OBJECTIVES:
- To understand the difference between vector and scalar quantities;
- To show vector addition and to understand the methods used to add vectors;
- To investigate the how different forces may be combined to produce equilibrium.

BACKGROUND:
We are all familiar with the notion that certain quantities have in some sense a “direction” associated with them, whereas others don’t. For example, in order to fully describe the velocity of a car one would need to give the magnitude of the velocity (also called the speed) and the direction. On the other hand, we would say that the temperature of an object requires no direction to specify it, only the magnitude, e.g. 15°C. We thus split up the categories of quantities we measure mostly into two different types (some more exotic measurements reveal other types of quantities called tensors); those which require a specification of direction (called vectors) and those which require no specification of direction (called scalars). Quantities such as mass, temperature and energy are scalars, while quantities such as displacement, velocity, acceleration etc. are vectors. Most importantly for us in this experiment, force is a vector quantity.

An important difference between scalar and vector quantities is how we combine or add vectors and scalars. Scalars add using ordinary arithmetic, e.g. if you put a 2 kg mass on a scales and then put a 3 kg mass on the total mass will be 5 kg. Adding vectors is slightly different however, due to the directional nature of the vector quantity. If you push on one side of an object with a force $F$, and someone else pushes on the opposite side of the object in the opposite direction with the same size force, then the two forces cancel out and the object doesn’t move. However, if you both push on the same side in the same direction, the effect is twice what you’d get just by pushing by yourself. Thus, depending on the direction in which the forces are applied, the result, i.e. the total force, can vary. Additionally, if one person pushes from the side and the other from the back, then the object will accelerate diagonally.

**Figure 1.** Directional nature of vector quantities (right and wrong ways to push an object)
Experimental set-up:

You are going to study the various forces acting on a body in equilibrium. As you know from lectures (or will find out in the near future), a body is in equilibrium (i.e. not accelerating) when the sum of all the forces acting on it are equal to zero. In this experiment you are going to study an object with 3 and 4 forces acting on it, and show that the vector sum of those forces is zero, using both the polygon and component methods (discussed in the analysis section below). The apparatus you will use will consist of a drawing board and a number of pulleys on a frame, masses, some twine and an object (in this case a small ring).

Figure 6a. Schematic diagram of apparatus  Figure 6b. Vector diagram of forces on object

When you initially put the apparatus together and put the masses on the twine, the ring will move slightly and then come to rest in an equilibrium position. When the ring in at rest, i.e. in equilibrium, the total force acting on it is zero.

The gravitational force (or weight) exerted on each mass is \( mg \) (where \( g \) is the acceleration due to gravity \( \sim 9.81 \text{ m/s}^2 \)). In figure 6a above there are 3 masses \( (m_1, m_2 & m_3) \), and each exerts a force on the object of size \( m_1g, m_2g & m_3g \), with the forces acting along the direction of the string connecting the mass to the object. The forces acting on the ring then look like those in figure 6b.

By taping a piece of graph paper on the drawing board behind the twine and marking the position of the three pieces of string (by marking 2 points for each piece, one close to the object and another close to the pulley), and then drawing vectors along these directions with lengths proportional to the masses (e.g. 1 cm \( \equiv 100 \text{ g} \)) you will have made a vector diagram of the forces on the object.

You can then use the polygon method or the method of components (discussed in the analysis section below) to show that the vector sum of these forces is zero.

Now answer questions A1 through A3 on the answer sheet
Procedure:

1. Preparation:
   - Tape a sheet of graph paper onto the face of the board.
   - Tie three pieces of thread to the ring and pass two of them over the upper pulleys leaving the third to hang free. Tie a loop at the end of each piece of thread and suspend a mass hanger from each loop.

2. Starting up:
   - Load each mass hanger with masses but keep the ring fairly central on the board.
   - With a pencil, mark on the paper the position of each piece of thread by taking a point at either end.
   - Note the masses (m₁, m₂ & m₃) suspended from each thread.
   - Remove the graph paper and draw vectors in the direction of the strings, with lengths directly proportional to the appropriate masses.

3. Making measurements:
   - Vary M₁, M₂ and M₃ and repeat this procedure for another different equilibrium situation.
   - Using the third pulley, follow the above procedure with 4 masses.

Note: Each person must submit his or her own set of 4 original vector diagrams on graph paper, stapled to the answer sheet (part A4 of answer sheet).

4. Data presentation:
   - For the first vector diagram you obtained, with 3 masses, use the polygon method to draw the resultant vector (on the same sheet of graph paper). Measure the length of the resultant vector (part A5 of answer sheet).
   - For the second vector diagram you obtained, again with 3 masses, calculate the x- and y-components of each vector (part A5 of answer sheet) and the x- and y-components of the resultant vector.
   - For the third vector diagram you obtained, this time with 4 masses, once again use the polygon method to draw the resultant vector (on the same sheet of graph paper). Measure the length of the resultant vector (part A6 of answer sheet).
   - For the fourth vector diagram you obtained, again with 4 masses, calculate the x- and y-components of each vector (part A6 of answer sheet) and the x- and y-components of the resultant vector.

Analysis:

Diagrammatic representation of vectors:
The directional nature of vectors means that a single number cannot describe them. An arrow is generally used to represent vectors diagrammatically. The length of the arrow is proportional to the vector magnitude, while the direction is parallel to the vector direction. This method of representing vectors also allows us to very easily see how vectors add.

The vector \( \mathbf{A} \) may be represented on a diagram by drawing an arrow the length of which is proportional to the magnitude of the vector quantity. The arrow is drawn in the direction of
the vector. Thus \( \mathbf{A} \) is a vector quantity and is represented in figure 2. The magnitude of vector \( \mathbf{A} \) is denoted as \( A \) or alternatively \( |\mathbf{A}| \).

![Figure 2. Vector A.](image)

*Adding vectors:*

The resultant of a number of vectors is the single vector which would have the same effect as all the original vectors together. \( \mathbf{R} = \mathbf{A} + \mathbf{B} + \mathbf{C} \) means that \( \mathbf{R} \) is the resultant of the vectors \( \mathbf{A}, \mathbf{B} \) and \( \mathbf{C} \). We can visualise how vectors are added in a geometric manner. We can actually perform the addition in two ways, either using a mainly geometrical methods (polygon method) or a more arithmetically based method (component method). **The two methods produce the exact same results.**

*Polygon Method:*

The resultant of two or more vectors may be obtained by drawing them "head-to-tail" (in any order). The line drawn to complete the triangle or polygon represents the resultant, which is pointed from the tail of the first vector to the arrow-head of the last vector.

![Addition of 2 vectors by the polygon method](image)

![Addition of 3 vectors by the polygon method](image)

*Components Method:*

We can simplify the way we look at a vector problem by splitting any vector up (also called resolving) into its components, i.e. by saying that any vector is the sum of two vectors, one along the \( x \)-axis and one along the \( y \)-axis, as in figure 4. These two vectors are called the \( x \)- and \( y \)-components respectively.
Simple trigonometry tells us that the sizes of the $x$- and $y$-component vectors of vector $A$ are:

$$A_x = A \cos(\theta) \quad \& \quad A_y = A \sin(\theta) \quad \text{(1 a & b)}$$

and also note that:

$$A = \sqrt{A_x^2 + A_y^2} \quad \& \quad \tan(\theta) = \frac{A_y}{A_x} \quad \text{(2 a & b)}$$

(since the shaded triangle is right-angled)

In figure 5, the resultant vector $R$ of two vectors $A$ & $B$ is shown. One may easily see that $R_x = A_x + B_x$ and that $R_y = A_y + B_y$. Using equations 2 (a & b) above we can work out the size of $R$ and the angle it makes with the $x$-axis. The component method may be easily extended to 3 dimensions, though in this experiment we will confine ourselves to 2 dimensions.