Unit 9 Voltage and power in AC circuits

- When the magnitude of the voltage waveform is specified in volts RMS (root mean square), the average power dissipated in a resistor, R, is given by:

$$\text{Power} \quad P = \frac{V_{RMS}^2}{R} = I_{RMS}^2 \times R \quad \text{watts}$$

- For a sinusoidal waveform:

$$\begin{aligned} V_{Amplitude} &= 1.4 \times V_{RMS} \\ V_{Peak-to-Peak} &= 2 \times V_{Amplitude} \end{aligned}$$

At any instant, the power dissipation in an electrical component is given by $P = V \times I$. For the special case of a resistor, we have Ohm's law, $V = I \times R$, and when we substitute for I, we get $P = \frac{V^2}{R}$. However, we also need to know the average power dissipation. This can be calculated, in the case of a sinusoidal waveform, by integrating the instantaneous power dissipation over a cycle and then averaging over the periodic time, T.

$$\begin{aligned} P_{Average} &= \frac{1}{T}\int_0^T \frac{V^2}{R} dt \\ &= \frac{1}{T}\int_0^T \frac{V_0^2 \sin^2(\frac{2\pi t}{T})}{R} dt \\ &= \frac{1}{T}\int_0^T V_0^2 \frac{1 - \cos\frac{4\pi t}{T}}{2R} dt \\ &= \frac{1}{T}\left[\frac{V_0^2 t}{2R} + \frac{V_0^2 T \sin\frac{4\pi t}{T}}{8\pi R}\right]_0^T \\ &= \frac{V_0^2}{2R} \\ \text{Let} \quad 1.414 \times V_{RMS} &= V_0 \\ \text{which gives} \quad P_{Average} &= \frac{V_{RMS}^2}{R} \\ \text{Form factor for sinusoids} &= 1.414 = \sqrt{2} \end{aligned}$$

When a meter is used to measure alternating voltage, the value indicated is usually in volts root mean square. When an oscilloscope is used to measure voltages, the most convenient operation is to measure the peak-to-peak voltage and use the form factor to convert to volts rms. It is important to specify the type of measurement and the type of voltage unit in use. If a nonsinusoidal voltage waveform is present then the integration shown above will be different and the form factor used in the conversion will be different from the 1.4 used with sinusoidal waveforms.

Some of the digital storage oscilloscopes on the market have facilities which allow you to display a waveform and also have an indication at the bottom of the screen which gives the numerical value of voltage magnitude in either *pp* or *rms* according to a selection on a menu. If you get an opportunity, you should try out this feature on a digital oscilloscope.

9.1 Example

9.1 Calculate the RMS voltage for the oscilloscope tracing of the voltage waveform shown in Figure 9.1. Calculate the frequency. The oscilloscope settings are 2 ms/division for the time base and 5 V/division for the Y amplifier.

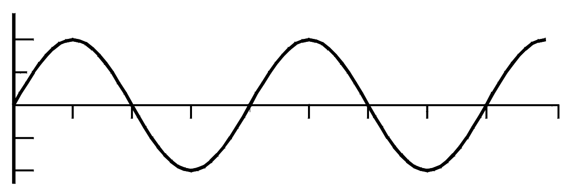

Figure 9.1: Example 9.1.

From the figure the amplitude is 2×5 V and the period is $4 \times 2 \times 10^{-3}$ s.

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\begin{aligned}
V_{RMS} &= \frac{2 \times 5}{1.41} = 7.07\,\mathrm{V} \\
\text{and} \quad \text{Frequency} &= \frac{1}{\text{Period}} = \frac{1}{4 \times 2 \times 10^{-3}} \\
&= 125\,\mathrm{Hz}
\end{aligned}
$$

9.2 Problems

9.1 Calculate the RMS voltage for a 6 V_{pp}, 1 kHz sinusoidal waveform. Will the RMS Voltage change if the frequency changes to 1.5 kHz?

9.2 Give a scaled sketch of the waveform which you would observe on an oscilloscope which is displaying an 11 V_{RMS}, 300 Hz sinusoidal waveform.

9.3 Plot a graph of a 400 kHz, 6 V amplitude square wave. Use either a square counting method or integration to calculate the RMS voltage. What is the form factor for a square wave?

9.4 A 500 Hz square waveform, of amplitude 8 V, is superimposed on a 9 V DC voltage. Calculate the RMS voltage. Is the RMS voltage greater when there is a DC component present in the signal?

9.5 Calculate the form factor for the trapezoidal voltage waveform shown in Figure 9.2.

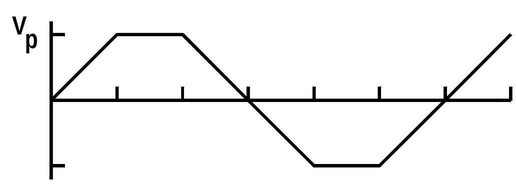

Figure 9.2: Problem 9.5.