Unit 19 Thévenin's theorem

- Any linear electronic system, having two output terminals, can be fully modelled by a voltage source, V_S, in series with an impedance, called the output impedance, Z_{out}.
- A voltage source gives a constant output voltage which is independent of the current drawn from the voltage source.

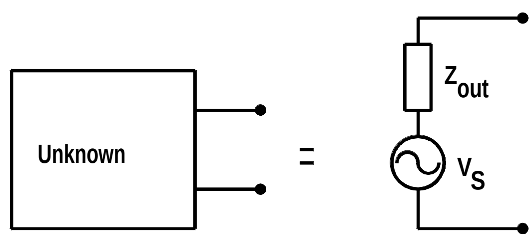

Figure 19.1: Thévenin equivalent circuit.

The voltage sources used in Thévenin analysis of circuits are devices which give a constant voltage at their output terminals. In the case of DC voltage sources, the voltage is constant and independent of the current which is drawn from the voltage source. In the case of AC voltage sources, the amplitude of the output voltage remains constant and the voltage varies sinusoidally at some fixed frequency. The Thévenin analysis method is the same for DC and AC circuits except that the complex impedance is used when capacitors or inductors are included in the circuit. We will therefore not make any fundamental distinction between the analysis of DC and AC circuits.

The concept of a voltage source is an idealization but there are a number of real examples which approximate to the ideal. Two good examples which approximate to the ideal voltage source are:

- The mains AC power supply. The RMS voltage at the wall socket remains at a constant 240 V independently of whether we draw no current or draw a $\frac{1}{4}$ A when we connect a 60 W bulb or draw 8 A when we connect a 2 kW electric fire.

- A lead acid car battery. The voltage at the battery terminals in a car remains constant at 12 V whether we operate the clock which draws 10 mA, the radio which draws 3 A, the headlights which draw 14 A or the starter motor which draws 120 A.

As an example of the Thévenin analysis method we consider how we can obtain the Thévenin model for a battery comprised of a number of dry cells in series.

If the output voltage from a fresh PP9 type battery is measured, a value of about 9 V is obtained. The voltmeter has a high resistance and therefore does not draw any significant current from the battery. There is therefore no voltage drop across the internal resistance of the battery and the voltmeter reading is then equal to that of the Thévenin voltage source.

If an ammeter is connected across the battery terminals, the battery is effectively shorted but an infinite current is not obtained. Typically the current will be 3 A. (At least for a short time until the battery becomes discharged!) The current is limited by the internal resistance of the electrolyte and electrodes of the battery.

We can model the PP9 battery by a 9 V voltage source and a resistance internal to the battery which is called the output resistance, R_{out}, which is physically equivalent to the resistance of the internal electrolytes and electrodes. We can then model these two measurements of the open circuit output voltage and the short circuit current with the circuits in Figure 19.2.

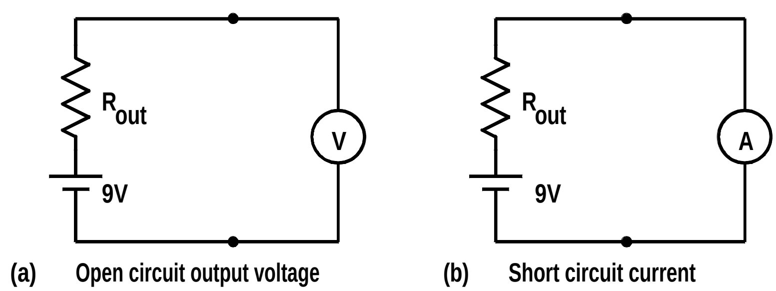

Figure 19.2: Measurement of Thévenin equivalent circuit of a PP9 battery.

From this simple model, we can see that:

$$R_{out} = \frac{V_{out\ open\ circuit}}{I_{out\ short\ circuit}}$$

For the PP9 battery example we therefore have $R_{out} = \frac{9\,\mathrm{V}}{3\,\mathrm{A}} = 3\,\Omega$.

In general, we do not normally short the output of a circuit with an ammeter in order to measure the short circuit current. The concept of short circuit current is, however, very useful for defining the output resistance.

If we return to the examples of voltage sources at the start of the discussion, we can say that the 240 V AC mains and the 12 V car battery voltage sources are really Thévenin sources in which the R_{out} is very small. If, for instance, a car battery had an output resistance of 1 Ω then the maximum current which it could drive through an external circuit would be $\frac{12\,\text{V}}{1\,\Omega} = 12\,\text{A}$ which would not even light the car headlights let alone operate the starter motor!

The output resistance of an electronic system can be measured by measuring the voltage at the output terminals as a function of the current drawn from the terminals. The short circuit current can then be obtained by extrapolation. It is in general not good practice to short out the output of an electronic system in order to measure the short circuit current, not only because of the possibility of damaging the electronics but also because many electronic circuits have special circuits included which detect a short circuit and close down the output before any damage is done. The electronic system is no longer linear and therefore one of the basic conditions for applicability of Thévenin's theorem no longer applies.

A circuit suitable for measuring the output resistance of an electronic system is shown in Figure 19.3 (a).

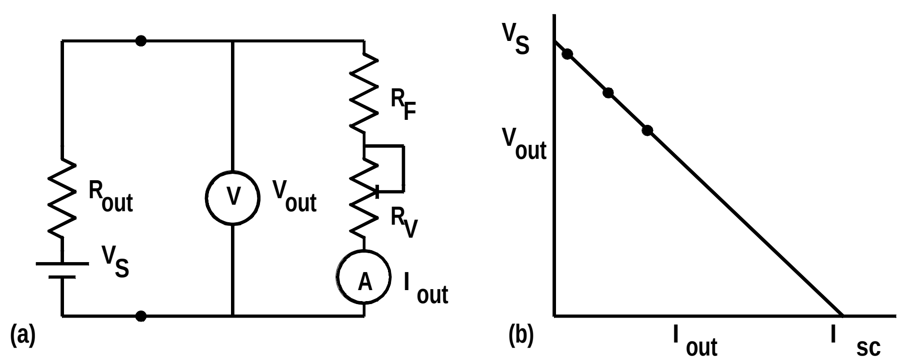

Figure 19.3: Practical measurement of Thévenin equivalent circuit.

The fixed resistor, R_F, controls the maximum current which can flow, since $I_{max} = \frac{V_S}{R_{out}+R_F}$. This I_{max} is specified by the manufacturer of the electronic system so as not cause any damage to the system. The variable resistor, R_V, allows the current to be varied so as to obtain a number of points on the V–I curve as shown in Figure 19.3 (b). The line through these points is then extrapolated to obtain the short circuit output current, $I_{short\ circuit}$.

It is not necessary to extrapolate to the short circuit current in order to obtain the output resistance as it is easily seen that the slope of the V–I characteristic gives the output resistance.

19.1 Examples

19.1 The table below shows the terminal voltages and currents which were measured when the specified resistance was connected across a PP3 battery. The circuit used is shown in Figure 19.4 (a). Calculate the Thévenin equivalent circuit for the PP3 battery.

Resistance	Output voltage V	Output current mA
500 kΩ	8.91	0.00
1 kΩ	8.82	9.18
470 Ω	8.75	18.70
180 Ω	8.56	49.10
100 Ω	8.39	85.70

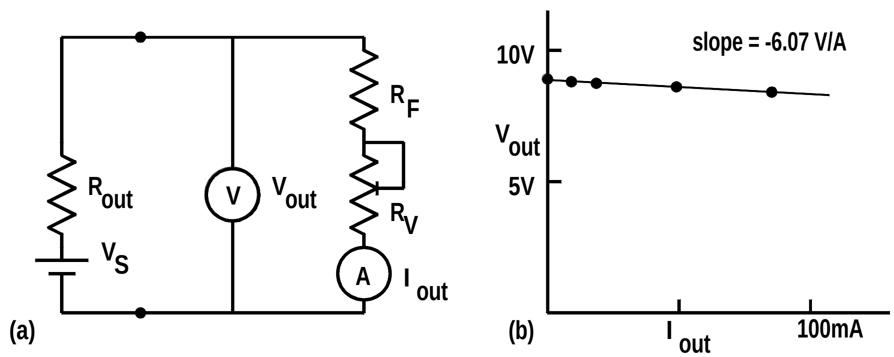

Figure 19.4: Example 19.1.

The output voltage is plotted against current as shown in Figure 19.4 (b). The slope of the curve is $-6.07\,\mathrm{VA}^{-1}$ which, when extrapolated, gives an intercept at $I_{short\ circuit} = \frac{8.91}{6.07} = 1.47\,\mathrm{A}$ giving:

$$R_{out} = \frac{V_{open\ circuit}}{I_{short\ circuit}} = \frac{8.91}{1.47} = 6.07\,\Omega$$

which is the slope of the V–I characteristic! So this PP3 can be modelled by an 8.91 V voltage source in series with 6.07 Ω.

19.2 Calculate the Thévenin equivalent of the circuit shown in Figure 19.5.

A voltmeter draws very little current from a circuit because of its high input resistance of 10 MΩ , so connecting a voltmeter across the output does not cause any significant current to flow in the 470 Ω resistor and there is then no significant voltage drop across the 470 Ω. Therefore the

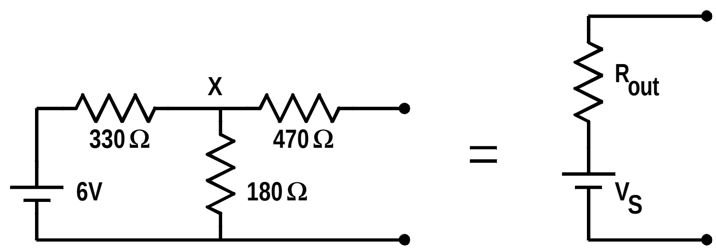

Figure 19.5: Example 19.2.

voltage at point X is the same as the voltage at the output terminals. We now have a potential divider of 330 Ω and 180 Ω in series which gives an open circuit output voltage of:

$$V_S = V_{open\ circuit} = \frac{180}{330 + 180} \times 6\,\mathrm{V} = 2.12\,\mathrm{V}$$

If an ammeter is connected across the output, the circuit shown in Figure 19.6 (b) results. Remember that an ammeter has an input resistance approaching zero ohms.

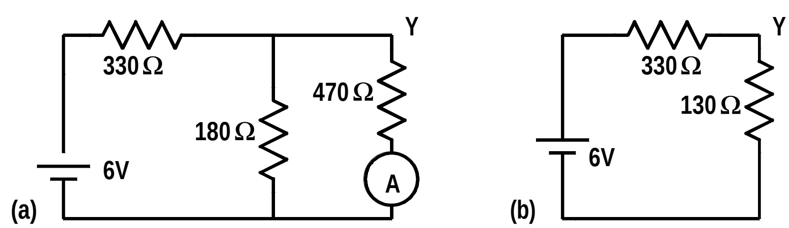

Figure 19.6: Example 19.2. Short circuit current.

The 180 Ω is now in parallel with the 470 Ω to make 130 Ω and this gives the voltage at point Y as:

$$V_Y = \frac{130}{330 + 130} \times 6\,\mathrm{V} = 1.7\,\mathrm{V}$$

This is the voltage across the 470 Ω and therefore the short circuit output current is $\frac{1.7}{470} = 3.61\,\mathrm{mA}$.

The output resistance for the Thévenin equivalent circuit is then:

$$R_{out} = \frac{V_{open\ circuit}}{I_{short\ circuit}} = \frac{2.12\,\mathrm{V}}{3.61\,\mathrm{mA}} = 586\,\Omega$$

We can then replace the circuit of Figure 19.6 (a) by its Thévenin equivalent as shown in Figure 19.7.

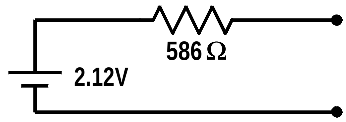

Figure 19.7: Example 19.2. Thévenin equivalent circuit.

19.3 Calculate the voltage which would be measured by a 10 MΩ input impedance voltmeter connected, as shown, between ground and point A in the circuit in Figure 19.8.

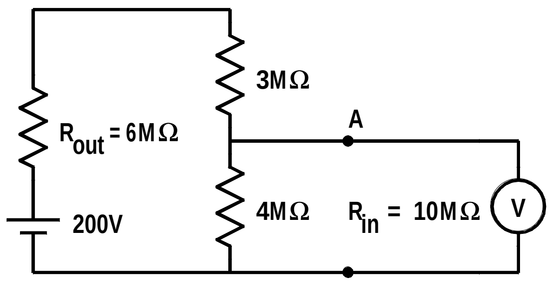

Figure 19.8: Example 19.3.

The input impedance of the voltmeter gives a potential divider lower arm of 4 MΩ in parallel with 10 MΩ, that is 2.86 MΩ. The upper arm of the potential divider is the output resistance of the voltage source, 6 MΩ in series with 3 MΩ.

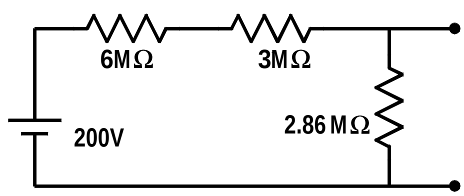

Figure 19.9: Effect of meter loading.

This effective circuit is shown in Figure 19.9. The voltage which is indicated by the voltmeter is then:

$$\frac{2.86}{2.86 + 6 + 3} \times 200\,\text{V} = 48.2\,\text{V}$$

You should note that connecting a voltmeter to the circuit can change the voltages at various points in the circuit. This is called **measurement loading**. It is usually only a significant problem when the resistances in the circuit are large (of the order of megohms).

19.2 Problems

19.1 The following measurements were made of the output terminal voltages and currents for an electronic circuit. Calculate the Thévenin equivalent for the circuit.

Voltage V	Current mA
16.0	0
15.8	320
15.3	1100
14.9	1770

19.2 Calculate the Thévenin equivalent of the circuit shown in Figure 19.10.

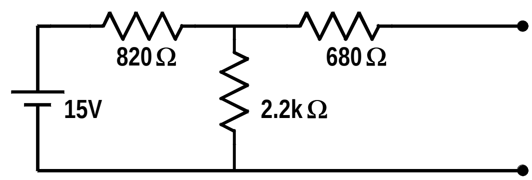

Figure 19.10: Problem 19.2.

19.3 The 6 V battery in Example 19.2 is replaced by a 9 V battery. Calculate and sketch the new Thévenin equivalent circuit.

19.4 Calculate the Thévenin equivalent for the circuit shown in Figure 19.11.

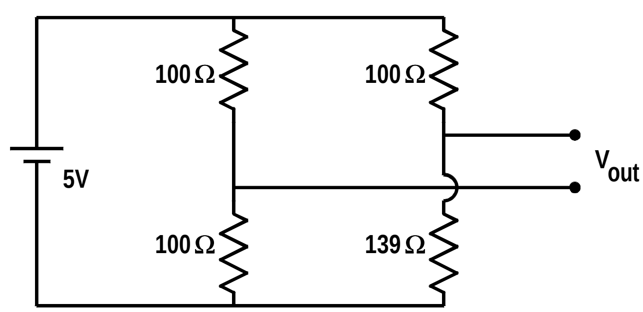

Figure 19.11: Problem 19.4.

19.5 Calculate the Thévenin equivalent for the circuit shown in Figure 19.12.

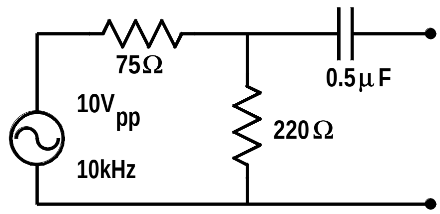

Figure 19.12: Problem 19.5.

19.6 Calculate the true voltages at points A and B in the circuit diagram shown in Figure 19.13. Calculate the voltages, relative to ground, which would be measured with a voltmeter having a 10 MΩ input resistance. You may assume that the 1000 V power supply has a negligible output resistance.

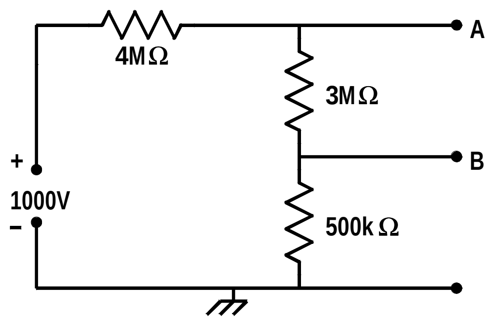

Figure 19.13: Problem 19.6.