Unit 15 Generalized potential divider

- Resistive or reactive components used in potential dividers give:

$$V_{out} = \frac{Z_2}{Z_1 + Z_2} V_{in}$$

- When the term $\frac{Z_2}{Z_1+Z_2}$ is put into the form $|A|\, e^{j\phi}$ then $|A|$ is the attenuation of the potential divider and ϕ is the phase shift.

There are two results from complex algebra which we will use extensively and which you may need to follow up in your mathematics textbook.

When a complex number, c, is in the form $c = a + jb$, the modulus and the phase angle for c and $\frac{1}{c}$ are given by:

$$|c| = |a + jb| = \sqrt{a^2 + b^2} \quad \text{and} \quad \tan\phi = \frac{b}{a}$$

$$\left|\frac{1}{c}\right| = \left|\frac{1}{a + jb}\right| = \frac{1}{\sqrt{a^2 + b^2}} \quad \text{and} \quad \tan\phi = \frac{-b}{a}$$

The resistors in the potential divider discussed in Unit 4 can be replaced by any combination of resistors, capacitors or inductors in series or parallel. A resultant impedance can then be calculated for each half of the potential divider. The current in each of the two impedances is given by $\frac{V_{in}}{Z_1+Z_2}$. This current flowing through Z_2 gives an output voltage $Z_2 I$.

The ratio of output to input voltage is then:

$$\frac{V_{out}}{V_{in}} = \frac{Z_2}{Z_1 + Z_2}$$

but since Z_1 and Z_2 are complex then $\frac{Z_2}{Z_1+Z_2}$ is usually also complex and has a magnitude less than 1.

If we express $\frac{Z_2}{Z_1+Z_2}$ in the form $|A|\, e^{j\phi}$ then $|A|$ gives the attenuation of the potential divider and ϕ gives the phase shift in radians.

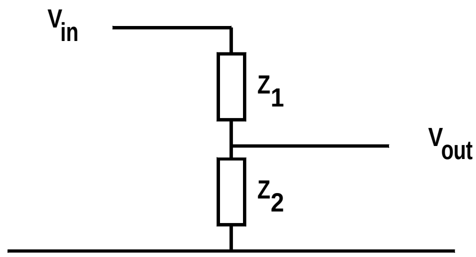

Figure 15.1: Generalized potential divider.

15.1 Example

15.1 Calculate the attenuation and phase shift in the RC network in Figure 15.2 where $R = 2.2\,\mathrm{k\Omega}$, $C = 0.1\,\mu\mathrm{F}$ and the frequency is 1.5 kHz.

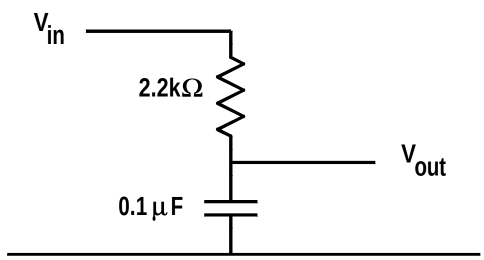

Figure 15.2: Example 15.1.

The network response is given by:

$$\begin{aligned}
\frac{V_{out}}{V_{in}} &= \frac{Z_2}{Z_1 + Z_2} \\
&= \frac{\frac{1}{j2\pi fC}}{R + \frac{1}{j2\pi fC}} \\
&= \frac{1}{1 + j2\pi fCR} \\
\text{First calculate} \quad \tan\phi &= -2\pi fCR \\
&= -2\pi 1500 \times 0.1 \times 10^{-6} \times 2200 \\
&= -2.07 \\
\text{So that} \quad \phi &= \tan^{-1}(-2.07) \\
&= -1.12\,\mathrm{rad} \text{ or } -64.3^\circ \\
\text{and the attenuation is} \quad \left|\frac{V_{out}}{V_{in}}\right| &= \frac{1}{\sqrt{1+\tan^2\phi}} = \frac{1}{\sqrt{1+2.07^2}} \\
&= 0.435 = 20\log 0.435\,\mathrm{dB} = -7.23\,\mathrm{dB}
\end{aligned}$$

If this circuit is constructed and the input and output waveforms are displayed on an oscilloscope then a trace similar to that in Figure 15.3 should be obtained.

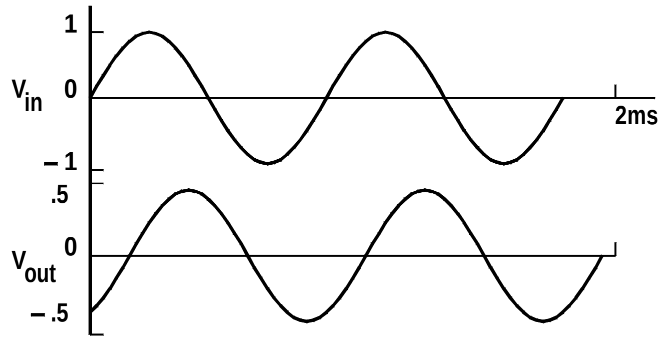

Figure 15.3: Input and output voltage waveforms for Example 15.1.

In the calculations we obtained a phase shift of -1.12 radians. In the oscilloscope diagram it can be seen that the output waveform is displaced to the right by 1.12 radians or 64° relative to the input voltage waveform. So we obtain the useful rule that:

- If the phase shift is positive then the output waveform is shifted to the left and is said to lead the input waveform.
- If the phase shift is negative then the output waveform is shifted to the right and is said to lag the input waveform.

15.2 Problems

15.1 Write down the expression for the output voltage waveform in Example 15.1 shown in Figure 15.3.

15.2 Calculate the attenuation and phase shift for the RC circuit shown in Figure 15.4 when $f = 500\,\text{Hz}$, $C = 22\,\text{nF}$ and $R = 10\,\text{k}\Omega$. Sketch the input and output voltage waveforms showing the amplitude and phase of the signals. Assume that the input signal is 1V_{pp}.

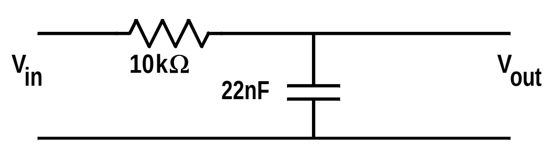

Figure 15.4: Problem 15.2.

15.3 Calculate the attenuation and phase shift for the CR circuit shown in Figure 15.5 when $f = 1.5\,\mathrm{kHz}$, $C = 0.1\,\mu\mathrm{F}$ and $R = 1.2\,\mathrm{k\Omega}$. Sketch the input and output voltage waveforms showing the amplitude and phase of the signals. Assume that the input signal is of amplitude 1 V.

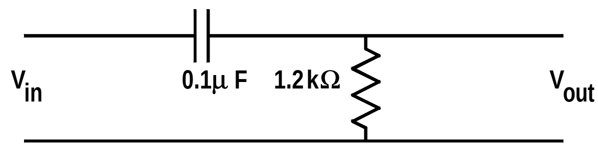

Figure 15.5: Problem 15.3.

15.4 Calculate the attenuation and phase shift for the LR circuit shown in Figure 15.6 when $f = 7\,\mathrm{kHz}$, $L = 10\,\mathrm{mH}$ and $R = 680\,\Omega$. Sketch the input and output voltage waveforms showing the amplitude and phase of the signals. Assume that the input signal is $1\,\mathrm{V_{pp}}$.

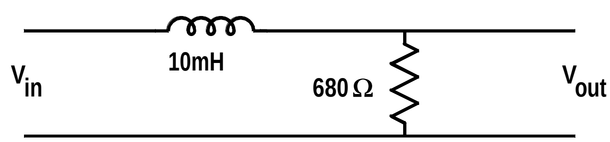

Figure 15.6: Problem 15.4.

15.5 Calculate the attenuation and phase shift for the RL circuit shown in Figure 15.7 when $f = 60\,\mathrm{kHz}$, $L = 1\,\mathrm{mH}$ and $R = 470\,\Omega$. Sketch the input and output voltage waveforms showing the amplitude and phase of the signals. Assume that the input signal is $1\,\mathrm{V_{pp}}$.

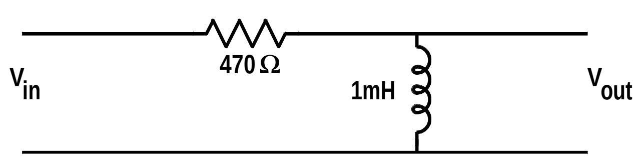

Figure 15.7: Problem 15.5.

15.6 Calculate the frequency for which the attenuation is 3 dB for the circuit in Figure 15.8. Calculate the phase shift in degrees at this frequency.

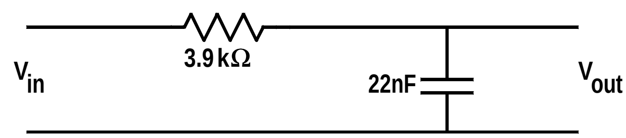

Figure 15.8: Problem 15.6.

15.7 Show that the bridge circuit in Figure 15.9 will be in balance, that is $V_A = V_B$, when $R_X = \frac{R_3 \times R_2}{R_1}$.

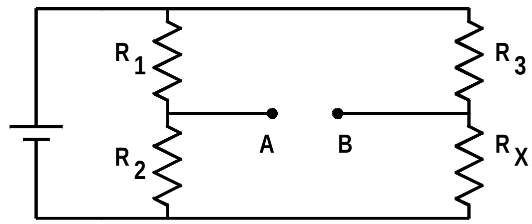

Figure 15.9: Problem 15.7.

15.8 Show that the Simple bridge in Figure 15.10 will be in balance when:

$$R_X = \frac{R_2 \times R_3}{R_1} \quad \text{and} \quad C_X = \frac{R_1}{R_3} \times C_2$$

Does the balance depend on the frequency of the voltage across the bridge? Note that the real and complex parts of the impedance equation must balance separately.

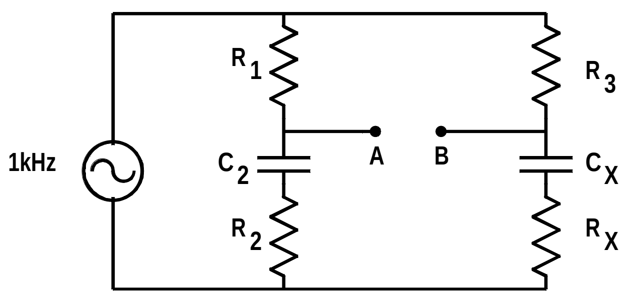

Figure 15.10: Problem 15.8.