Unit 11 Inductors

- The unit of inductance is the henry and the circuit symbol used for the inductor is:

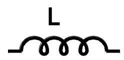

Figure 11.1: Circuit symbol for inductor.

- A rate of change of current of one amp per second through an inductor of one henry gives a voltage across the inductor of one volt.
- If the current which flows through an inductor is given by:

$$I = I_0 \sin(2\pi f t)$$

then the voltage across the inductor is:

$$V = L 2\pi f I_0 \sin(2\pi f t + \frac{\pi}{2})$$

An inductor is basically a coil of wire, with or without an iron or ferrite core. The current through the wire sets up a magnetic field for which the flux lines loop through the coil. Energy is stored in this magnetic field. If the current in the coil is changed then energy is either added to or removed from the energy stored in the magnetic field. Therefore in carrying out calculations on inductors, we are concerned with the changes in the current through the inductor.

If a current is set flowing in a coil made from superconducting metal (very low temperatures) then the current will flow indefinitely and the magnetic field of the coil will remain constant. This feature is used in the superconducting magnets, operated at liquid helium temperatures, which are found in the magnetic resonance tomography scanners used in hospitals.

At room temperatures, the resistance of the wires will usually cause the current to die away in a short time and the energy stored in the magnetic field will be dissipated in resistive heating of the coil.

The laws of electromagnetic induction, which derive from Lenz's law, give the relationship between the rate of change of current in a coil and the induced emf:

$$\mathcal{E} = -L\frac{dI}{dt}$$

where the unit of inductance, H, is the henry.

If an external voltage is applied across an inductor so as to give a current in the inductor which varies sinusoidally in accordance with:

$$I = I_0 \sin(2\pi f t)$$

then from Lenz's law, this externally applied voltage is opposed by the induced emf:

$$\begin{aligned}
V &= -\mathcal{E} \\
&= L\frac{dI}{dt} \\
&= L\frac{d}{dt}\left(I_0 \sin(2\pi f t)\right) \\
&= L 2\pi f I_0 \cos(2\pi f t) \\
&= L 2\pi f I_0 \sin(2\pi f t + \frac{\pi}{2}) \\
&= L\omega I_0 \sin(\omega t + \frac{\pi}{2}) \\
\text{where} \quad \omega &= 2\pi f \quad \text{is the angular frequency}
\end{aligned}$$

Therefore, for a sinusoidal current through an inductor, the voltage across the inductor is 90° out of phase with the current waveform.

There is one significant difference between inductors and resistors or capacitors. It is possible to combine inductors in series or parallel and calculate a single equivalent inductor only when the inductors are sufficiently separated so that no inductor is in the magnetic field of another inductor.

There are two main types of inductor:

- **Air cored inductors** where the wire used is stiff enough to support itself or else is wound on a plastic or paper supporting former. For air cored inductors, the inductance in nH $= 10^{-9}$ H is given by:

$$L(\text{nH}) = \frac{N^2 d^2}{0.46d + 1.02b}$$

  where N is the number of turns, d is the coil diameter in mm and b is the length of the coil in mm.

- **Ferrite cored inductors** have higher values for the inductance because of the higher magnetic permeability of the ferrite core. The ferrite core manufacturer will usually give a formula for the inductance which takes into account the ferrite type, size etc. A typical formula would be:

  $$L(\text{nH}) = N^2 A_L$$

  where L is the inductance in nH, N is the number of turns of wire in the coil and A_L is the inductance factor which is provided by the manufacturer of the ferrite core. Ferrite cores usually have an upper operating frequency due to the limited frequency response of the ceramic ferrite material.

11.1 Example

11.1 A sinusoidally varying current of 2 mA amplitude and frequency 3 kHz passes through a coil having an inductance of 10 mH. Calculate the voltage across the inductor and graph the current and voltage waveforms.

$$\begin{aligned} V &= L\frac{dI}{dt} = L\frac{d}{dt}(I_0 \sin(2\pi f t)) = L2\pi f I_0 \sin(2\pi f t + \frac{\pi}{2}) \\ &= 0.01 \times 2\pi 3 \times 10^3 \times 2 \times 10^{-3} \times \sin(2\pi 3 \times 10^3 \times t + \frac{\pi}{2}) \\ &= 0.377 \sin(18849t + 1.57) = 0.377 \cos(18849t) \end{aligned}$$

The current and voltage waveforms are shown in Figure 11.2.

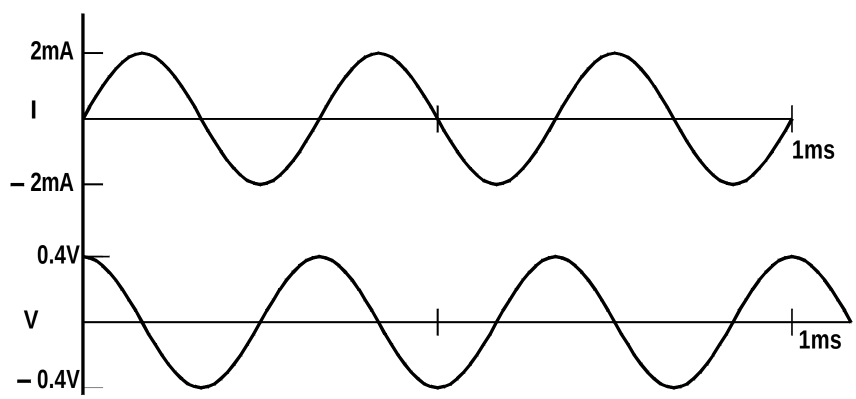

Figure 11.2: Example 11.1. Current and voltage waveforms for inductor.

Compare these current and voltage waveforms for an inductor shown in Figure 11.2 to the voltage and current waveforms for a capacitor shown in Figure 10.2 and you will see that the phase shifts of the current waveforms are of opposite sign for capacitors and inductors.

11.2 Problems

11.1 If the current in a 0.35 H inductor changes at a steady rate of $0.8\,\mathrm{A\,s^{-1}}$, calculate the voltage across the inductor. Assume that the inductor has zero resistance.

11.2 A voltage of 20 V is applied across a 0.015 H inductor. Calculate the initial rate of change of current in the inductor.

11.3 Calculate the inductance of an air cored inductor having 30 turns, a diameter of 12 mm and a length of 20 mm.

11.4 An inductor of value $2.9\,\mu$H is required for a radio frequency tuning circuit. If the inductor is to be wound from 0.5 mm diameter wire which is stiff enough to give a self supporting coil, calculate suitable dimensions for the coil and the number of turns required. Is there a single, unique answer?

11.5 Calculate the number of turns required for an inductor having a value of $590\,\mu$H constructed using a ferrite core type for which the manufacturer quotes an inductance factor, $A_L = 250$.

11.6 A sinusoidal voltage of 6 V amplitude at 1 kHz is applied across a 200 mH inductor. Calculate the current in the inductor and plot the current and voltage waveforms. Obtain the expression for the current waveform.

11.7 If the amplitude of a sinusoidal current waveform in a 30 mH inductor is 12 mA and the frequency is 2 kHz, calculate the amplitude of the voltage across the inductor. Plot the current and voltage waveforms and show the phase difference. Obtain the equations for the current and for the voltage.