PS403 – Digital Signal Processing

Lectures & Tutorials

Will aim to cover materials in 30 classes.
(Total Contact Time: 30 hours)

We have 3 slots per week.

The recommended textbook is

DSP with Computer Applications
by
Paul A Lynn and Wolfgang Fuerst [2nd Edition – Wiley]

PS403 - Digital Signal processing

Indicative Syllabus

- Signal Sampling: Shannon's theorem, Nyquist concepts, etc
- Linear DSP systems: Scope, definitions and concepts
- Analysing DSP system in the time domain: responses, etc
- Analysing DSP system in the frequency domain: Discrete Fourier Series (DFS) and Discrete Fourier Transform (DFT)
- The Z-transform and its applications in DSP
- Non-recursive (Finite Impulse Response - FIR) filter design
- Recursive (Infinite Impulse Responses - IIR) filter design
- *The Fast Fourier Transform (FFT) and its applications in DSP*

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Lecture notes as pdf files on LOOP.

* Lecture notes will also be uploaded onto my webpage

<u>Research Absences</u>
<u>Semester 2, 2023</u>

None so far.

PS403 - Digital Signal processing

I. Signal Sampling and Averaging

Key Text:

Digital Signal Processing with Computer Applications (2nd Ed.)

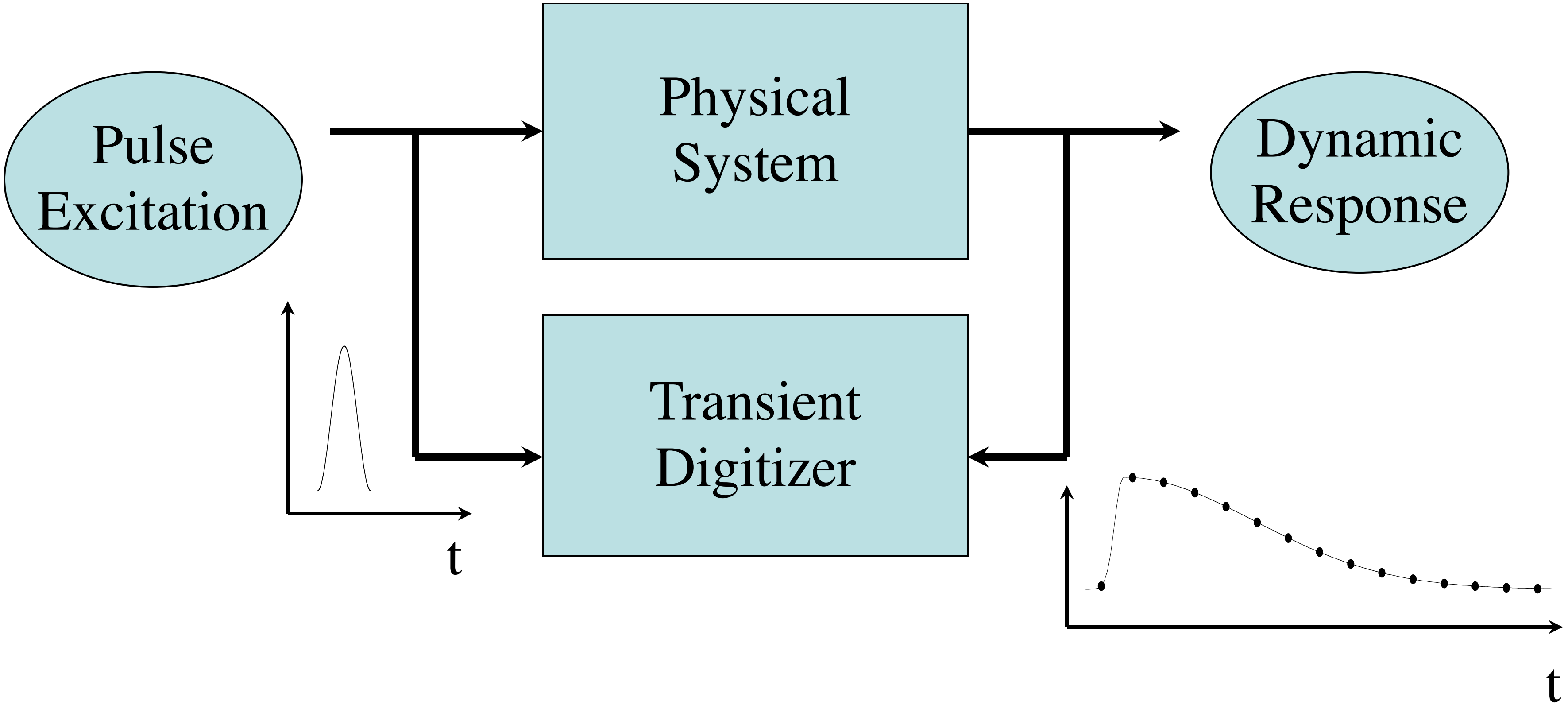
Paul A Lynn and Wolfgang Fuerst, (Publisher: John Wiley & Sons, UK)

We will cover in this section

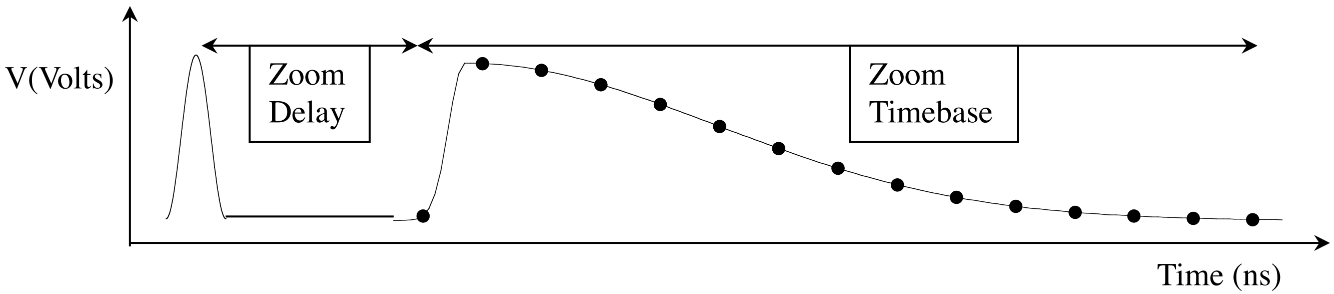
Signal Sampling and Averaging

Sampling Frequency and Shannon's theorem

Signal Sampling and Averaging



Terminology



Terminology

Zoom Delay: Time interval from trigger pulse to first sampled point on waveform

Zoom Timebase: Time interval form first to final sampled points

Sampling interval: Time interval between successive sampled points on the waveform

Frame: A complete set of stored waveform samples

Samples per point: The number of times that you sample a particular point on the waveform (used to improve signal to noise ratio)

Principle: when a noisy signal is sampled many times, the random noise component averages to zero while the signal component averages to its original form.

Point/Frame Averaging

$$s_j(t_i) + n_j(t_i)$$

Noise is additive !

Now sample each point 'M' times or equivalently make 'M' frames and add them together. The average value of the noisy signal at time t_i is:

$$\sum_{j=1}^{j=M}\left[\frac{s_j(t_i) + n_j(t_i)}{M}\right]$$

For a stable signal all values, $s_1(t_i) = s_2(t_i) = \ldots.\ s_j(t_i) = \ldots$ and so the signal just averages to itself. However the noise values are randomly distributed about zero and average to zero.

$$\sum_{j=1}^{j=M}\left[\frac{s_j(t_i)}{M}\right] = \overline{s(t_i)} = s(t_i)$$

$$\sum_{j=1}^{j=M}\left[\frac{n_j(t_i)}{M}\right] = \overline{n(t_i)} \rightarrow 0$$

Noise Power Reduction - Power SNR

Variance of input signal component (1 sample) = $$\sigma_{si}^2 = \overline{s_j^2(t_i)} - \overline{s_j(t_i)}^2 = 0$$

Variance of output signal (M sample average) = $$\sigma_{so}^2 = \sum_{j=1}^{j=M} \frac{\left[\overline{s_j^2(t_i)} - \overline{s_j(t_i)}^2\right]}{M} = 0$$

Variance of input noise (1 sample) = $$\sigma_{ni}^2 = \overline{n_j^2(t_i)} - \overline{n_j(t_i)}^2$$

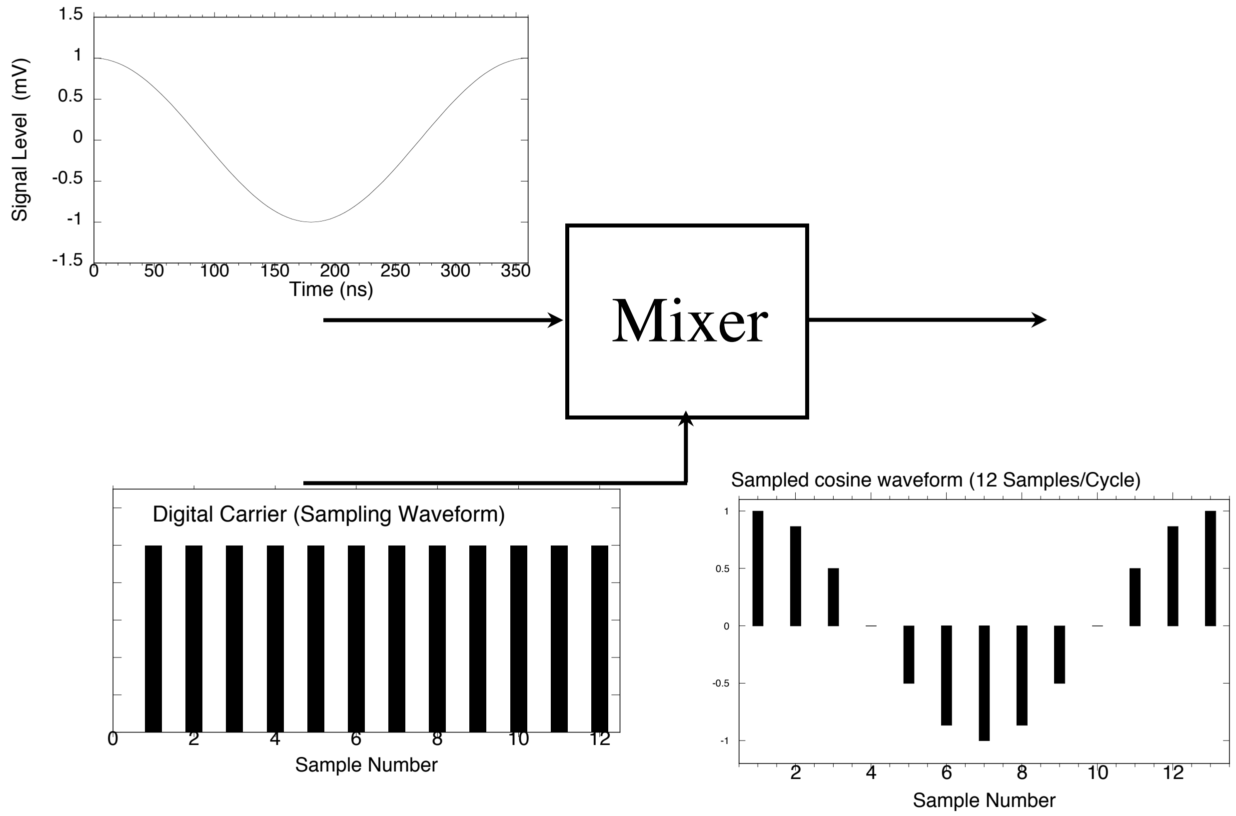
Variance of output noise (M sample average) = $$\sigma_{no}^2 = \sum_{j=1}^{j=M} \frac{\left[\overline{n_j^2(t_i)} - \overline{n_j(t_i)}^2\right]}{M}$$

So although $\overline{n_j(t_i)}^2 = \mathbf{0}$, $\overline{n_j^2(t_i)} \neq 0$ and hence $\sigma_{no}^2 = \dfrac{\sigma_{ni}^2}{M}$

When a noisy signal is sampled and averaged 'M' times, the signal power to noise (SNR) ratio is increased by a factor of M and the signal (voltage) to noise ratio by a factor of √M.

Shannon's Sampling Theorem

A waveform may be reproduced from its sampled form provided that the sampling frequency is greater than twice the highest frequency component of the waveform



Sampling Frequency

$$f_s = \frac{1}{T_s}$$

T_s is the so-called 'sampling interval'.

$$A_0 + \sum_{n=1}^{n=\infty} A_n Cos(2\pi n f_s t)$$

Fourier expansion of the 'digital carrier'

$$A_1 Cos(2\pi f_m t)$$

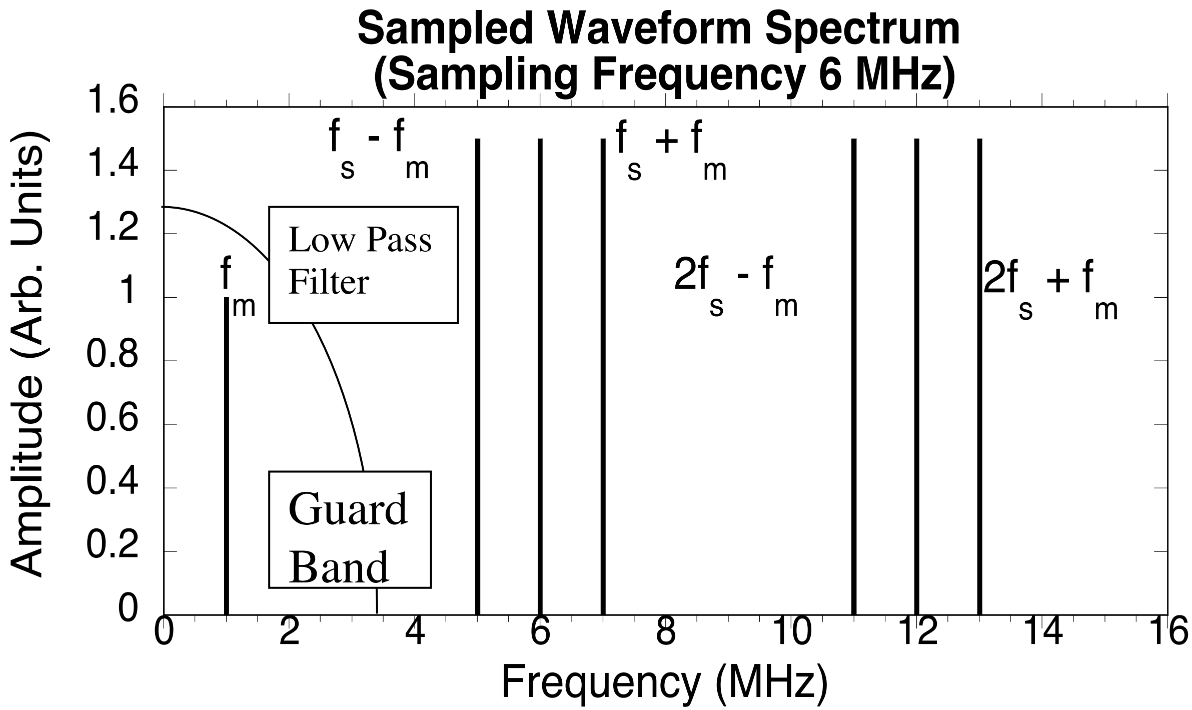
System dynamic response - 'info signal'

So the sampled waveform looks like

$$A_1 A_0 Cos(2\pi f_m t) + \sum_{n=1}^{n=\infty} \frac{A_1 A_n}{2} \left[Cos 2\pi(nf_s + f_m)t + Cos 2\pi(nf_s - f_m)t \right]$$

So the sampled waveform consists of the original waveform at frequency f_m plus a series of with sidelines (bands) centred about the sampling frequency and its harmonics.

Sampled Sinusoid Spectrum



In the example above the 'information signal' is simply a single tone at a frequency f_m = 1 MHz - ironically this standard example uses the archetypal 'no information' signal. The sampling frequency is 6 MHz or 6 Samples/ Cycle.

So in order to reproduce the original cosine waveform at f_m = 1MHz we simply need to low pass filter the sampled waveform

Aliasing

However we have to ensure that the cutoff frequency of the filter lies between f_m and f_s - f_m so that we pass the 1 MHz signal and reject all high frequencies, i.e., in order to reproduce the original signal from its sampled form we require $\mathbf{f_m < f_s - f_m}$ or: $f_s > 2f_m$

In practice a real signal will have to contain frequencies up to some cutoff f_{max} and so Shannon's theorem generalizes to: $f_s > 2f_{\max}$

Terminology

Sampling Interval: Time between successive samples on a waveform

Sampling frequency: Reciprocal of the sampling interval

Guard band: the frequency interval between the highest frequency component on the signal to be sampled and first lowest sideband.

Aliasing: Occurs when the guard band drops to zero or goes negative, i.e., when:

$$f_s \leq 2f_{\max}$$

We should also consider impulse response and deconvolution (signal restoration) in this section but time will not permit – defer to another day.