OPTICAL TWEEZERS AND SPANNERS

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**Introduction:**

Optical tweezers are a technique for the non-intrusive manipulation of macroscopic objects by laser light. The laser light is collimated and directed towards the sample object which then can be trapped in the waist of the beam. If the force exerted by the laser beam is large enough it is possible to move objects with great precision by moving the beam slightly. Arthur Ashkin, a physicist at the Bell Telephone Laboratories in the USA, pioneered the technique in the early 1970’s. A variety of methods were developed by Ashkin and his team that used laser beams to manipulate micron-sized particles. Eventually they were able to trap particles in 3-D with a single laser beam and this was the modern optical tweezers. Ashkin was presented with the Rank Prize in 1993 in recognition of this achievement.

Optical spanners are a relatively new development in the physics world. In 1996, Miles Padgett & Les Allen from the University of St. Andrews in Scotland [1] discovered a new dimension to the optical tweezers – particles could be rotated. First proposed in 1994, it was this added degree of rotational control that led to the term “Optical spanner”. An optical spanner only works with a special type of laser beam called a Laguerre-Gaussian beam. These beams have a circular cross section, but do not have any intensity along the beam axis, so the beams cross-sectional profile is ring shaped. They have a helical wavefront and their Poynting vector spirals about the axis of the beam. The Laguerre-Gaussian beam is described in greater detail later on.

In this presentation we attempt to give the reader a picture of how these instruments actually work, initially through a simple ray optics picture and then through a more qualitative & rigorous discussion of the intensity gradient associated with a laser beam that is experienced by a sample which is really the more accurate analysis.

![Fig. 1 – The experimental layout of an optical spanner.](M.J Padget et al, Optics Letters Vol.22 No.1 1 January 1997.)
**The Physics**

The physics can be explained with a complicated mathematical analysis using Maxwell equations and quantum theory. A simple ray optics model will also provide a certain degree of understanding considering that a ray of light carries momentum in its propagating direction.

Momentum must, as always be conserved in this model. If we look at the ray shown in Fig.1, we see that it is travelling purely in the y-direction with no x component. All it’s momentum is directed in the y-direction.

![Fig.2 Light Ray Momentum ρ](image)

In Fig.3 we see the same ray as in Fig.2 meet a sphere. These spheres in experiment are usually made of Teflon or silica and they are surrounded by a medium of lower refractive index. These spheres mimic the properties of living transparent cells well. The lower refractive index will cause the ray to refract into the sphere. As we can see from the diagram the ray takes on momentum in the x-direction once it makes contact with the sphere. But there was no initial momentum in this plane so the sphere moves in the opposite direction to the ray to compensate and keep the x-directed momentum equal to zero.

![Movement of sphere once interacts with the ray.](image)

The momentum of a wave is linked to the De Broglie hypothesis which relates the wavelength to the momentum. Each photon carries momentum (proven to be in the direction of the ray) so the momentum of the ray is proportional to the intensity.
Ray 1       Ray 2

In Fig.4, Ray 1 carries more momentum than Ray 2. So given that the angles of incidence/refraction for each of the rays are the same then the sphere will move to the left simply because there is more momentum to ‘compensate’ for from the bending of Ray 1 than Ray 2.

We can generalise the behaviour in Fig.3 and say that the transparent particle will always move with the light intensity gradient i.e. it will always move towards the regions of greater intensity. This is the basis of the optical tweezers. It uses a tightly focused laser beam passed through a microscope with its lenses. The beam converges to its characteristic beam waist (Fig.5a) radius which is around 1 wavelength in diameter with high quality equipment. The intensity profile of the beam has a Gaussian shape (Fig.5b) near this region and this produces the intensity gradient that is needed to keep the particle in place.

The intensity gradient of the beam is toward the beam waist and the spheres will feel a force which keeps them tightly bound in this region.

There are complications involved with scattering and reflections from the sphere. Any light that is reflected will cause the sphere to gain a momentum in the opposite direction to the reflected wave. Most of the reflected light travels back along the beam and this gives the sphere a forward momentum. This is a design consideration: how to over come this force (from reflection) that drives the particle away along the beam path. One way that this has been overcome is with a Laguerre-Gaussian beam. These beams have a circular cross-section but the intensity is zero along the beams axis of propagation On-axis rays do not contribute to the axial trapping force so obviously their removal doesn't affect the trapping force. Therefore there is no intense central light to be back reflected. In practice, the steepest possible light gradients are required so that the gradient force is greater than the scattering force at the beam waist, thus ensuring that the sample object will not be thrown out of the beam as described above.

Looking at a more rigorous analysis of the Optical Tweezer, we account for the effect momentum transfer from photons to the spheres and the Lorentz force that the electromagnetic radiation exerts on dipoles within the material. An equilibrium of the two effects leads to trapping.
The electromagnetic force on a dipole in a polarizable material is \( f(r,t) \):

\[
f(r,t) = p(r,t)\nabla E(r,t) + \frac{\partial p(r,t)}{\partial t} \times B(r,t)
\]

where \( p \) is the dipole moment, \( E \) is the electric field and \( B \) is the magnetic field of the incident wave. A derivation of this result is given in the appendix (1).

The approach to understanding these effects mathematically adopted by Barton et al says that the net radiation force on the sphere is the integral of the dot product of the outward directed vector \( n \) and Maxwell’s stress tensor \( T \) over a surface around the particle.

\[
\langle F \rangle = \int n T dS , \quad \langle F \rangle \text{ denotes the time average of the force.}
\]

\[
T = \frac{1}{4\pi} \left[ \varepsilon E E + H H - \frac{1}{2} \left( \varepsilon E + H \right)^2 \right] I
\]

\( T \) and \( I \) are vector tensor quantities along with \( E \) and \( H \) (\( H \) is the magnetic field).

An interesting outcome of continuing this analysis is that the optical power needed varies as a function of the diameter of the trapped sphere. (Especially when the particle is optically levitated) The power drops at certain diameters that correspond with the excitation of certain radial modes of the general EM wave solution. The wavelengths involved are integral numbers of the given diameter. For further reading see “Theoretical determination of the Net Radiation..”[2].

The physics of the Optical Spanner cannot be explained well with ray diagrams. This only works with a special laser beam called a "Laguerre-Gaussian" beam. These beams have a circular cross-section but the intensity is zero along the beams axis of propagation. This gives the beam a ringed shaped profile as shown in Fig. (6a) and its wavefronts are helical in shape, Fig. (6b). This means that the beams phase changes with its position on the wavefront. \( S \) is the Poynting Vector. See appendix (2). This beam is usually obtained by converting a Hermite-Gaussian beam using cylindrical lenses.

![Fig. (6a)](image)
![Fig. (6b)](image)

(From “Optical Tweezers & Spanners” – Physics World September 1997.)

The spanner transfers some of the angular momentum that its beams carry to the sphere. It involves the fact that a photon carries an intrinsic angular momentum called the orbital angular momentum. However, in a light beam further angular momentum can be added by elliptically polarising the beam. Experimentally this has been done by using a quarter-wave-plate [3]. This extra momentum is termed the spin angular momentum. In a circularly polarised beam the extra momentum comes as \( L h \) where \( L \) is the number of times that the phase of the wave changes by \( 2\pi \) moving once around the wavefront. Using a Laguerre-Gaussian beam with \( L = 1 \) (for which the orbital angular momentum is \( h \)) the sum of the angular and orbital momentum are between 0, \( h \) and \( 2h \). Using the quarter-wave-plate with the microscope-spanner set up we can watch \( h \) a specimen (often a silica sphere) rotate with speeds proportional to 0, \( h \) and \( 2h \). That is with zero rotational speed, a given rotational speed and twice that speed. At the zero rotational speed, we can see that the orbital and spin angular momentum of the photons cancel each other. This has been cited as proof that the orbital
Applications:

Optical tweezers, spanners and scalpels have found many uses in the biotech laboratory, mainly because the forces involved (~ piconewtons) are very similar to the kind of forces experienced by biological cells. But they also have found other applications too.

The laser scalpel is the latest tool to be developed from this 'family' and is based upon a pulsed laser beam whose intensity is very carefully controlled. The scalpel can be used to destroy selected cells in various micro-organisms, cut DNA or open cells walls.[5]

One of the big advantages with the laser tweezers is the fact that the laser light is sterile, which is vital when dealing with micro-organisms. Scissors can be used to cut open a virus or a cell and organelles can be added or removed with the tweezers. In an age or genetic engineering such technology is very useful. If for example, a sample is put within a sealed cell it gives scientists an intrinsically sterile laboratory in which they are able to manipulate it in any way they wish at absolutely no risk to themselves or others. In addition, as the power of the laser beam can be minutely controlled and the trapping force acts on a large volume of the target object, not just on one specific spot of its surface no excessive force has to be applied to fragile and vulnerable biological systems.

A variation of this technique is currently used in the Scottish Parasite Diagnostic Laboratory to study and sort parasites from water samples. Eventually they hope to be able to isolate specific parasites and move them to a particular area of a petri dish where they can later be removed for analysis.[6]

The only disadvantage about these laser tools is the risk of damaging or destroying the sample cells due to excessive laser power, a process called ‘optiction’. In practice, trapping lasers focus from 10 mW to 1 W of light into the microscope, producing huge fluxes at the sample plane, ranging from 106–108 W/cm2. To diminish the possibility of optical damage called “optiction” arising from these enormous power levels, a laser whose wavelength lies in the near infrared region is generally used. This is because visible light is heavily absorbed in naturally occurring pigments found in biological material, while far infrared light is absorbed by water. Most biological specimens, but not all are fairly transparent in between, over the near infrared region from 700–1300 nm. For this reason, the Nd: Yag laser with a wavelength of 1064nm is used. Recently, Berns et al [7] conducted an investigation into "optiction" over a range of wavelengths using a titanium-sapphire laser. They showed that at 700 nm the laser produced significantly less damage than the Nd: Yag laser with a wavelength of 1064nm, which is good for future experiments that will require higher power or smaller sample sizes. There are still some problems to be ironed out though before this method becomes universally adopted.

There are a few limiting factors for the optical spanner. These mainly relate to the sample shape and the viscosity of the liquid that the sample is immersed in. The angular velocity is limited mainly by the rotational speed which increases the drag force on the specimen in the liquid. So a sphere of radius r undergoing a torque $\tau$ has a limited angular velocity $\omega_{\text{lim}}$ given by $^{[8]}$:

$$\omega_{\text{lim}} = \frac{\tau}{8\pi\eta r^3}$$

where $\eta$ is the Reynolds number which determines the viscosity of the liquid.
A more physics related application of this technology is **Scanning Force Microscopy**. The mechanical cantilever holding the stylus in conventional scanning force microscopy can be replaced with a single beam optical trap. Scanning force microscopy involves scanning a sharp stylus across a sample. The movement of the tip is due to tip surface interactions…it gives an image of the surface. The resolution of the instrument is highly dependent on the resonance frequency and a lowest possible spring constant of the cantilever system. The optically trapped particle will move across the surface with a lower spring constant $10^{-4}$-$10^{-5}$ N/m. (Conventional constants are $\sim >10^{-3}$) This will much higher resolution but the system can only be scanned over transparent subjects.

Ken Mackay, working at the CRNS Laboratoire in France is currently studying how these optical tools may be used to move magnetic particles. His goal is to create tiny electromagnets capable of producing magnetic fields of 10 Tesla or more. Current magnets of this strength are very expensive and large, but he hopes that his will be around the size of an average printed circuit board.

The ability to non-intrusively and non-destructively manipulate various types of biological cells has opened up a huge range of future applications. In particular, optical tweezers and scalpels offer new approaches to some biological problems which are not possible with current methods. The possibilities are endless, and clearly the future is bright for these optical tools.

![Fig.7 - An example of the precision achievable with optical tweezers.](http://www.phys.umu.se/laser)
Appendix:

1. Derivation of $f(r,t) = p(r,t)\nabla E(r,t) + \frac{\partial p(r,t)}{\partial t} \times B(r,t)$ \hspace{1cm} (1)

This equation is divided up into 2 parts. On the RHS the first term comes from the Electric fields effect on the dipole, the second term belongs to the Lorentz force effect. The Electric field force term is derived below.

The electrostatic force $f_x$ due to the interaction of the EM radiation’s Electric field is given by

\[
 f_x = q \left[ E \left( \frac{\partial E}{\partial x} + \frac{\partial E}{\partial y} + \frac{\partial E}{\partial z} \right) \right] = q \left( \frac{\partial E}{\partial x} + \frac{\partial E}{\partial y} + \frac{\partial E}{\partial z} \right) = p \nabla E
\]

A similar analysis will yield the Lorentz term given in (1) above.

2. The Poynting vector is the flow of energy per unit area and per unit time through a cross-sectional area perpendicular to the wave's propagation direction. Due to the high frequencies electromagnetic waves exhibit, the time variation of the Poynting vector changes rapidly so we look at the average value of it. This gives us the intensity of radiation at this point.

\[
 I = S_{AV} = \frac{(EB)_{MAX}}{2\mu_0c}
\]

(Intensity in a vacuum)

Where:
- $\mu$ = Magnetic moment.
- $C$ = Speed of light.
- $B$ = Magnetic field strength.
- $E$ = Electric field strength.
Bibliography:


