Unit 10 Capacitors

- The voltage, in volts, across a capacitor, in farads, is related to the charge, in coulombs, by:

$$Q = C \times V$$

- When capacitors are connected in parallel the resultant capacitance is given by:

$$C_p = C_1 + C_2 + C_3 + \cdots$$

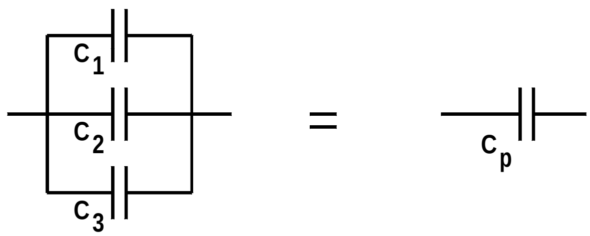

Figure 10.1: Capacitors in parallel.

- When a sinusoidal voltage of the form:

$$V = V_0 \sin(2\pi f t)$$

is applied across a capacitor, the current is 90° or $\frac{\pi}{2}$ radians out of phase with the driving voltage and is given by:

$$I = CV_0 2\pi f \sin(2\pi f t + \frac{\pi}{2})$$

The fundamental defining equation for a capacitance, $Q = CV$, can be differentiated to obtain the current:

$$I = \frac{dQ}{dt} = C\frac{dV}{dt}$$

Therefore the current through a capacitor is proportional to the capacitance and to the rate of change of the voltage across the capacitor. But you should note that the electrons that enter one lead of the capacitor are not the same

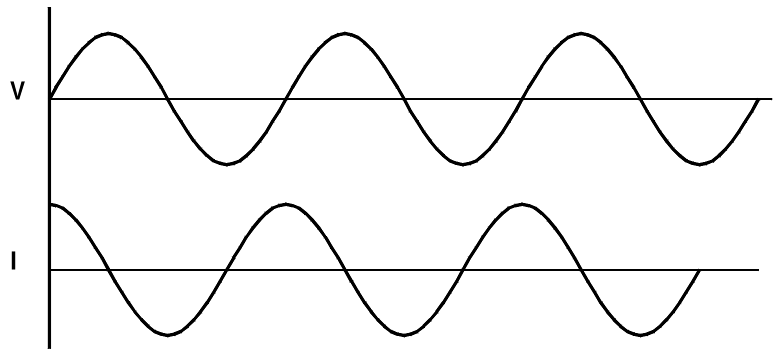

Figure 10.2: Steady state voltage and current waveforms in a capacitor.

electrons that emerge from the other lead. There is, however, a net transfer of charge through the capacitor.

Suppose we apply a sinusoidal voltage waveform across a capacitor.

$$\begin{aligned} V &= V_0 \sin(2\pi f t) \\ \text{We then get} \quad I &= C\frac{dV}{dt} \\ &= CV_0 2\pi f \cos(2\pi f t) \\ &= CV_0 2\pi f \sin(2\pi f t + \frac{\pi}{2}) \end{aligned}$$

Therefore, for a sinusoidal waveform, the voltage across a capacitor and the current through a capacitor are 90° or $\frac{\pi}{2}$ out of phase with each other. From the definition of phase angle in Unit 8 we see that the phase angle of the current waveform is positive with respect to the voltage waveform which we take as the reference and the current waveform therefore leads the voltage waveform.

In Unit 6, we saw that the power dissipation in a resistor is given by $P = V \times I$. We use the same equation to calculate the power dissipation in a capacitor. We have, for the instantaneous power dissipation:

$$\begin{aligned} P &= V \times I \\ &= V_0 \sin(2\pi f t) \times CV_0 2\pi f \sin(2\pi f t + \frac{\pi}{2}) \\ &= CV_0^2 2\pi f \sin(2\pi f t) \sin(2\pi f t + \frac{\pi}{2}) \\ &= CV_0^2 2\pi f \sin(2\pi f t) \cos(2\pi f t) \\ &= CV_0^2 \pi f \sin(4\pi f t) \end{aligned}$$

The average power is then obtained by integration:

$$P_{Ave} = \frac{1}{T}\int_0^T CV_0^2 \pi f \sin(4\pi f t) dt = 0$$

so that no power is dissipated in a capacitor.

10.1 Example

10.1 If a 15 $V_{\text{Amplitude}}$, 20 kHz sinusoidal voltage is applied across a 0.1 μF capacitor, calculate the current in the capacitor.

The current is calculated as follows:

$$\begin{aligned} I &= CV_0 2\pi f \sin(2\pi f t + \frac{\pi}{2}) \\ &= 0.1 \times 10^{-6} \times 15 \times 2\pi \times 20 \times 10^3 \times \sin(2\pi \times 20 \times 10^3 \times t + \frac{\pi}{2}) \\ &= 0.188 \sin(1.256 \times 10^5 \times t + 1.57)\ A_{\text{Amplitude}} \end{aligned}$$

and the expected oscilloscope traces of the input voltage and output current waveforms are shown in Figure 10.3.

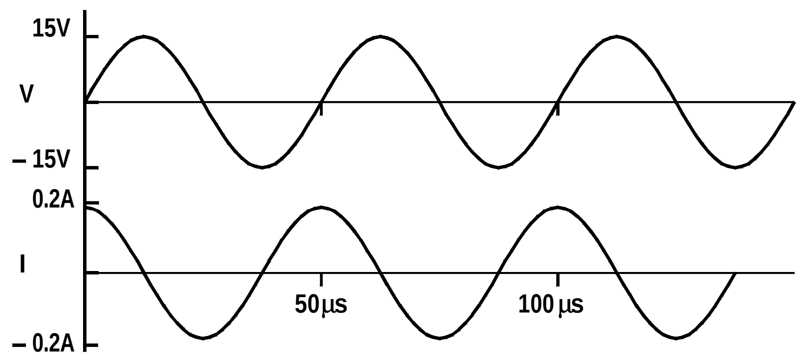

Figure 10.3: Example 10.1.

10.2 Problems

10.1 A 100 μF capacitor is connected across the 220 V_{RMS}, 50 Hz mains supply. Calculate the RMS current which flows in the capacitor. (Such capacitors are sometimes used for power factor correction and help to keep the voltage and current in phase when large motors are in use.)

10.2 A voltage of 3 V_{pp} at 10 MHz is applied across a 2 nF capacitor. Calculate the current in the capacitor. If the frequency is increased to 15 MHz, calculate the new current.