Unit 5 Resistor networks

Very great simplifications can often be made in determining resistor networks if the following four procedures are utilized:

- Redraw the circuit in a different shape while maintaining the same connections or circuit topology.
- If, by symmetry arguments or otherwise, you can find nodes which are at the same potential, then these nodes can be connected and significant circuit simplifications made.
- For components connected in series, use the fact that the same current flows in all the components.
- For components connected in parallel, use the fact that the voltage across all the components is the same.

The currents and voltages in any network of resistors can be determined by the application of the principles and methods collectively known as Kirchhoff's laws. The difficulty is that significant computation, either hand calculation or computer programming, is often required which takes time and does not always give insight into the characteristics of the circuit. If the insights which come from understanding a circuit, as distinct from simply calculating the answer, are missing, then it is unlikely that new or original circuits will be developed.

Very often a circuit is encountered in texts or manuals which is unfamilar and whose performance is unknown. In such cases, you should try redrawing the circuit in a different shape so that the component connection or topology is maintained. It will often be found that the circuit is one with which you are already familiar. Another variant of this approach is to split a large, complex circuit into blocks, each having a known characteristic.

Many resistor networks, particularly those constructed from identically valued resistors, have axes of symmetry and therefore have nodes which must be at the same potential. If two nodes which are at the same potential are connected together, no current flows from node to node so there will be no change caused to the voltages in the circuit by making the connection. The change made to the circuit may, however, make significant simplifications to the analysis of the circuit.

In particular, Example 5.3 shows how a circuit can be analyzed by redrawing the circuit rather than by formal numerical analysis.

5.1 Examples

5.1 Calculate the resistance between points A and B in Figure 5.1.

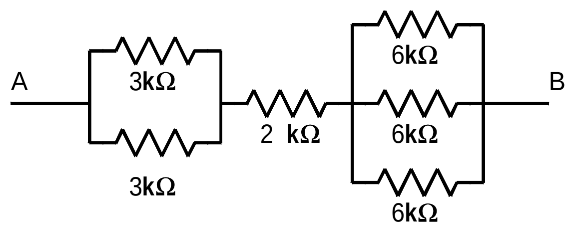

Figure 5.1: Example 5.1.

$$
\begin{aligned}
R_{AB} &= \frac{3\,\mathrm{k}\Omega}{2} + 2\,\mathrm{k}\Omega + \frac{6\,\mathrm{k}\Omega}{3} \\
&= 1.5\,\mathrm{k}\Omega + 2\,\mathrm{k}\Omega + 2\,\mathrm{k}\Omega \\
&= 5.5\,\mathrm{k}\Omega
\end{aligned}
$$

5.2 Calculate the voltages at points A, B, C, D, E in the resistive ladder network shown in Figure 5.2.

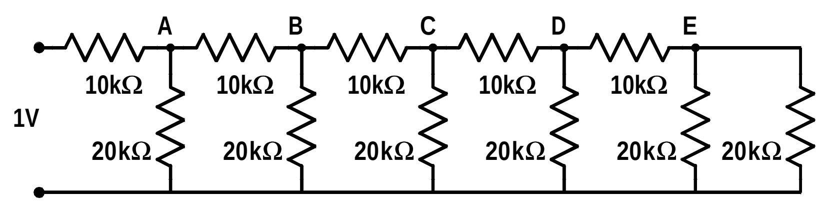

Figure 5.2: R–$2R$ resistive ladder.

This problem is derived from circuits used in the R–$2R$ type of digital to analog converters which we will meet later in Unit 54. The analysis is best carried out using a recursive technique as follows:

Consider two $10\,\mathrm{k}\Omega$ resistors in series as shown in Figure 5.3 (a). Looking from the left they appear as a $20\,\mathrm{k}\Omega$ resistor.

If one of the $10\,\mathrm{k}\Omega$ resistors is replaced by two $20\,\mathrm{k}\Omega$ resistors in parallel, we get the circuit in Figure 5.3 (b). This circuit still has a $20\,\mathrm{k}\Omega$ input

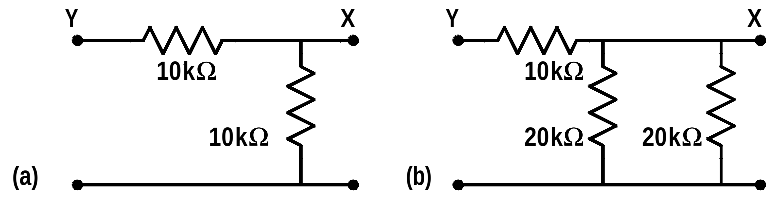

Figure 5.3: Equivalent circuits.

resistance. Also the voltage at point X is half of the voltage at point Y since we have effectively two equal resistors in a potential divider.

Now replace the right hand 20 kΩ resistor in Figure 5.3 (b) by a copy of the original circuit which has an input resistance of 20 kΩ as shown in Figure 5.4.

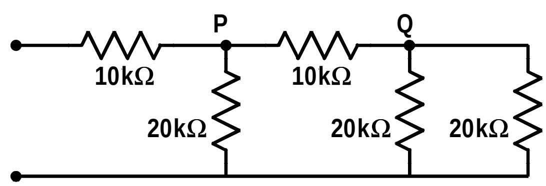

Figure 5.4: Make a substitution.

At point Q, the voltage is half of the voltage at point P because of the two 20 kΩ resistors in parallel acting as a 10 kΩ and forming a potential divider with the 10 kΩ between P and Q. At point P, the 20 kΩ and the 20 kΩ input resistance of the circuit to the right act as a 10 kΩ to form a potential divider so that the voltage at P is half of the voltage at the input to the circuit.

We can then see that the voltage at each node, A, B, C, D and E, in Figure 5.2 is half of the value at the earlier node. The voltages are therefore:

$$V_A = \frac{1}{2}\,\text{V}, \quad V_B = \frac{1}{4}\,\text{V}, \quad V_C = \frac{1}{8}\,\text{V}, \quad V_D = \frac{1}{16}\,\text{V}, \quad V_E = \frac{1}{32}\,\text{V}$$

5.3 Determine the resistance between nodes A and B in the resistor network in Figure 5.5, given that all of the resistors are of value 1 kΩ.

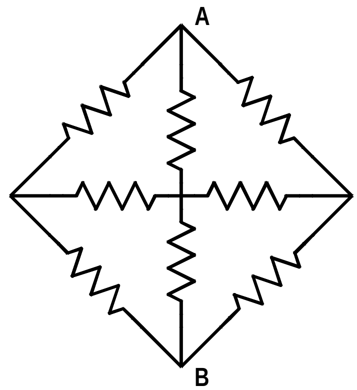

Figure 5.5: Example 5.3.

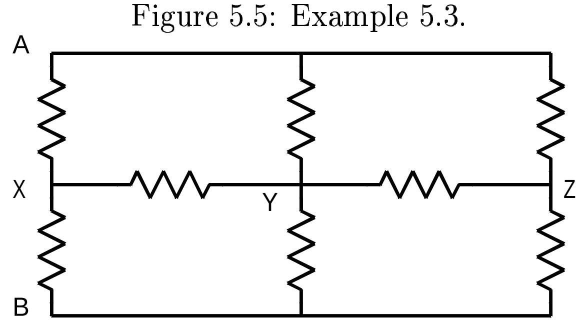

Figure 5.6: Redraw the circuit.

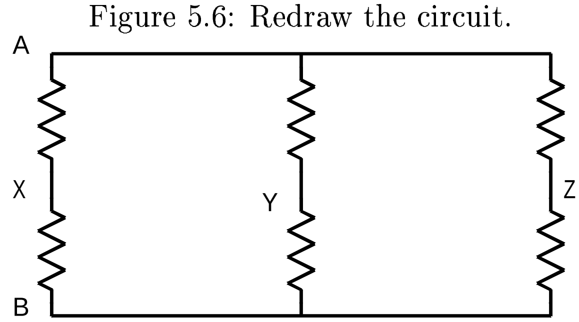

Figure 5.7: The nodes X, Y and Z are at the same potential, therefore replacing the cross resistors will not affect the circuit because no current will flow in the cross resistors. Therefore the cross resistors can be omitted.

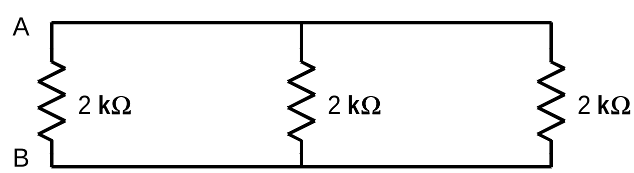

Figure 5.8: Combine series 1 kΩ resistors and then combine the parallel 2 kΩ resistors to obtain a single 666 Ω resistor.

5.2 Problems

5.1 Consider a cube having 1 kΩ resistors along the 12 edges of the cube as shown in Figure 5.9. Determine the resistance between diagonally opposite corners of the cube, A and B, as shown in the diagram.

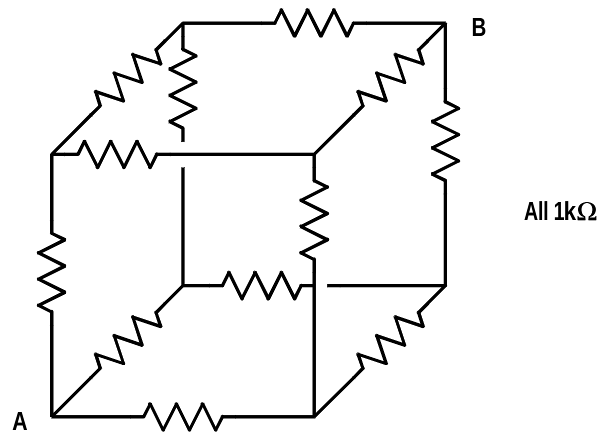

Figure 5. 9: Problem 5. 1.

5.2 Determine the resistance bet ween the points A and B in the circuit in Figure 5. 10, given that all of the resistors are of value 1 kΩ.

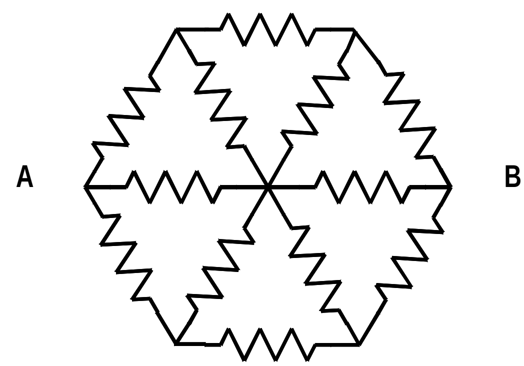

Figure 5.10: Problem 5.2.

5.3 Calculate the voltages at nodes A, B, C, D, E in the circuit shown in Figure 5.11.

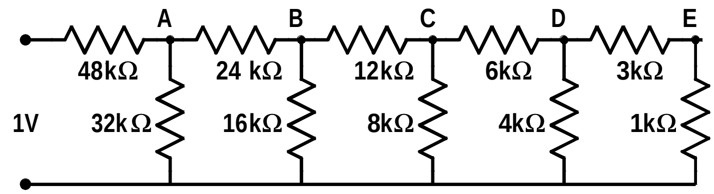

Figure 5.11: Problem 5.3.

5.4 The space between two conductive strips is occupied by a square lattice of $n \times n$ locations as shown in Figure 5.12. Metal balls are placed at random in the lattice. The diameter of the balls is such that they can touch a ball in an adjacent location or the conductive strip. At what percentage occupancy of the lattice will there be a 50% probability of a conductive path being established between the two conductive strips?

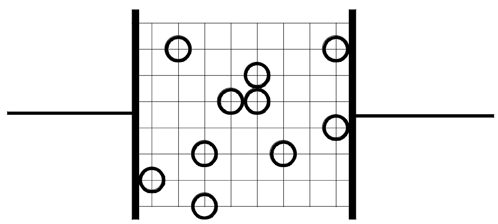

Figure 5.12: Problem 5.4.

Note that the solution to this problem is nontrivial and a computer simulation approach might be more appropriate than an analytic approach. The problem might also be suitable for use as a group exercise. The keyword library index term for further information is percolation theory. (See Chapter 5, *Introduction to Percolation Theory 2nd edn* by Dietrich Stauffer and Amnon Aharony, Taylor & Francis, 1992.)