Unit 3 Resistors in parallel

- When resistors are connected in parallel, the same voltage difference or potential difference is present across all of the resistors.

$$\text{That is} \quad V_p = I_{Total}R_p = I_1R_1 = I_2R_2 = I_3R_3 = \cdots$$

$$\text{but} \quad I_{Total} = I_1 + I_2 + I_3 + \cdots$$

$$\text{Therefore} \quad \frac{1}{R_p} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \cdots$$

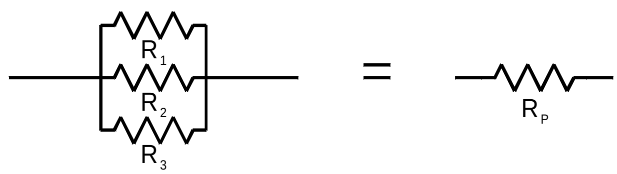

Figure 3.1: Resistors in parallel.

When current, flowing through a circuit, meets a number of resistors connected in parallel, the current divides between the resistors with part of the current flowing in each resistor. The sum of the currents in each of the resistors equals the total current flowing around the circuit. The voltage difference between the point where the current divides and the point where the current recombines is the same for all of the resistors and therefore the voltage difference across all of the resistors is the same. This gives the two equations in the summary: the IR value for each of the resistors is equal to the voltage across the resistors and the total current is the sum of the currents in the individual resistors. We can therefore consider a single resistor which we call R_p and is the equivalent parallel resistor which will allow the same total current to flow with the same voltage across the resistor as the n resistors connected in parallel.

A good analogy is to consider a river flowing past a number of islands. The total flow divides up into a number of smaller streams in the channels between the islands and recombines downstream. The change in level or head loss between a point upstream of the islands and a point downstream of the islands is the same for all possible routes taken by a cork floating in the stream.

3.1 Examples

3.1 Calculate the equivalent resistance for the parallel resistor circuit shown in Figure 3.2.

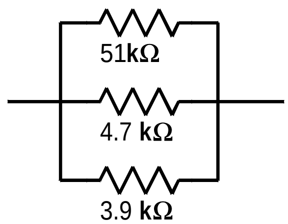

Figure 3.2: Example 3.1.

$$
\begin{aligned}
\frac{1}{R_p} &= \frac{1}{51\,\mathrm{k\Omega}} + \frac{1}{4.7\,\mathrm{k\Omega}} + \frac{1}{3.9\,\mathrm{k\Omega}} \\
&= 4.89 \times 10^{-4}\,\Omega^{-1} \\
\text{Therefore} \quad R_p &= 2046\,\Omega \\
&= 2.046\,\mathrm{k\Omega}
\end{aligned}
$$

3.2 In the circuit shown in Figure 3.3, the current in the 820 Ω resistor is measured to be 2.5 mA. Calculate the battery voltage, V_B, and also calculate the total current flowing through the battery. Calculate the current in the 3.9 kΩ resistor.

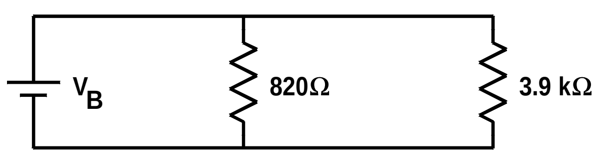

Figure 3.3: Example 3.2.

$$
\begin{aligned}
V_B &= 2.5\,\mathrm{mA} \times 820\,\Omega \\
&= 2.5 \times 10^{-3} \times 820\,\mathrm{V} \\
&= 2.05\,\mathrm{V}
\end{aligned}
$$

$$\begin{aligned}\frac{1}{R_p} &= \frac{1}{820}+\frac{1}{3900}\\ &= 1.48\times10^{-3}\,\Omega^{-1}\end{aligned}$$

$$\begin{aligned}\text{Therefore}\quad R_p &= 678\,\Omega\\ \text{Hence}\quad I_{Total} &= \frac{2.05\,\text{V}}{678\,\Omega}\\ &= 3.03\times10^{-3}\,\text{A}\\ &= 3.03\,\text{mA}\\ \text{and}\quad I_{3.9\,\text{k}\Omega} &= \frac{2.05\,\text{V}}{3.9\times10^{3}\,\Omega}\\ &= 5.26\times10^{-4}\,\text{A}\\ &= 0.526\,\text{mA}\quad\text{or}\quad 526\,\mu\text{A}\end{aligned}$$

3.2 Problems

3.1 Use the equation for the potential difference and the equation for the total current stated in the summary to derive the equation for the equivalent parallel resistor stated in the summary.

3.2 Calculate the equivalent resistance for the parallel resistor circuit shown in Figure 3.4.

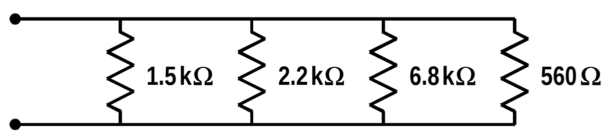

Figure 3.4: Problem 3.2.

3.3 If the current in the 470 Ω resistor in Figure 3.5 is measured to be 1.9 mA, calculate the battery voltage, V_B, and also calculate the total current flowing through the battery. Calculate the currents in the 6.8 kΩ and 5.6 kΩ resistors.

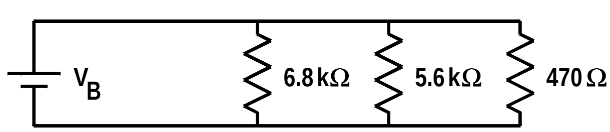

Figure 3.5: Problem 3.3.

3.4 If the total current flowing in the circuit in Figure 3.6 is measured to be 45 mA, calculate the voltage setting of the variable voltage power supply.

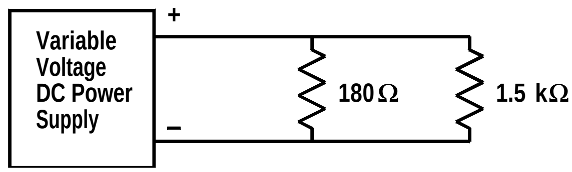

Figure 3.6: Problem 3.4.

3.5 In the circuit in Figure 3.7, the battery voltage is 10 V. Calculate the total current drawn from the battery. Calculate the current in each of the resistors.

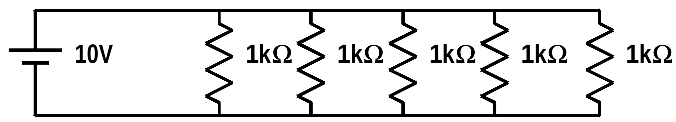

Figure 3.7: Problem 3.5.

3.6 A value of 3.6 Ω was measured for the resistance of 90 m of 16/0.2 mm insulated equipment wire (16 strands each of 0.2 mm diameter). Calculate the cross sectional area of the wire. Calculate the resistance of 1 m of a single strand of the wire. (Insulated equipment wire of this cross section of copper conductor would typically be rated to carry a maximum current of 3 A without overheating.)