Unit 50 Noise

- Noise signals present within a bandwidth B are specified in units of:

$$\text{Volts per } \sqrt{\text{Hz}} \quad \text{or} \quad \text{Amps per } \sqrt{\text{Hz}}$$

- The thermal or white noise from a resistor, R, at temperature T within a bandwidth B is:

$$V_{noise} = \sqrt{4kTRB}$$

- The shot noise associated with a DC current I is:

$$I_{noise} = \sqrt{2eIB}$$

- Flicker noise (sometimes called 'one over f noise') is comparable in magnitude to thermal noise at about 100 Hz and has a spectrum which varies as $\frac{1}{f} = \frac{1}{\text{Frequency}}$.

If the gain of an electronic amplifier, with no signal present at the input, is gradually increased, a point is reached when the zero signal at the output starts to fluctuate or become noisy. This effect can be easily noted on audio amplifier systems when the noise appears as a hiss from the loudspeaker when the volume is turned up fully with no tape or CD inserted. The phenomenon can also be observed as snow on the screen of older TV sets when there is no signal at the input or when no antenna is plugged in. (More modern sets give the illusion of being noise free because of internal muting circuits which turn off the screen display when there is no signal at the input.)

Some of the origins of these noise signals are shown in Figure 50.1. On the left, we have man-made signals, loosely called interference. Good design and shielding of sensitive equipment in grounded metal cabinets or Faraday cages can prevent signals from reaching sensitive equipment. Legislation also requires manufacturers to reduce the spurious electromagnetic interference (EMI) emitted by equipment such as computers and household and office equipment. In the case of very sensitive equipment, moving the equipment to a quieter location can help to reduce the noise. However, there is an inherent limit to how quiet or noise free a system can be made. There

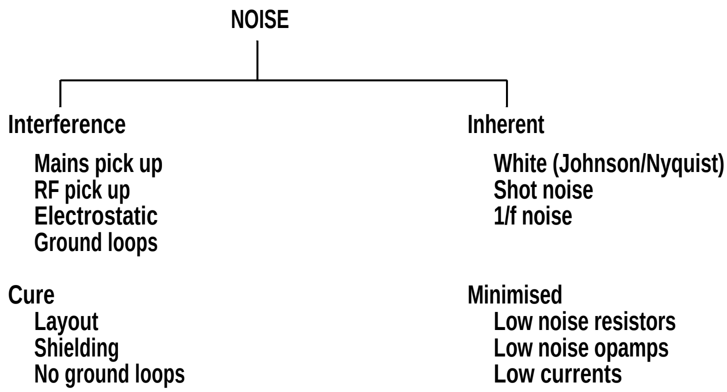

Figure 50.1: Noise sources.

are three unavoidable noise generation mechanisms present in all electronic systems as shown on the right in Figure 50.1.

Before examining the mechanisms in detail we must specify the units in which noise in electronic systems is measured.

By analogy with optical systems, we have a noise power spectrum. Light which has a power or intensity distribution which is uniform over the spectrum is called white light. Light which is more intense at longer (red) wavelengths or lower frequencies is said to be pink. We therefore have the concept of white noise having a power distribution spread equally over all frequencies.

We also have the concept of spectral density. If the power in a frequency range $\delta f = f_1 - f_2$ is measured, we can then plot the power per unit frequency as a function of frequency.

Since we are working with electronic systems in which we more conveniently measure the voltages and in which the power is proportional to the square of the voltage we use:

$$P = \frac{V^2}{R}$$

Then by measuring the noise voltages with an oscilloscope or voltmeter we have a quantity which is proportional to $\sqrt{P}$.

Consider the spectral density of this noise. At any given frequency, the units of power spectral density are watts per Hz bandwidth.

Therefore if we take the square root of the power spectral density we get the units of the noise voltage spectral density as:

$$\text{Volts per } \sqrt{\text{Hz}}$$

White noise was theoretically analyzed by Nyquist and experimentally measured by Johnson. Consider a resistor which is at a temperature T K. The electrons within the resistor move randomly with a kinetic energy appropriate to kT where k is Boltzmann's constant. Connect a resistor at each end of a matching transmission line as shown in Figure 50.2.

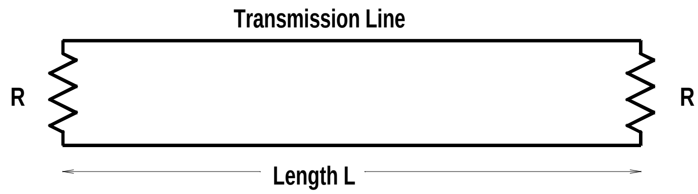

Figure 50.2: Transmission line carrying noise.

A good example of a transmission line is a coaxial cable used to carry radio or TV signals. Such a line has a characteristic impedance of $75\,\Omega$ which is determined by the diameters of the outer shield and the central conductor. So we will consider a coaxial cable terminated at each end by a $75\,\Omega$ matching resistor and we will also assume that the coaxial cable acts as a perfect loss free transmission line which does not itself introduce any noise.

The thermal motions of the electrons in the resistor cause the resistor to radiate electromagnetic signals into the transmission line and the resistor absorbs any signal travelling down the line to the resistor. After a short time, thermal equilibrium is reached and the spectral power distribution on the line is the same as the spectral distribution emitted by the resistor. Now cut the resistors from the line and leave the electromagnetic signal trapped on the line, bouncing back and forth between the ends.

What is the energy distribution of this signal on the transmission line? Or to put the question differently, what are the modes or frequencies that can be present as propagating waves on the line? The question is very similar to asking what modes of vibration can be present on a violin string or in an organ pipe.

If the length of the line is L and the velocity of the waves is C then the modes will be integer multiples of the fundamental, $f_0 = \frac{C}{2L}$.

The number of modes, N, between the frequency $f_1 = p\frac{C}{2L}$ and the frequency $f_2 = q\frac{C}{2L}$, where p and q are integers, is:

$$N = p - q = \frac{f_1}{\frac{C}{2L}} - \frac{f_2}{\frac{C}{2L}} = (f_1 - f_2)\frac{2L}{C}$$

Each mode has an energy kT associated with it and therefore the energy, W, on the line in the frequency range f_1 to f_2 is:

$$W = NkT = (p - q)kT = (f_1 - f_2)\frac{2L}{C}kT$$

This energy was delivered into the line by the two resistors in one line transit time $\frac{L}{C}$ and therefore the power from each resistor is:

$$P = \frac{W}{\frac{2L}{C}} = \frac{(f_1 - f_2)\frac{2L}{C}}{\frac{2L}{C}}kT = (f_1 - f_2)kT = \overline{i^2}R$$

where $\overline{i^2}$ is the mean square noise current.

The mean square noise voltage which drives this current through the two resistors in series is then (by squaring Ohm's law and making a substitution for $\overline{i^2}R$):

$$\overline{v^2} = \overline{i^2}(2R)^2 = 4\overline{i^2}RR = 4(f_1 - f_2)kTR = 4kTRB$$

where $B = f_1 - f_2$ is the bandwidth.

The noise voltage is then:

$$v_{noise} = \sqrt{4kTRB}$$

Shot noise. An electric current is numerically equal to the number of electrons flowing past a given point per unit time multiplied by the electronic charge. If N electrons on average flow in a given time interval then the statistical fluctuation in this number is $\sqrt{N}$ so that a specific measurement will be in the range $N \pm \sqrt{N}$. If the bandwidth of the measuring amplifier is B then the sample time is given by $\Delta t = \frac{1}{B}$.

$$\text{The average DC current is} \quad I = \frac{Ne}{\Delta t} = NeB$$

$$\text{The fluctuation in the current is} \quad \Delta I = eB\sqrt{N} = eB\sqrt{\frac{I}{eB}} = \sqrt{eIB}$$

A more detailed argument based on statistical mechanics shows that there is an extra factor of 2 so that the shot noise is then:

$$\Delta I = \sqrt{2eIB}$$

Flicker noise or $\frac{1}{f}$ noise. As the name suggests, this noise mechanism is most significant at low frequencies when $\frac{1}{f}$ is large. This can be seen in Figure 50.3 where the noise voltages associated with flicker noise are masked by shot noise at frequencies above some corner frequency f_0. Typically this corner frequency is at about 100 Hz.

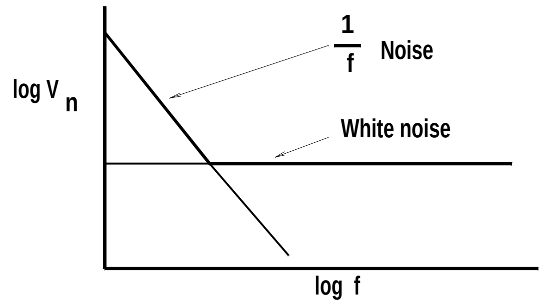

Figure 50.3: Noise spectra.

The precise mechanism by which flicker noise is generated is not fully understood but some progress has been made in modelling flicker noise by using the techniques of chaos theory and intermittency.

Flicker noise is a particular problem with DC amplifiers and very low frequency amplifiers when slow variations in signals on time scales of seconds are being examined. One technique for minimizing or avoiding flicker noise is to chop the signal at say 1 kHz and thus shift the the signal up in frequency and out of the flicker noise domain. Amplifiers which use this technique are called chopper amplifiers. The amplifiers used in medicine to measure the voltages associated with the heart (ECG) and brain (EEG) are examples of where flicker noise can be a problem.

50.1 Example

50.1 Calculate the thermal noise from the 500 kΩ resistor at 20°C which would be observed on an oscilloscope connected to the output of the circuit shown in Figure 50.4 (a). Assume that the op-amp does not introduce any noise. The V_S in the circuit diagram is the Thévenin equivalent of the noise source.

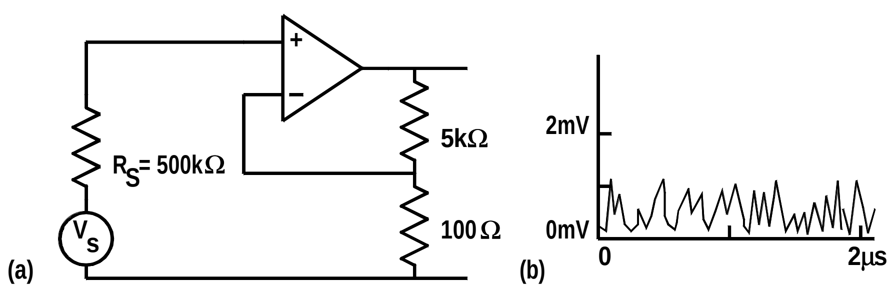

Figure 50.4: Example 50.1.

The gain of the amplifier is $A = 1 + \frac{5000}{100} = 51$ or 34 dB.

Plot the 34 dB line on the op-amp frequency response curve shown in Figure 49.1 and the bandwidth is obtained to be about 30 kHz.

The thermal noise from the 500 kΩ resistor is given by:

$$\begin{aligned} V_{noise} &= \sqrt{4kTR_s} \times \sqrt{\text{Bandwidth}} \\ &= \sqrt{4 \times 1.38 \times 10^{-23} \times 293 \times 5 \times 10^5 \times 3 \times 10^4} \\ &= 16\,\mu\text{V}_{\text{RMS}} \end{aligned}$$

This noise signal is amplified by a factor of 51 to give an output signal of 0.87 mV_{RMS}. On a good oscilloscope, this will appear as a fine 'grass' on the display when the scope is set at near maximum sensitivity. This is shown in Figure 50.4 (b)

50.2 Problems

50.1 Calculate the shot noise voltage within a 15 MHz bandwidth due to a current of 0.8 A flowing through a 50 Ω resistor. Could this noise be observed with an oscilloscope?

50.2 A radio receiver has a bandwidth of 10 kHz and a 300 Ω antenna input connection. The equivalent noise temperature at the antenna input due to the electronic circuits of the receiver is specified by the manufacturer to be 600 K. Calculate the voltage signal at the antenna terminals which will give a 10:1 signal to noise ratio.

50.3 A strain gauge is a metal foil resistor whose resistance changes when it is strained. Two such gauges are bonded onto opposite sides of a cantilever beam of thickness, T, which is loaded to a radius of curvature, D, as shown in Figure 50.5 (a). For such gauges $\frac{dR}{R} = G \times \frac{dL}{L}$ where G is the gauge factor and dL and dR are the changes in gauge length and resistance. The gauges used have unstrained resistance of 120 Ω and gauge factor, $G = 2.1$. Calculate the noise voltage which is present at the output of the bridge circuit shown in Figure 50.2 (b). Calculate the output voltage from the bridge for a given fractional change in the radius of curvature of the beam. Also calculate the fractional change in the radius of curvature which gives a signal to noise ratio of 1.

50.4 The noise generated in the internal circuits of a 741 op-amp can be described by two noise sources v_n and i_n connected to the input to an ideal noiseless op-amp as shown in Figure 50.6 (a). These sources

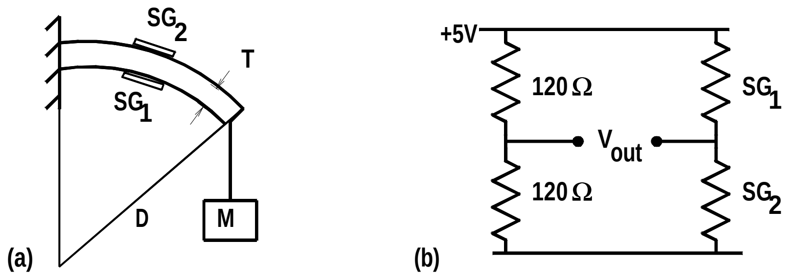

Figure 50.5: Problem 50.3. Strain gauge bridge.

have values of $4 \times 10^{-15}\,\mathrm{V}^2_{\mathrm{RMS}}\ \mathrm{Hz}^{-1}$ and $5 \times 10^{-25}\,\mathrm{A}^2_{\mathrm{RMS}}\,\mathrm{Hz}^{-1}$. Use the principle of superposition and the two rules for op-amps in Unit 39 to calculate the noise signal at the output of the amplifier shown in Figure 50.6 (b). Note that the resultant, V_R of two random noise sources, V_1 and V_2 is given by $V_R = \sqrt{V_1^2 + V_2^2}$.

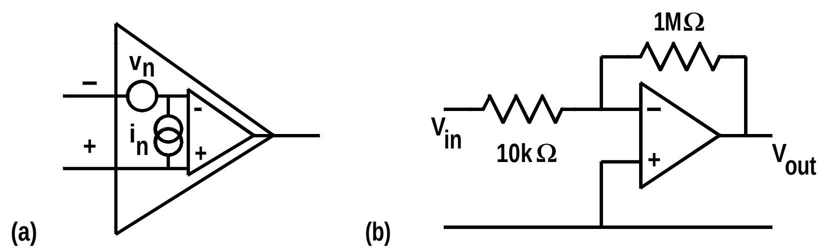

Figure 50.6: Problem 50.4. Op-amp noise evaluation.

50.5 Analyze the operation of the active full wave rectifier shown in Figure 50.7 and calculate the average output signal for a sinusoidal signal of 100 mV amplitude at 3 kHz and for a white noise signal, bandwidth limited to 3 kHz and 0.4 $\mathrm{V_{RMS}}$. Sketch the output waveforms.

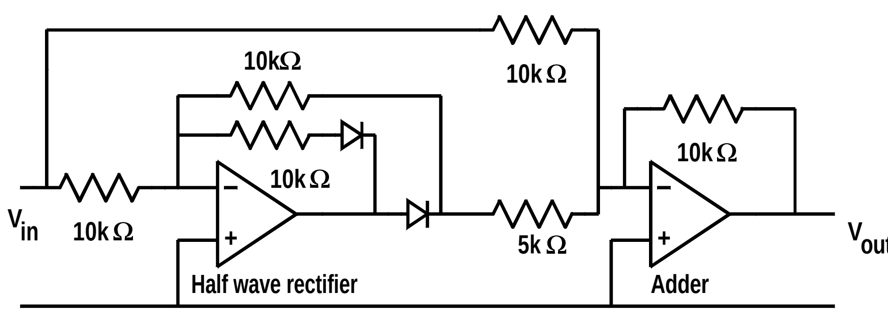

Figure 50.7: Problem 50.5. Precision full wave rectifier.