Physics is one of the most fundamental sciences: it is the foundation of all engineering and technology, e.g. electronics, computing, etc...

Physics is an experimental science: natural phenomena or physical systems, e.g. light propagation, free fall, ... are measured and quantified with numbers.

A model is then constructed from basic principles (basic physical laws) that should provide numbers equal to the measured ones and predict the behaviour of the system in different conditions.

I-1 UNITS

All quantities are measured with respect to some reference standard, e.g. metres, seconds, amps, etc... Such a standard defines a UNIT of the quantity.

The currently admitted system of units is called the SI system. Two types: the base units and the derived units (see p.1453)

<table>
<thead>
<tr>
<th>Physical quantity</th>
<th>SI base Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>length (L)</td>
<td>metre (m)</td>
</tr>
<tr>
<td>mass (M)</td>
<td>kilogram (kg)</td>
</tr>
<tr>
<td>time (T)</td>
<td>second (s)</td>
</tr>
<tr>
<td>electric current (I)</td>
<td>ampere (A)</td>
</tr>
<tr>
<td>temperature (T)</td>
<td>kelvin (K)</td>
</tr>
<tr>
<td>amount of matter</td>
<td>mole (mol)</td>
</tr>
</tbody>
</table>

Derived units, e.g.: Volt (V), Ohm (Ω), ...

In some countries (USA) or some engineering applications, the Imperial (British) system of units is still in use: inch, mile, pound, BTU (British Thermal Unit), ... Try to think directly in SI units!

All the derived units can be expressed in terms of the base units. e.g.,:

<table>
<thead>
<tr>
<th>Force (Newton, N)</th>
<th>N = m.kg.s⁻²</th>
</tr>
</thead>
<tbody>
<tr>
<td>Resistance (Ohm, Ω)</td>
<td>Ω = V.A⁻¹</td>
</tr>
<tr>
<td>Capacitance (Farad, F)</td>
<td>F = C.V⁻¹=A.s.V⁻¹</td>
</tr>
<tr>
<td>Force (Newton, N)</td>
<td>N = Kg.m.s⁻²</td>
</tr>
</tbody>
</table>
This is extremely useful to check that a physical formula is correct. 

*Detailed Example: ENERGY, SI units JOULE.*

\[ \text{[Energy]?} \]

We know that kinetic energy \( K \)
\[ K = \frac{1}{2}mv^2, [K] = M(\text{L}T^{-1})^2 = ML^2T^{-2} \]

**Use this result to show that the product of the gas pressure \( P \) by the volume \( V \) of its container is equivalent to energy.**

\[
\begin{align*}
PV &= \text{pressure} \times \text{volume} = \frac{\text{force}}{\text{area}} \times \text{volume} \\
&= \frac{\text{mass} \times \text{acceleration}}{\text{area}} \times \text{volume} \\
&= \frac{M(\text{L}T^{-2})}{\text{L}^2} \times \text{L}^3 = ML^1T^{-2}L^1 = ML^1T^{-2}
\end{align*}
\]

- Any equation relating physical quantities must therefore be dimensionally consistent. Terms can only be added or equated if they have the same units. e.g.

\[
\text{distance (m) = speed (\frac{\text{m}}{\text{s}}) x time (s)}
\]

- All *measured* quantities carry an error (*uncertainty*). The standard notation is:

\[
(\text{value} \pm \text{error})\text{units}
\]

| e.g., \((54.784 \pm 0.014)\text{mm}\)

- The above result is given with 5 significant figures, i.e. the number of meaningful digits.

- It is essential to **round off numbers** in order to preserve the correct number of significant figures.
Example: speed of 20.7 m/s during 1.74 s. Calculate the distance travelled?

\[ D = 20.7 \text{ m/s} \times 1.74 \text{ s} = 36.018 \text{ m} \] (5 significant figures).

Acceptable answers would be:
36.0 m (3 significant figures) or 36.02 m (4)

• Very often results are expressed using scientific notation with powers-of-ten.
  e.g. 384,000,000 m = 384 million metres = \( 3.84 \times 10^8 = 384 \times 10^6 \text{ m} \)

### Units Prefixes

<table>
<thead>
<tr>
<th>Prefix</th>
<th>Symbol</th>
<th>Factor</th>
</tr>
</thead>
<tbody>
<tr>
<td>exa</td>
<td>E</td>
<td>(10^{18})</td>
</tr>
<tr>
<td>peta</td>
<td>P</td>
<td>(10^{15})</td>
</tr>
<tr>
<td>tera</td>
<td>T</td>
<td>(10^{12})</td>
</tr>
<tr>
<td>giga</td>
<td>G</td>
<td>(10^9)</td>
</tr>
<tr>
<td>mega</td>
<td>M</td>
<td>(10^6)</td>
</tr>
<tr>
<td>kilo</td>
<td>k</td>
<td>(10^3)</td>
</tr>
<tr>
<td>hecto</td>
<td>h</td>
<td>(10^2)</td>
</tr>
<tr>
<td>deca</td>
<td>d</td>
<td>(10^1)</td>
</tr>
<tr>
<td>centi</td>
<td>c</td>
<td>(10^{-2})</td>
</tr>
<tr>
<td>milli</td>
<td>m</td>
<td>(10^{-3})</td>
</tr>
<tr>
<td>micro</td>
<td>µ</td>
<td>(10^{-6})</td>
</tr>
<tr>
<td>nano</td>
<td>n</td>
<td>(10^{-9})</td>
</tr>
<tr>
<td>pico</td>
<td>p</td>
<td>(10^{-12})</td>
</tr>
<tr>
<td>femto</td>
<td>f</td>
<td>(10^{-15})</td>
</tr>
<tr>
<td>atto</td>
<td>a</td>
<td>(10^{-18})</td>
</tr>
</tbody>
</table>

• See page 5 for typical orders of magnitude, e.g.
  - size of a molecule: \(1 \text{ nm}\)
  - earth’s radius: \(6\times10^6 \text{ m}\)

### I-2 VECTORS AND ELEMENTARY VECTOR ALGEBRA

• Physical quantities are either:
  - **SCALAR**, they are described completely by a single number with a unit. e.g., mass, time, density,...
  - **VECTOR**, they also have a direction associated with them. A vector is characterised by a direction and a magnitude (+ a starting point and an endpoint).

Example: the **displacement** vector
• Quite clearly the displacement vector is not equal to the total distance traveled.

• We have: \( \vec{A} = \vec{B} = -\vec{C} \)

• The magnitude (modulus) of a vector is its “length”
The notation is: Magnitude of \( \vec{P} = P = |\vec{P}| \)

• It is always a positive quantity and is not equal to the vector itself!

• Two successive displacements (\( \vec{A} \) and \( \vec{B} \)) are equivalent to one vector \( \vec{C} \) such that:

\[
\vec{C} = \vec{A} + \vec{B},
\]

\( \vec{C} \) is called the vector sum or resultant of \( \vec{A} \) and \( \vec{B} \).

• Geometric construction:
• One has $\vec{C} = \vec{A} + \vec{B} = \vec{B} + \vec{A}$. However, $|\vec{C}| \neq |\vec{A}| + |\vec{B}|$

• **Vector subtraction** is defined similarly. Noting that:
  \[ \vec{D} = \vec{A} - \vec{B} = \vec{A} + (-\vec{B}) \]

• A vector can be multiplied by a scalar quantity:
  \[ \vec{M} = 3\vec{A} = \vec{A} + \vec{A} + \vec{A} \]

• The displacement $\vec{M}$ is three times as long as the displacement $\vec{A}$

• A vector is measured in space with respect to a Cartesian (rectangular) system of coordinates:

• $\vec{i}$ and $\vec{j}$ are unit vectors along $0_x$ and $0_y$ resp.

• $A_x$ and $A_y$ are called the **components** of $\vec{A}$ along $0x$ and $0y$. 
• Vectorially: \( \vec{A} = A_x \hat{i} + A_y \hat{j} \)

• The positive direction for measuring the angle \( \theta \) is from \( 0_x \) anticlockwise. Thus, from basic trigonometry:

\[
\frac{A_x}{A} = \cos \theta \quad \text{and} \quad \frac{A_y}{A} = \sin \theta
\]

• From Pythagoras, we get the magnitude of \( \vec{A} \) in terms of its components: \( |\vec{A}| = \sqrt{A_x^2 + A_y^2} \), and its direction with respect to \( 0_x \):

\[
\tan \theta = \frac{A_y}{A_x} \quad \text{and} \quad \theta = \arctan \frac{A_y}{A_x}
\]

• One should be careful when using inverse circular functions (arccos, arcsin, arctan). See example on page 15.

• The vector sum can now be written as a function of the basic components:

\[
\vec{R} = \vec{A} + \vec{B} = (A_x + B_x) \hat{i} + (A_y + B_y) \hat{j}
\]

\[
\begin{align*}
R_x &= A_x + B_x \\
R_y &= A_y + B_y
\end{align*}
\]

i.e.,

• The SCALAR or dot product of two vectors \( \vec{A} \) and \( \vec{B} \) is defined by:

\[
\vec{A} \cdot \vec{B} = AB \cos \phi = |\vec{A}| |\vec{B}| \cos \phi
\]

The scalar product of two vectors is a **number**

• We also have (dot product in terms of components): \( \vec{A} \cdot \vec{B} = A_x B_x + A_y B_y \), since \( \hat{i} \cdot \hat{j} = 0 \)
• We can generalize all the previous results in a **three-dimensional system of coordinates** (space):

![Diagram of three-dimensional coordinates](image)

• In particular, we have:

\[
\vec{A} = A_x \hat{i} + A_y \hat{j} + A_z \hat{k}
\]

\[
\vec{B} = B_x \hat{i} + B_y \hat{j} + B_z \hat{k}
\]

\[
\vec{A} \cdot \vec{B} = A_x B_x + A_y B_y + A_z B_z
\]

\[
\vec{A} \cdot \vec{B} = AB \cos \Phi
\]

• The **VECTOR or CROSS** product of two vectors is denoted by:

\[
\vec{C} = \vec{A} \times \vec{B}
\]

• It is a **vector**. Its direction is **perpendicular to both** \(\vec{A}\) and \(\vec{B}\).

![Diagram of cross product](image)

• The magnitude of the cross product vector is \(\vec{C} = AB \sin \phi\)

• Unlike the dot product, the cross product is such that:

\[
\vec{C} = \vec{A} \times \vec{B} = -\vec{B} \times \vec{A}
\]
The direction of $\hat{C}$ is found by using the right-hand-rule.

Since a large number of the physical quantities used in Physics are vectors, the dot product and cross product play essential roles in the description of their properties.

e.g., Work done by a force, Torque, Angular Momentum, Electromagnetism (Maxwell’s equations), etc...