EXPERIMENT P4

INVESTIGATION OF ELASTICITY

Objectives

1. To investigate the elasticity of various materials by examining to what extent they obey Hooke's Law.
2. To investigate periodic motion for a spring.

Background

Elasticity is a physical property of material objects that determines how easily an object may be deformed by stretching, bending, or compressing and still return to its original shape. It is said to be more elastic if it restores itself more precisely to its original configuration. A rubber band is easy to stretch, and snaps back to near its original length when released, but it is not as elastic as a piece of piano wire. The piano wire is harder to stretch, but would be said to be more elastic than the rubber band because of the precision of its return to its original length. A real piano string can be struck hundreds of time without stretching enough to go noticeably out of tune.

A spring is an example of an elastic object - when stretched, it exerts a restoring force which tends to bring it back to its original length. Automobile suspensions, playground toys and even retractable ball-point pens employ springs. Most springs have an easily predicted behaviour when a force is applied i.e. as the spring is extended or compressed. Hooke's Law, as commonly used, states that the force F a spring exerts on a body is directly proportional to the displacement Δl of the system (extension of the spring).

$$F = -k\Delta l$$

where k is the spring constant and the magnitude depends on the spring, being large for stiff springs and small for easily stretched springs. For wires or columns, the elasticity is generally described in terms of the amount of deformation (strain) resulting from a given stress (Young's modulus).

Hooke's Law applies as long as the material stress (applied force) does not pass a certain point known as its proportional limit. Beyond this point there is no longer a linear relationship between the applied force and the spring extension, but up a point called the *elastic limit* the spring will still return to its original length once the force is removed. However, if the spring is stretched <u>beyond</u> its elastic limit, it does not return to its original length upon removal of the applied force but remains permanently deformed (like bending a paper clip).

Now answer questions A1 through A2 on the answer sheet

Experiment 1: Hooke's Law

Materials for which the deformation is proportional to the applied force are said to obey Hooke's Law. It is the objective of this present experiment to examine a range of materials generally classified as elastic and to investigate to what extent they obey Hooke's Law.

Apparatus

Rubber cord	Metre stick	mass hanger
Steel spring	Retort stand	10 g and 100 g masses

Procedure

1. Fix one end of the rubber cord to the retort stand and the other to a mass hanger.
2. Measure the relaxed length l of rubber cord with no mass attached.
3. Determine a suitable range of masses over which to measure the extension of the rubber cord.
4. Successively apply the masses (M) onto the rubber cord and record the length l and the extension (Δl) in each case to get at least 8-10 readings.
5. Take care not to overload the cord otherwise permanent deformation can result.
6. Plot a graph of extension (in mm) versus mass (in kg) for the rubber cord. Indicate on the graph the region (if any) where Hooke's Law is satisfied. For this region, measure the slope (S) in mm kg^{-1}.
7. Connect the steel spring to the retort stand and repeat steps 1-6 for the steel spring.

Note: Graphs should be plotted in accordance with the guidelines given in Appendix 1.

Now answer questions A3 through A9 on the answer sheet

Analysis

When a mass M is suspended from a spring, the spring extends by an amount Δl so that the downward force (Mg) is balanced by the upward restoring force Fs of the spring (see Figure 1). The mass is at rest in its equilibrium position.

i.e. $Fs = -Mg$

Because extension is linearly proportional to load (as you discovered in Expt. 1 -- Hooke's Law) it follows that the restoring force Fs must be linearly proportional to extension. Thus,

$$Fs = -k\Delta l$$

$$\therefore \quad Mg = k\Delta l$$

$$\text{i.e.} \quad \Delta l = (g/k)M$$

k is a constant -- termed the spring constant

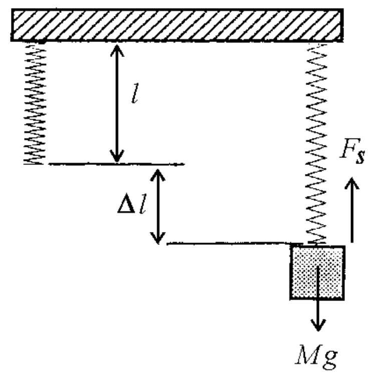

Figure 1. Spring at equilibrium.

Therefore the slope of the graph Δl vs M (as plotted in Experiment 1) enables the ratio g/k to be determined.

Experiment 2 Periodic Motion

If an elastic object is stressed and released it will oscillate periodically about its equilibrium or rest position. Examples of such objects are a musical string instrument, a saw blade clamped at one end or a mass attached to a spring.

It is the purpose of this experiment to investigate how the periodic time of an oscillating spring may be used to deduce a value for g, the acceleration due to gravity.

Apparatus

Steel spring	Stopclock	mass hanger
Metre stick	Retort stand	100 g and 10 g masses

Procedure

1. Set up the steel spring on the retort stand and attach a 100 g mass. Measure the time for 30 oscillations and take the average value to get the periodic time T. Include the mass of the hanger in your value of M.
2. Gradually increase the mass M, measuring T in each case. Record 8-10 readings taking care not to over-stretch the spring.
3. Plot a graph of T^2 (in s^2) vs M (kg) on graph paper.
4. From the slope of the T^2 vs M graph, determine g the acceleration due to gravity using Equation 4. (S is already known from Experiment 1).
5. From the intercept determine the effective mass m of the spring. Pay particular attention to units here.
6.

Now answer questions A10 through A17 on the answer sheet

Analysis

The motion of a body that oscillates back and forth is defined as *Simple Harmonic Motion* if there exists a restoring force F that is opposite and directly proportional to the distance x that the body is displaced from its equilibrium position. If Hooke's Law holds for a spring, then the motion of masses vibrating up and down on the spring should be simple harmonic motion. If the mass, when hanging from the spring, is given a small additional displacement x from its equilibrium position and then released, the spring will exert a net force $F = -kx$ which tends to restore the mass to its equilibrium position. The constant of proportionality k is called the spring constant and can be found by subjecting the spring to an applied force and measuring the amount that the spring stretches.

It is clear that at all positions of the mass's motion, the net force on the mass M is directed towards the equilibrium position. As a result the mass M undergoes repetitive vertical oscillations about the equilibrium position. The periodic time for these vibrations may be determined in the following way.

$$F = M\, d^2x/dt^2 \qquad \text{(Newton's 2nd Law)} \quad \text{and} \quad F = -kx$$

$$\therefore \qquad M\, d^2x/dt^2 = -kx$$

$$\text{or} \qquad M\, d^2x/dt^2 + kx = 0 \qquad (1)$$

We can show, by direct substitution, that $x = A \sin\omega t$ satisfies Equation 1 if A (maximum Amplitude) and ω (angular frequency) are constants.

$$dx/dt = \omega A \cos \omega t$$

and

$$d^2x/dt^2 = -\omega^2 A \sin \omega t$$

Therefore Equation 1 becomes

$$-M\omega^2 A \sin \omega t + kA \sin \omega t = 0$$

If this is to be true for all t we must have

$$\omega^2 = k/M \qquad \text{i.e. } \omega = \sqrt{\frac{k}{M}}$$

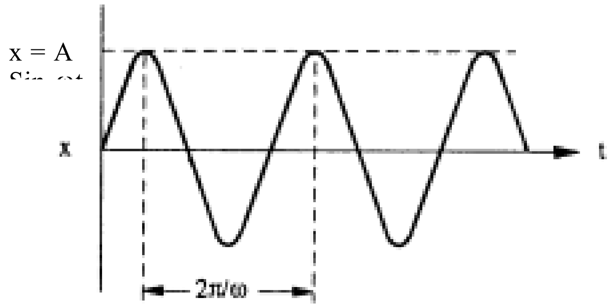

Figure 2. Simple Harmonic Motion (SHM).

We can help to visualise the motion by plotting x against t as in figure 2.

We see that the motion repeats itself with a periodic time, with the time for one oscillation called the period T, given by:

$$T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{M}{k}} \qquad (2)$$

The frequency f of the oscillations is the number of oscillations per unit time and is the reciprocal of the period, f=1/T, and is given by:

$$f = \frac{1}{2\pi}\sqrt{\frac{k}{M}}$$

We have shown already that the slope S of the *Δl* vs M graph is equal to g/k in Experiment 1.

Thus, $\quad k = g/S \quad$ and $\quad T = 2\pi\sqrt{\frac{MS}{g}}$

We have ignored the mass of the spring itself in the above analysis. This may be taken into account by writing:

$$T = 2\pi\sqrt{\frac{(M+m)S}{g}} \qquad (3)$$

where m is the "effective mass" of the spring.

On squaring Equation we obtain

$$T^2 = \left(\frac{4\pi^2 S}{g}\right)M + \frac{4\pi^2 Sm}{g} \tag{4}$$

This equation is in the form y = mx + c. Therefore, if the periodic time T is measured for various masses M, a graph of T^2 vs M should give a straight line

- the slope of which is $4\pi^2$ S/g (knowing S enables g to be determined)
- the intercept of which is $4\pi^2$mS/g (knowing S and g enables m to be determined.)

Reference

Giancoli, Physics, Fifth Edition, Chapter 9.
Young and Freedman, University Physics, Ed. 9, Chapters 6 and 13.