Unit 21 Principle of superposition

- In a linear system, with several causes acting to give a combined effect, the net effect is determined by adding the individual effects.
- In electronic circuits, the individual effect of a voltage or current source is determined by removing all of the other voltage and current sources and replacing the voltage sources by short circuits and the current sources by open circuits.

The use of the principle of superposition can lead to major simplifications in the analysis of electronic circuits because most analog circuits are linear or at least they are linear in a small region near the operating point. The advantage of using analysis methods based on the principle of superposition is that a direct and proportional link is maintained between the application of an input and the effect at the output. This gives a significantly greater understanding of the circuit operation than the methods based on Kirchhoff type analysis where the link between cause and effect is frequently obscured by the computations.

You should, however, remember that there are limits to linearity. If one man can dig a certain size hole in 10 hours, two men can dig the same hole in five hours but 20 men could not dig it in half an hour because they would get in each others' way and the job is therefore no longer linear!

Suppose we need to obtain the current in R_3 in Figure 21.1 (a). The circuit is redrawn twice with only one voltage source remaining in each version (or 10 times if there are 10 voltage sources). The gaps where the voltage sources were located are shorted out and the current in R_3 due to each of the voltage sources is calculated as indicated in Figure 21.1 (b) and Figure 21.1 (c). The current which flows when both of the voltage sources are present is the algebraic sum of the currents which flow in R_3 when each of the voltage sources is present.

In dealing with problems using the principle of superposition it is important that you draw the modified circuit at each stage. It is also important that you indicate the defined direction of current flow on the diagram. There is no problem if the initial assignment of the current direction is incorrect as all that will happen is that the sign of the current flow will be reversed.

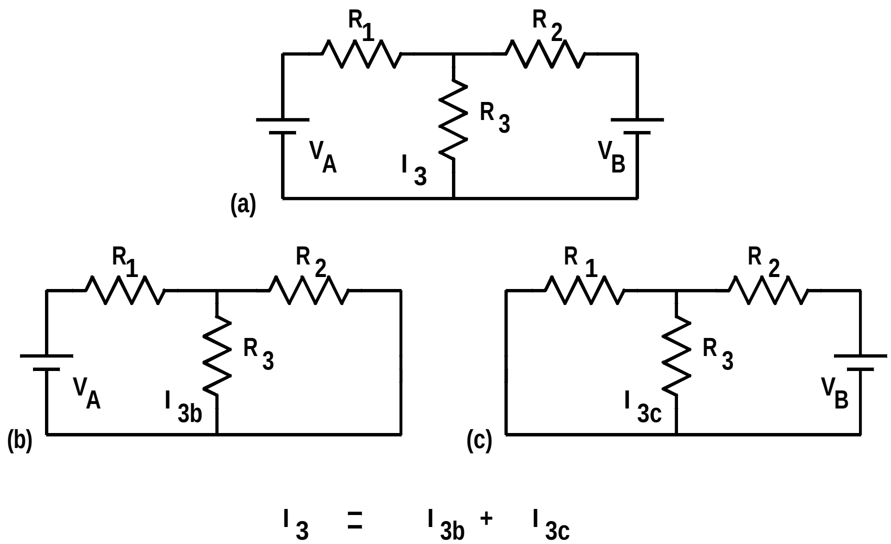

Figure 21.1: Splitting of circuit before applying principle of superposition.

21.1 Examples

21.1 Calculate the current in the $8\,\Omega$ resistor in Figure 21.2 (a).

There are two voltage sources present so there are two sets of calculations to be carried out. These are indicated in the two vertical sets of circuits in Figures 21.2 (b) to (d) and (e) to (g).

In (b) the 10 V battery has been removed and replaced by a short.

In (c) the $3\,\Omega$ resistor has been removed since it does not influence the current in the $8\,\Omega$ resistor.

In (d) the $5\,\Omega$ and $8\,\Omega$ have been replaced by their parallel equivalent resistor $3.07\,\Omega$.

By application of the potential divider principle we calculate the voltage across the $3.07\,\Omega$. This is the same as the voltage across the $8\,\Omega$.

The voltage across the $8\,\Omega$ due to the 6 V source is:

$$V_{6,8} = \frac{3.07}{3.07 + 4} \times 6\,\mathrm{V} = 2.60\,\mathrm{V}$$

This gives the current in the $8\,\Omega$ resistor due to the 6 V source as:

$$I_{6,8} = \frac{2.60}{8} = 0.33\,\mathrm{A}$$

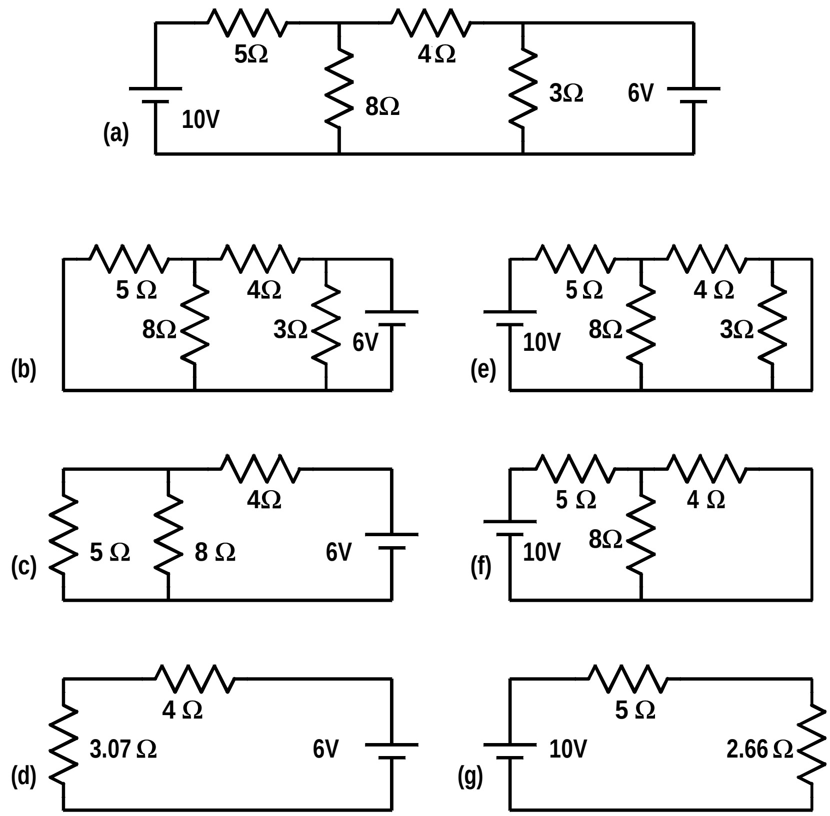

Figure 21.2: Calculate the effect of each cause separately.

On the right hand side of the diagram, replacement of the 6 V source by a short circuit gives Figure 21.2 (e).
The 3 Ω is shorted out and therefore has no effect. We then get the circuit in (f).
Combine 8 Ω and 4 Ω in parallel to get 2. 66 Ω.
Using the potential divider, the voltage across the 2. 66 Ω and therefore across the 8 Ω in (g) is:

$$V_{10,8} = \frac{2.66}{2.66 + 5} \times 10 = 3.48\,\mathrm{V}$$

The current in the 8 Ω due to the 10 V source is then:

$$I_{10,8} = \frac{3.48}{8} = 0.43\,\mathrm{A}$$

The total current in the 8 Ω is therefore $0.33 + 0.43 = 0.76$ A.

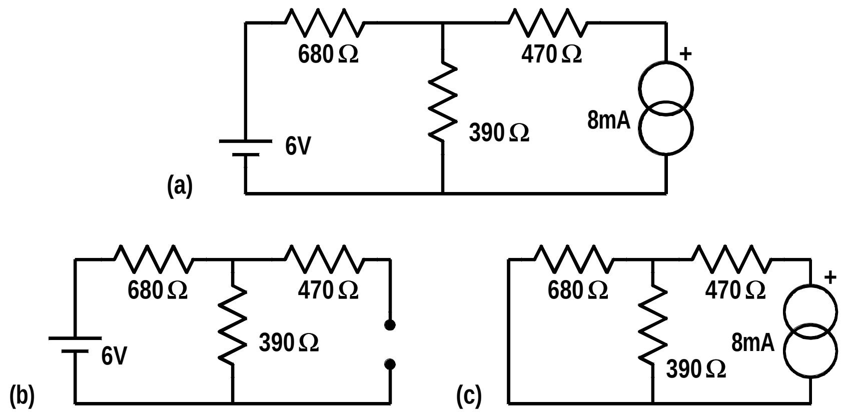

Figure 21.3: Example 21.2.

21.2 Calculate the current in the 390 Ω resistor in Figure 21.3 (a).

Redraw the circuit to eliminate the current source as shown in (b). The current source is replaced by an open circuit. Redraw the circuit to eliminate the voltage source as shown in (c). The voltage source is replaced by a short circuit.
In (b) the current in the 390 Ω due to the 6 V voltage source is:

$$I_{6,390} = \frac{6\,\text{V}}{680\,\Omega + 390\,\Omega} = 5.6\,\text{mA}$$

In (c) the voltage source has been eliminated and replaced by a short and we see that the 680 Ω is in parallel with the 390 Ω to give 248 Ω. The 8 mA passes through the 248 Ω and gives a voltage of $248 \times 0.008 = 1.98$ V which appears across the 390 Ω and 680 Ω in parallel.
The current in the 390 Ω due to the 8 mA current source is:

$$I_{8,390} = \frac{1.98}{390} = 5.1\,\text{mA} \quad \text{giving} \quad I_{total} = 5.6 + 5.1 = 10.7\,\text{mA}$$

21.2 Problems

21.1 Calculate the current in the 270 Ω resistor in Figure 21.4.

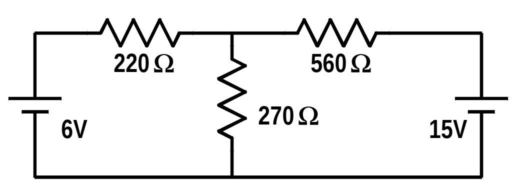

Figure 21.4: Problem 21.1.

21.2 Calculate the current in the $100\,\Omega$ resistor in Figure 21.5.

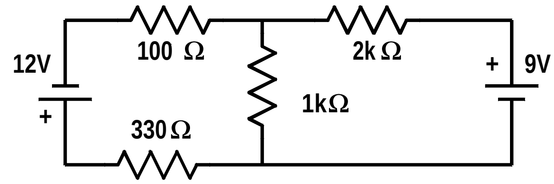

Figure 21.5: Problem 21.2.

21.3 Calculate the voltage across the $1\,\mathrm{k}\Omega$ resistor in Figure 21.6.

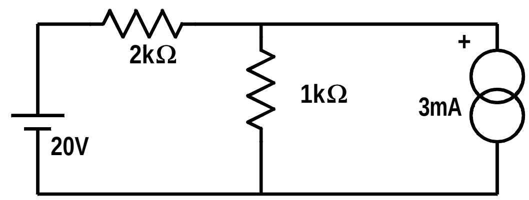

Figure 21.6: Problem 21.3.

21.4 Calculate the voltage wrt ground at point X in Figure 21.7.

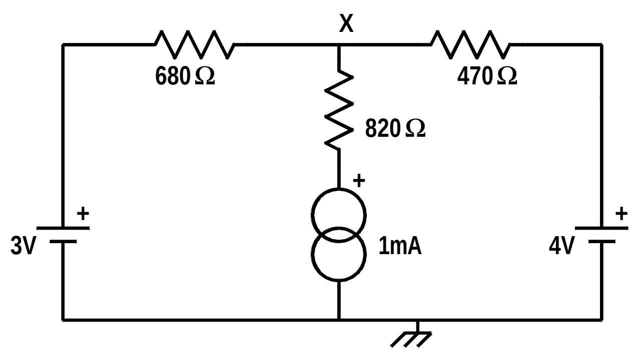

Figure 21.7: Problem 21.4.

21.5 Explain why voltage sources are replaced by short circuits and current sources are replaced by open circuits when the principle of superposition is used in the analysis of circuits.