Unit 13 Complex impedance diagram

- The impedance of components can be represented on a complex impedance diagram by vectors drawn from the origin.
- When components are connected in series, the resultant impedance is obtained from the vector sum of the impedances of the individual components.

In drawing the complex impedance diagram the following rules are used:

- A resistance is represented by a vector of magnitude R ohms drawn along the positive x axis.
- A capacitor is represented by a vector of magnitude $\frac{1}{\omega C}$ ohms drawn along the negative y axis.
- An inductance is represented by a vector of magnitude ωL ohms drawn along the positive y axis.

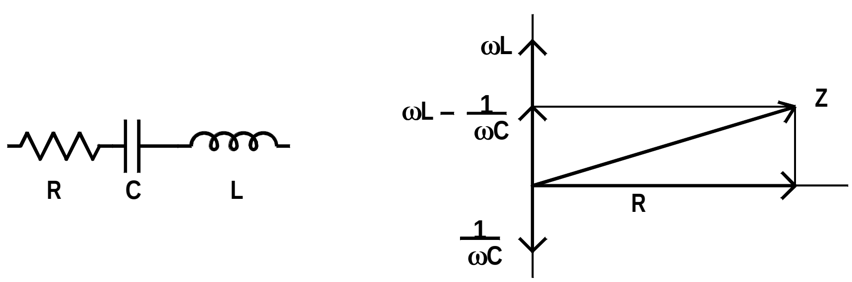

Figure 13.1: Complex impedance diagram for RCL in series.

The resultant impedance, Z, of the three components connected in series is the vector sum as shown in the diagram.

The magnitude of the impedance of the resultant is obtained from:

$$|Z| = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}$$

The phase angle, ϕ, is obtained from:

$$\phi = \tan^{-1}\left(\frac{\omega L - \frac{1}{\omega C}}{R}\right)$$

So we now have the result that the voltage and the current are related by:

$$\begin{aligned} V &= ZI \\ &= |Z|\, e^{j\phi} I \\ \text{or} \quad V_0 e^{j\omega t} &= |Z|\, I_0 e^{j(\omega t + \phi)} \end{aligned}$$

The sign of the phase angle can be determined by using the equation for ϕ given above or by using the diagram shown in Figure 13.2.

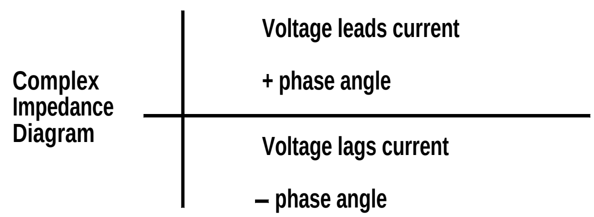

Figure 13.2: Identification of the sign of the phase angle.

In this diagram, the current waveform is taken as the reference and the sign of the phase angle is + when the resultant impedance lies in the top half of the diagram and − when the resultant impedance lies in the bottom half of the diagram. This is probably best remembered by saying that a current through a component gives a voltage across the component.

If $I = I_0 \sin(2\pi f t)$ then $V = V_0 \sin(2\pi f t + \phi)$.

13.1 Example

13.1 Calculate the complex impedance of 100 nF in series with 820 Ω at a frequency of 3.5 kHz.

The magnitude of the impedance is given by:

$$\begin{aligned} |Z| &= \sqrt{820^2 + \left(\frac{1}{2\pi 3.5 \times 10^3 \times 100 \times 10^{-9}}\right)^2} \\ &= 938\,\Omega \end{aligned}$$

and the phase angle is given by:

$$\begin{aligned}\phi &= \tan^{-1}\left(\frac{\frac{-1}{2\pi fC}}{R}\right) \\ &= \tan^{-1}\left(\frac{-1}{2\pi fCR}\right) \\ &= -0.51 \text{ radians} \quad \text{or} \quad -29^\circ\end{aligned}$$

The voltage, V, and the current, I, will then be related by the equation:

$$V = 938 \times e^{-0.51j} I$$

Note that when exponential notation is used the value for ϕ is always in radians.

13.2 Problems

13.1 Verify that the waveform diagrams shown in Figures 10.2 and 11.2 give signs of the phase angle which are in agreement with the signs of the phase angles obtained from Figure 13.2.

13.2 Plot on a complex impedance diagram the impedances of a 2.2 kΩ resistor and a 0.2 μF capacitor for a frequency of 1.2 kHz. Use the plot to estimate the magnitude of the impedance and the phase angle when these two components are connected in series.

13.3 Plot the complex impedance diagram for 3.3 kΩ in series with 47 mH at a frequency of 1.3 kHz. Calculate the magnitude of the impedance and the phase angle.

13.4 Plot the locus or path of the tip of the impedance vector for a resistor of 1.5 kΩ in series with a capacitance of 0.1 μF as the frequency is varied from 100 Hz to 50 kHz.

13.5 An inductance of 0.1 mH, a resistance of 680 Ω and a capacitance of 0.22 μF are connected in series. Sketch the path followed on the complex impedance diagram by the tip of the resulting complex impedance vector as the frequency is varied from 10 kHz to 90 kHz.