Unit 16 Bode plot and frequency response

- The corner frequency, f_c, in hertz for a first order RC or RL filter is given by:
$$f_c = \frac{1}{2\pi CR} \quad \text{or} \quad \frac{R}{2\pi L}$$
- The filter response is approximated, on a dB versus log frequency plot, by two straight lines, one of slope 0 and the other of slope $\pm 20\,\text{dB}$ per decade and drawn through the point at the corner frequency and 0 dB, $(\log f_c, 0\,\text{dB})$.
- The attenuation at the corner frequency is $-3\,\text{dB}$ and the phase shift is $\pm\frac{\pi}{4}$ radians or $\pm 45°$.

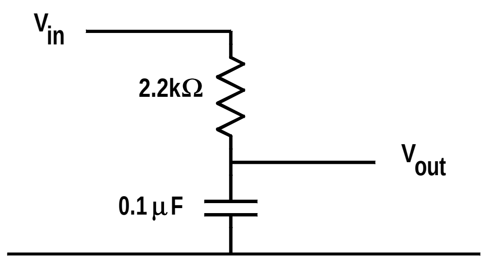

Figure 16.1: First order RC filter.

The frequency response of a filter such as that shown in Figure 16.1 is obtained by repeating the calculation of the attenuation carried out in Example 15.1 for a range of frequencies. When the response in decibels, dB, as a function of the log of the frequency, is plotted, a response curve similar to that shown in Figure 16.2 is obtained.

In doing these calculations the frequencies used were 10, 20, 50, 100, 200, 500, 1000, 2000, 5000, 10,000 Hz and the resulting points are shown on the plot. You should note that the sequence 1, 2, 5 and multiples of 10 give roughly equally spaced points on a log frequency plot. In the laboratory you should try to use these multiples where possible.

There is one special frequency for first order filters which is given by $f_c = \frac{1}{2\pi RC}$ Hz and is called the corner frequency. For this example the corner

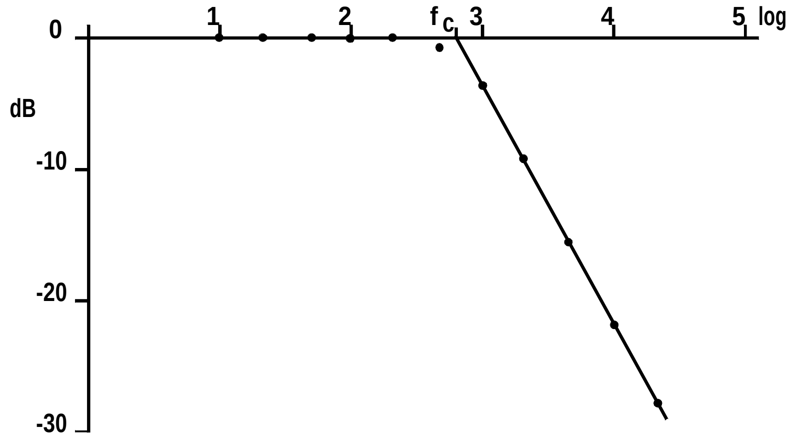

Figure 16.2: Amplitude response of first order filter.

frequency $f_c = \frac{1}{2\pi 2200\times 0.1\times 10^{-6}} = 724\,\text{Hz}$. Then $\log(724) = 2.86$ and this is marked on the log frequency axis as f_c.

If the point ($\log f_c$, 0 dB) is plotted, then the response curve can be approximated by two straight lines through this point as shown on the plot. One of the lines is at a constant 0 dB and is the frequency axis. The second line has a slope of -20 dB per decade, that is it drops by 20 dB for each decade in frequency or each change of 1.0 on the log frequency scale. The powerful feature of this Bode plot method is that you only need one number, the corner frequency, to be able to give a reasonably accurate plot of the frequency response of an RC or RL circuit. You do not need to do, for each frequency, the detailed computations which were done in order to obtain the points plotted in Figure 16.2.

At the corner frequency $\tan\phi = 2\pi f_c CR = 1$ and therefore the attenuation is:

$$\frac{1}{\sqrt{1+\tan^2\phi}} = \frac{1}{\sqrt{1+1}} = \frac{1}{\sqrt{2}} = 0.707$$

Expressed in dB this becomes $20\log 0.707 = -3\,\text{dB}$ so a small rounding off of the sharp corner made by the straight lines at the intersection accommodates this $-3\,\text{dB}$ error at the corner frequency.

The second half of the Bode plot is the phase response. We have seen that the phase shift is $-45°$ at the corner frequency. The phase shift is $0°$ for frequencies where the response curve is flat. Where the response is falling at -20 dB per decade, the phase shift is approximately $\pm 90°$, depending on whether an RC or a CR filter is being used, so we have the approximate response as shown in the phase plot in Figure 16.3. This is a straight line

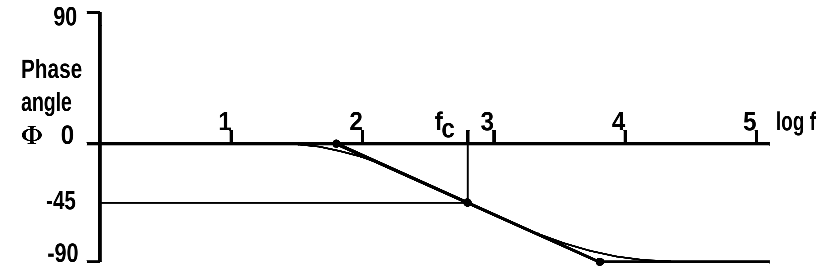

Figure 16.3: Phase plot.

approximation from 0° and either $0.1f_c$ or $10f_c$ and a second point at 45° and f_c.

When the amplitude response and the phase response are combined on one diagram with the log frequency scale the pair of curves is called a Bode plot. The full Bode plot is shown in Figure 16.4.

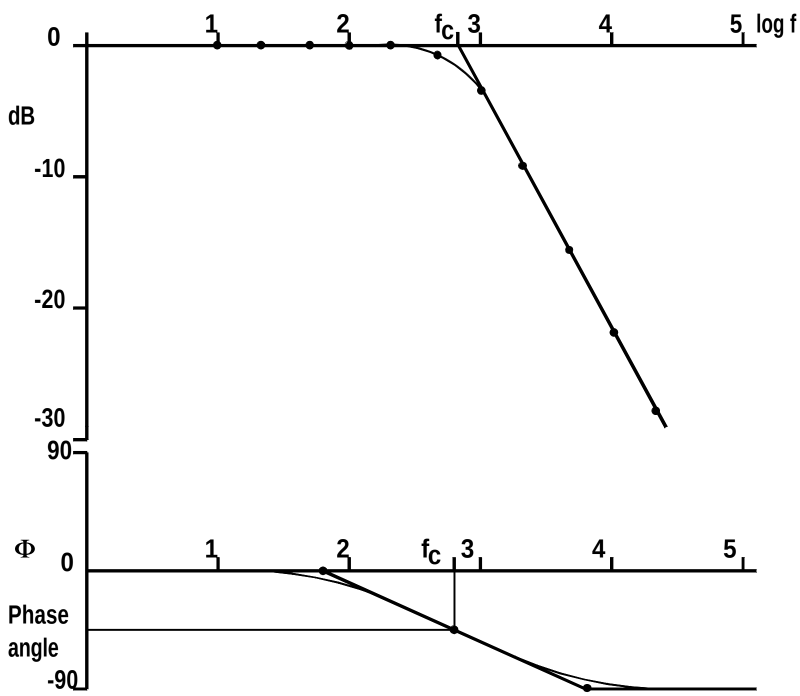

Figure 16.4: Bode plot for RC low pass filter.

The RC circuit which we have discussed is a low pass circuit. A CR circuit where the capacitor and resistor are interchanged is a high pass circuit. When you have an RC circuit or a CR circuit the type of response can be easily determined by remembering that at low frequencies a capacitor is essentially an open circuit and at high frequencies a capacitor is a short circuit. These approximations are illustrated in Figure 16.5.

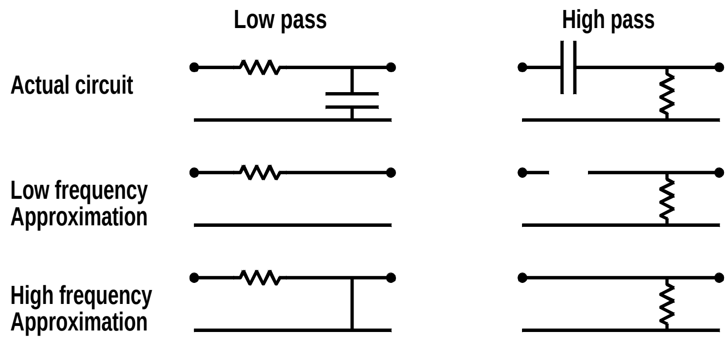

Figure 16.5: Extreme responses of filters.

We have discussed the Bode plots in terms of electronic circuits but Bode plots also have extensive applications in the analysis of instrumentation and control systems. For instance, a thermometer may have a response time constant of 30 seconds. This gives a corner frequency of $f_c = \frac{1}{2\pi 30} = 5.3\,\text{mHz}$. Such a thermometer would only be of use in situations where the temperature varies slowly as it would be necessary to wait for at least three time constants, $3 \times T = 90\,\text{seconds}$, to allow the temperature to stabilize before the thermometer gives a valid measurement of temperature.

16.1 Problems

16.1 Calculate the response of the filter shown in Figure 16.1 for the frequencies 10 Hz, 724 Hz, 1 kHz and 10 kHz and verify the response curve shown in Figure 16.2.

16.2 Calculate the attenuation at frequencies of 100, 1000, 10,000, 100,000 Hz for the circuits shown in Figure 16.6 and verify that the attenuation on the falling part of the response curves changes by 20 dB for each factor of 10 in frequency.

Figure 16.6: Problem 16.2.

16.3 Construct the Bode plot for the circuit shown in Figure 16.7.

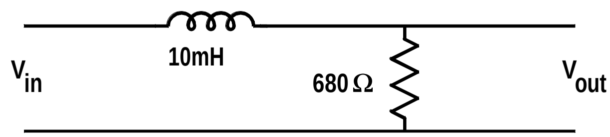

Figure 16.7: Problem 16.3.

16.4 Construct the Bode plot for the circuit shown in Figure 16.8.

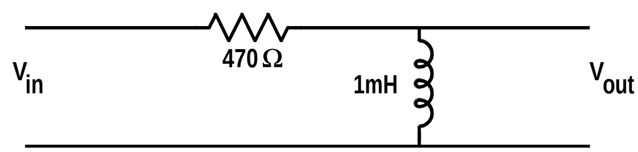

Figure 16.8: Problem 16.4.

16.5 Construct the Bode plot for the circuit shown in Figure 16.9.

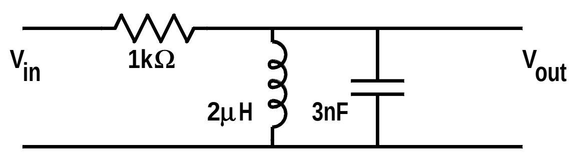

Figure 16.9: Problem 16.5.

16.6 When the heat flux incident on a radiant heat detector is modulated at 12 Hz, it is found that the output voltage is 0.707 of the output voltage when there is no modulation. Sketch the frequency response (Bode plot) for the heat detector.