Unit 8 AC and DC waveforms

Three numbers are used to specify a sinusoidal waveform:

- Amplitude of the waveform, either voltage, V_0, or current, I_0
- Frequency of the waveform, f, in hertz (Hz) or angular frequency, ω, (radians per second)

  OR

  The period, T, in seconds (s) given by $T = \frac{1}{f}$
- The phase, ϕ, in degrees or radians, measured with respect to a reference waveform of the same frequency:

$$\begin{aligned} \omega &= 2\pi f \\ T &= \frac{1}{f} \\ V &= V_0 \sin(2\pi f t + \phi) \quad \text{for voltage waveforms} \\ I &= I_0 \sin(2\pi f t + \phi) \quad \text{for current waveforms} \end{aligned}$$

Calculations are usually made using radian mode.

The sign of the phase shift is the same as the sign of the output waveform measured at the time of a positive going zero crossing of the reference sine wave. The phase angle of the lower waveform in Figure 8.1 is negative.

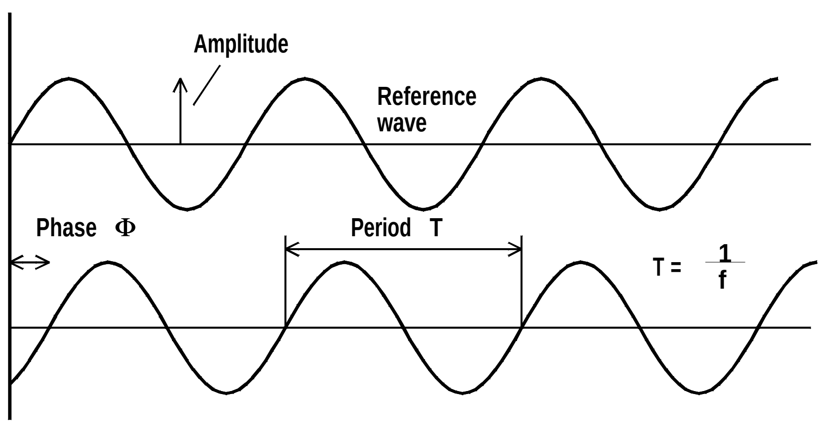

Figure 8.1: Specification of sinusoidal waveforms.

The single cells or batteries of cells in series which we have used in the circuit diagrams give constant output voltage resulting from the conversion of chemical energy into electrical energy. This is a very inefficient process and would not be capable of supplying the electrical energy used in a modern society.

The conversion of mechanical energy to electrical energy in a generator is a much more efficient process and also does not leave a residue of used batteries to be disposed of! The operation of a generator depends on Lenz's law which states that when the magnetic flux through a conducting loop changes, a voltage is generated or induced in the conductor which is proportional to the rate of change of the magnetic field, B, the area of the loop, A, and the number of turns of conductor, N. The induced emf, $\mathcal{E}$, is then given by:

$$\mathcal{E} = -NA\frac{dB}{dt}$$

The sign of the induced emf is such that it would drive a current in the conducting loop which would oppose the change in magnetic flux intersecting the loop. The work done by the mechanical motor or hydroelectric source of power in moving the loop in the magnetic field is converted into electrical energy at the output of the generator. The rotary motion of the conducting loop causes the magnetic flux direction through the loop to alternate which gives an alternating voltage waveform at the output of the generator and a consequent alternating current. In electronics we will often use alternating voltages which do not originate from rotation of coils in magnetic fields but the description of the waveforms is the same.

The power companies in Europe supply electrical current at a frequency of 50 Hz. In America the frequency is 60 Hz. In some specialist applications such as aircraft power systems the line frequency can be higher, 400 Hz, since this reduces the weight of transformers and generators. On some railway systems DC or low frequency AC is sometimes used since DC or low frequency AC electrical motors give greater torques at low speeds than can be obtained from normal AC induction motors. In electronics the frequencies used extend over a much wider range, loosely described as audio for 1 kHz, RF (radio frequency) for 100 MHz and microwave for 10,000 MHz.

The one great advantage of using AC is that it is possible to change the voltage either up or down by the use of a transformer without any significant loss of power. Voltage conversion in DC is much more difficult, inefficient and expensive in terms of the equipment required (motor-generators or inverters).

The term, AC, stands for alternating current and therefore the use of the term AC as a descriptor for a voltage is not a strictly valid usage but you will encounter the term AC voltages instead of the more correct term, alternating voltages.

Most electronic circuits are analyzed in terms of their response to sinusoidal waveforms. However, there are many situations where other waveforms are used. The most common are square, triangle and sawtooth waveforms, as shown in Figure 8.2. The mathematical description of these waveforms is not as simple as the sinusoidal waveform and the response of circuits to these waveforms is not as susceptible to mathematical analysis.

In specifying the phase, ϕ, of a sine wave, a reference sine wave of the same frequency is used. When this reference wave crosses zero in the positive going direction, for example at the origin in the diagram in Figure 8.1, the value of the sign of the second wave is observed. If the sign is positive then the phase, ϕ, is positive; if the sign is negative, as it is in Figure 8.1, then the phase is negative. If the second wave has a positive value of phase, it is said to **lead** the reference; if the second wave has a negative value of phase, it is said to **lag** the reference wave. Therefore, if the magnitude of the phase shift in Figure 8.1 is 60° or 1.05 radians, the equation which describes the second wave in Figure 8.1 is:

$$Y = Y_0 \sin(2\pi f t - 1.05)$$

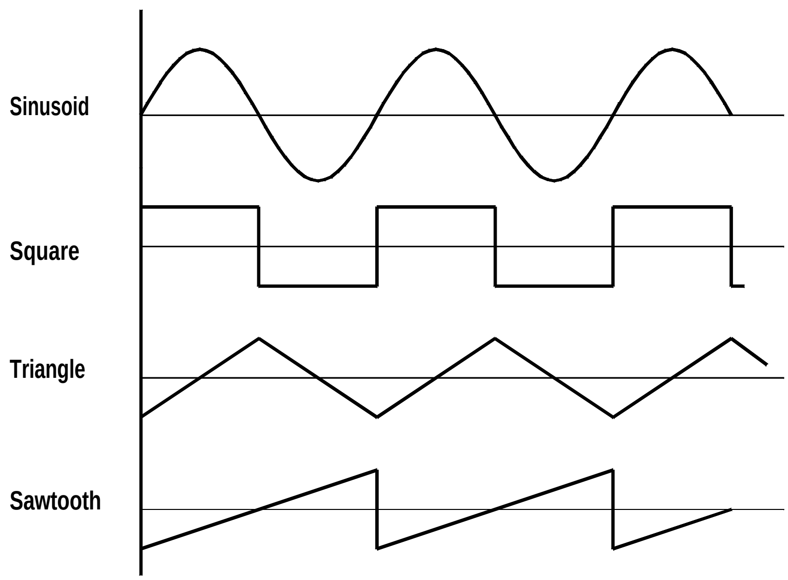

Figure 8.2: Common waveforms.

8.1 Example

8.1 Figure 8.3 shows a sketch of a sinusoidal voltage waveform displayed on an oscilloscope which has a Y axis setting of 2 V/division and a time axis setting of 5 ms/division. Calculate the amplitude, frequency, angular frequency and period of the waveform and write down the equation for the voltage waveform.

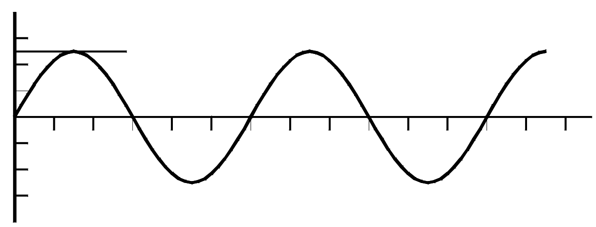

Figure 8.3: Example 8.1.

The maximum of the waveform is at 2.5 divisions and therefore the amplitude of the waveform is $2.5 \times 2\,\text{V} = 5.0\,\text{V}$.

The period of the waveform is 6 divisions and therefore $T = 6 \times 5 \times 10^{-3}$ seconds $= 30$ ms.

The frequency is $f = \frac{1}{T} = \frac{1}{30\times10^{-3}}\,\text{Hz} = 33.3\,\text{Hz}$.

The angular frequency is $\omega = 2\pi f = 209.3$ radians per second.

There is no reference waveform so we take the phase $\phi = 0$.

The equation which describes the waveform is therefore:

$$V = 5.0\sin(209.3t + 0) = 5.0\sin(209.3t)\ \text{volts}$$

8.2 Problems

8.1 A particular sinusoidal voltage waveform has parameters as follows:

$$\begin{aligned} \text{Voltage amplitude} \quad V_0 &= 13\,\text{volts} \\ \text{Frequency} \quad f &= 1.5\,\text{kHz} \\ \text{Phase} \quad \phi &= +0.6\ \text{radians} \end{aligned}$$

Give a scaled sketch of a reference voltage waveform and of this voltage waveform for time from $t = 0$ to $t = 10$ ms. Calculate the times when the voltage is zero. Calculate the period of the waveform.

8.2 The magnetic field, B, between the pole pieces of a permanent magnet is 0.15 tesla. A circular search coil of diameter 12 mm and containing 200 turns of wire is placed between the poles. Calculate the average voltage induced in the coil when the coil is withdrawn from between the poles in a time of 0.1 seconds.

8.3 Convert 47° to radians.

Convert 2.45 rad to degrees.

8.4 A waveform is specified as having a periodic time, $T = 2.6\,\text{ms}$. The voltage is $+5\,\text{V}$ for times from $n \times T$ to times $(n + \frac{1}{2})T$ and the voltage is $-5\,\text{V}$ for times from $(n + \frac{1}{2}) \times T$ to times $(n + 1) \times T$ where $n = 0, 1, 2, 3, \ldots$ Give a scaled sketch of the voltage waveform.

8.5 A particular sinusoidal voltage waveform has an amplitude of $V_0 = 7.3\,\text{V}$, a frequency of 170 Hz and a phase of $\phi = 0.4\,\text{rad}$.
Calculate the values of the voltage at times 0 s, 14 ms, 2.4 s and 10.2 s.

8.6 A triangular voltage waveform has an amplitude of 4.3 V, a phase angle of $-0.1\,\text{rad}$ measured with respect to a reference waveform and a frequency of 280 Hz.
Sketch the waveform and calculate the values of the voltages at times 2.0 ms, 10.2 ms and 180.0 ms.

8.7 A square voltage waveform has an amplitude of 4 V, a periodic time of 120 ms and crosses the zero volts level in the positive going direction at time $t = 0.1\,\text{s}$.
Sketch the voltage waveform and calculate the value at $t = 490\,\text{ms}$.