

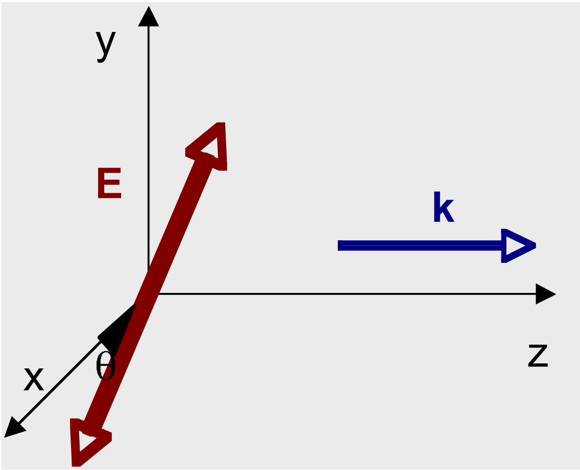
- If **k** is along Oz, but **E** is in x-y plane making angle θ with Ox
- Resulting polarisation?



IV-Crystal Optics

- If **k** is along Oz, but **E** is in x-y plane making angle θ with Ox
- Traveling wave is a sum of the normal modes: each travels at (c_0/n_1) and (c_0/n_2) resp.
- The phase difference after a distance d travelled through the crystal:

$$\varphi = \frac{2\pi}{\lambda_0}(n_2 - n_1)d$$



•The output wave is elliptically polarised.

•Crystal acts as a wave retarder (see Jones vector)

•Retardation plates are polarisation state converters

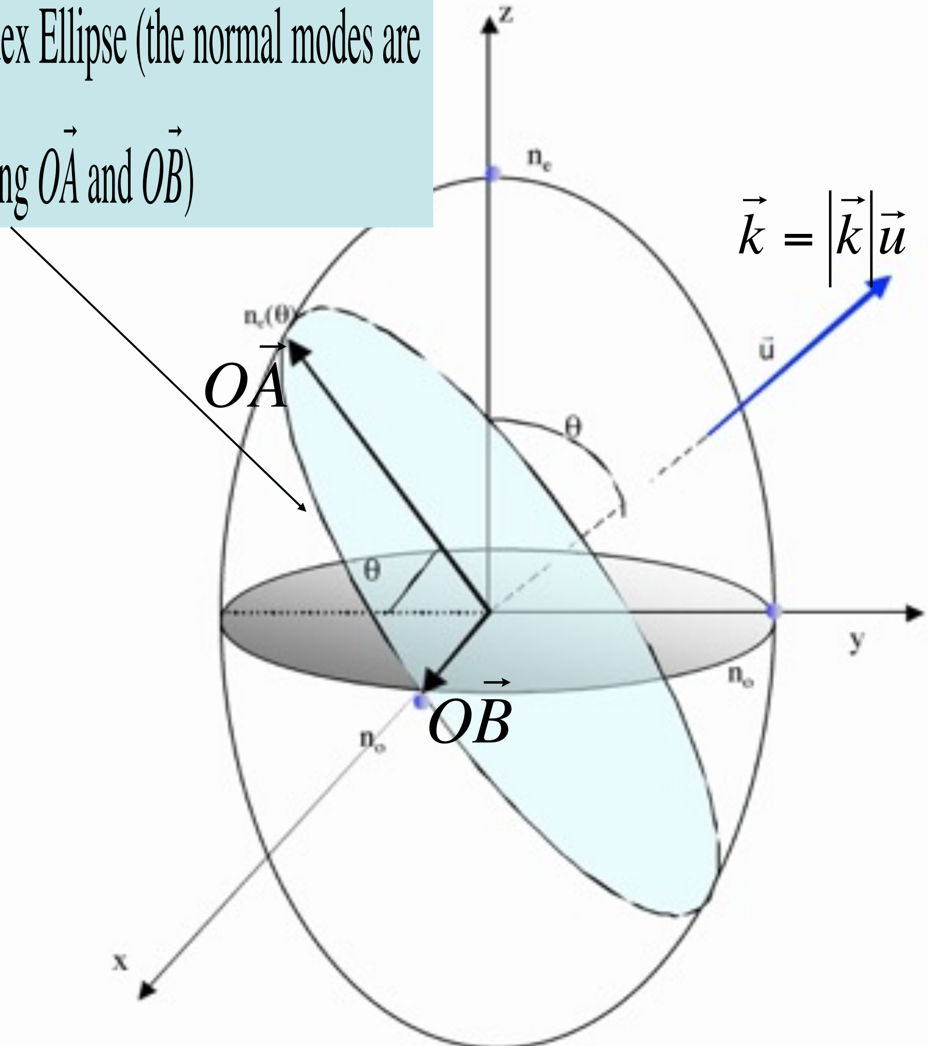
IV-Crystal Optics

- Propagation in arbitrary direction (case of uniaxial crystals only): ***k*** makes angle θ with respect to O_z (optic axis)
- The normal modes are two linearly polarised and orthogonal directions **OA** and **OB**.
- They form the semi-axes of the Index Ellipse and define the Ordinary (OB) and Extraordinary (OA) waves resp.
- O wave travels at c_0/n_O, E wave travels at $c_0/n_e(\theta)$ (uniaxial crystal).
- Simple geometry is used to calculate $n_e(\theta)$

IV-Crystal Optics

Index Ellipse (the normal modes are along $\vec{OA}$ and $\vec{OB}$)

$$\vec{k} = \left|\vec{k}\right|\vec{u}$$



The Index Ellipse for uniaxial crystals

To calculate the value of the index $n_e(\theta)$ of the E wave:

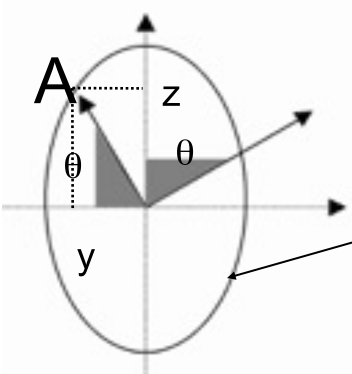
$$n_e^2(\theta) = z^2 + y^2$$

$$\frac{z}{n_e(\theta)} = \sin\theta$$

$$\frac{y^2}{n_O^2} + \frac{z^2}{n_e^2} = 1 \text{ (equ. of ellipse)}$$

Yield:

$$\frac{1}{n_e^2(\theta)} = \frac{\cos^2\theta}{n_O^2} + \frac{\sin^2\theta}{n_e^2}$$



Equation of ellipse

$$\frac{y^2}{n_0^2} + \frac{z^2}{n_e^2} = 1$$

IV-Crystal Optics

- For $\theta = 0$ (k along z), $n_O = n_e(\theta)$, there is no birefringent behaviour (hence the name uniaxial).
- Retardation plates have optic axis in the plane of the plate surface. The desired state of polarisation is obtained by adjusting the thickness (see p.64)

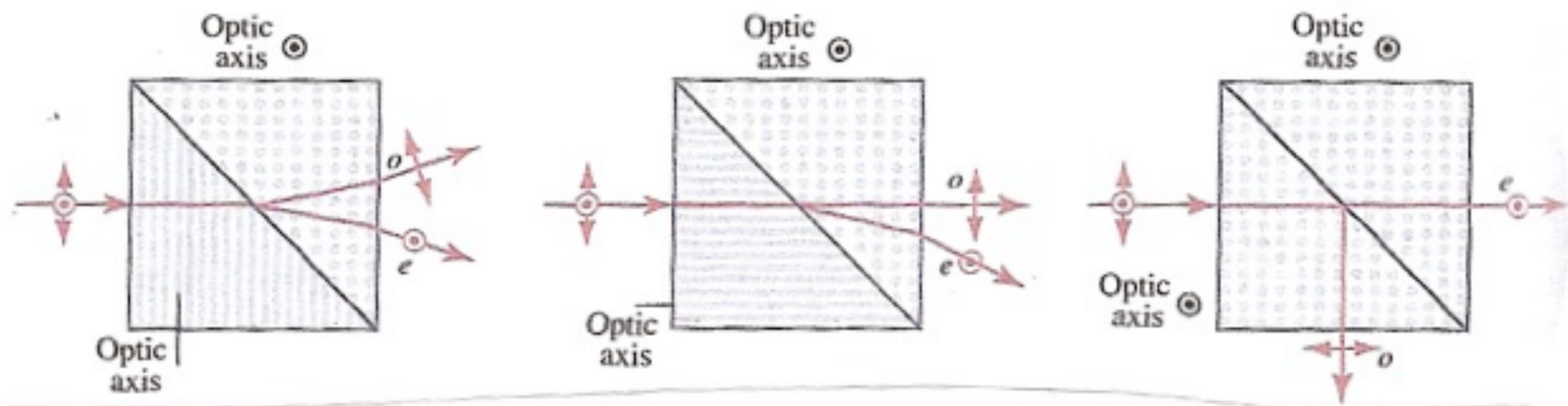
V POLARIZATION DEVICES: POLARIZERS

- Linear polarizer:
  - Transmits components of E field along the direction of its transmission axis
  - Blocks the orthogonal component
  - Can be achieved by:
    - Dichroic materials (selective absorption); Polaroid sheet
    - Selective reflection from isotropic media; Brewster’s angle
    - Selective reflection/refraction in anisotropic media; Polarizing beamsplitters

V POLARIZERS: POLARIZING BEAMSPLITTERS

- Ordinary and extraordinary waves refract at different angles in anisotropic crystal: polarized light can be obtained from unpolarized light.
- Typically two cemented prisms made of uniaxial materials with different orientations:
  - Wollaston prism
  - Rochon prism
  - Glan-Thompson prism

V POLARIZERS: POLARIZING BEAMSPLITTERS



Wollaston prism | **Rochon prism** | **Glan-Thompson prism**

O-ray is totally internally reflected at cement interface

V POLARIZERS: Wave Retarders

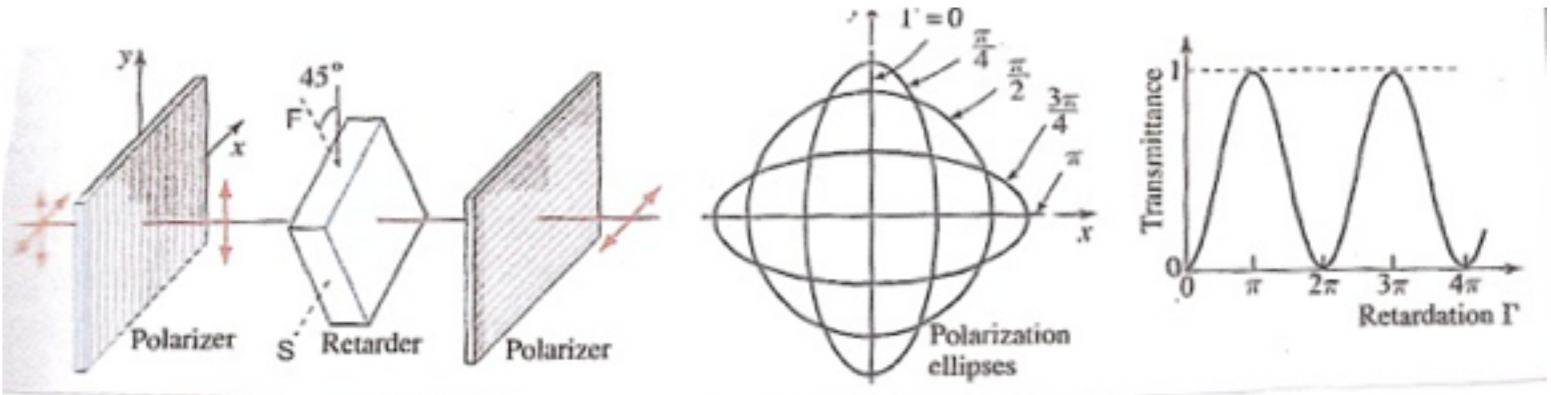
- Convert one polarisation into another
- Normal modes are linearly polarised along the fast n_f and slow n_s axes.
- Constructed from anisotropic materials in the form of plates: light is made to travel along one of the principal axis
- Retardation is directly proportional to plate thickness:

$$\Gamma = \frac{2\pi}{\lambda}\left(n_f - n_s\right)d$$

V POLARIZERS: Wave Retarders

- Retardation is directly proportional to the thickness of the plate
- Retardation is inversely proportional to the wavelength
- Thin sheet of mica:
  - Indices: 1.599 and 1.594 at 633 nm (He-Ne laser) $\Gamma / d \approx 15.8$ rad/mm
  - Sheet of 63.3 microns yields $\Gamma \approx \pi$ rad

V Wave Retarders: Light intensity control



- Wave retarder placed between 2 cross-polarisers whose axes are at 45 deg. with respect to the axes of the retarder.
- Intensity transmittance of this device is:

$$I_T = \sin^2(\Gamma/2)$$

- Intensity can be changed by altering the retardation (see Electro-Optics) via use of electro-optic anisotropic crystals