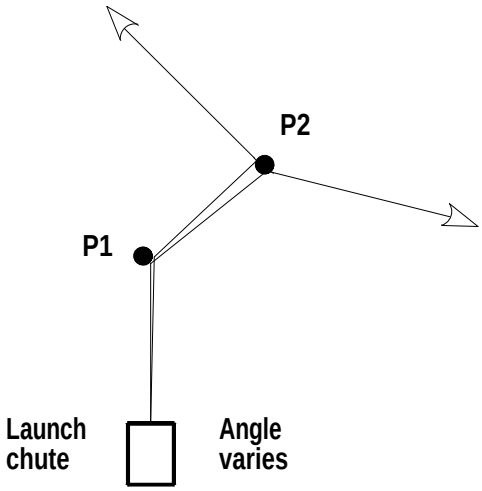
- Very sensitive dependence on the initial conditions
- Exponential divergence
- A chaotic system must have at least three degrees of freedom if the system state is never to repeat.
- Characteristic route to chaos.
- Period doubling.
- Many chaotic system orbits projected onto a plane form a strange attractor.



Pinball machine

One of the properties of chaotic systems is an exponential divergence characterized by a Lyanopov exponent.

The most common Route to chaos is the period doubling route.
Frogs living in a pond.
For reproduction rate, r,

$$x_{n+1} = r \times x_n$$

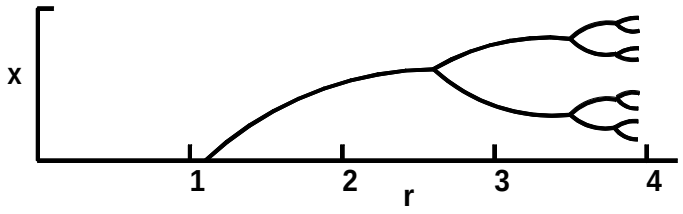
Population stable when $r = 1$.
If $r > 1$ there is a population explosion
If $r < 1$ the frogs become extinct.

Frogs eat flies
More frogs gives less flies
Flies available for food given by $(1 - x_n)$

$$x_{n+1} = r \times x_n \times (1 - x_n)$$

$$x_{n+1} = r \times x_n \times (1 - x_n)$$

$r =$	1	1.5	2.0	2.5	3.0	3.5	3.55
1001	0	0.33	0.5	0.6	0.674	0.501	0.540
1002	0	0.33	0.5	0.6	0.659	0.874	0.882
1003	0	0.33	0.5	0.6	0.674	0.383	0.370
1004	0	0.33	0.5	0.6	0.659	0.826	0.827
1005	0	0.33	0.5	0.6	0.674	0.501	0.506
1006	0	0.33	0.5	0.6	0.659	0.874	0.887
1007	0	0.33	0.5	0.6	0.674	0.383	0.354
1008	0	0.33	0.5	0.6	0.659	0.826	0.812
1009	0	0.33	0.5	0.6	0.674	0.501	0.540
1010	0	0.33	0.5	0.6	0.659	0.874	0.882



Bifurcation points on the graph
'Period doubling route to chaos'.
Sequence of numbers for $r = 4$ is not a set of random numbers because each number is computed or determined from the previous number and the system is therefore said to exhibit deterministic chaos.

- In an electronic system distortion is characterized by higher Fourier harmonics appearing in the output but this is not chaotic behaviour.

- Period doubling is characterized by subharmonics appearing in the output and this is a signature of the imminence of chaos.

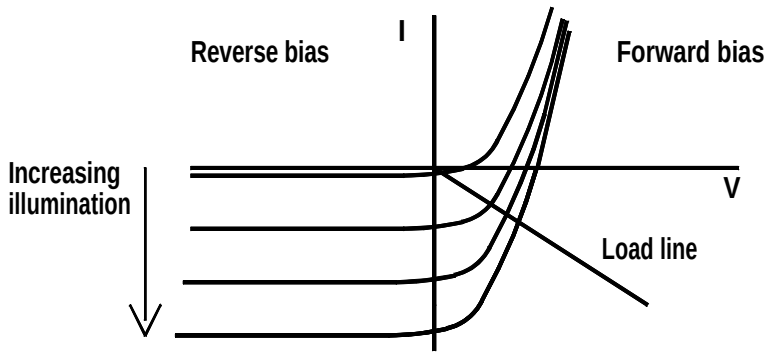
- Linear systems do not exhibit chaotic behaviour.

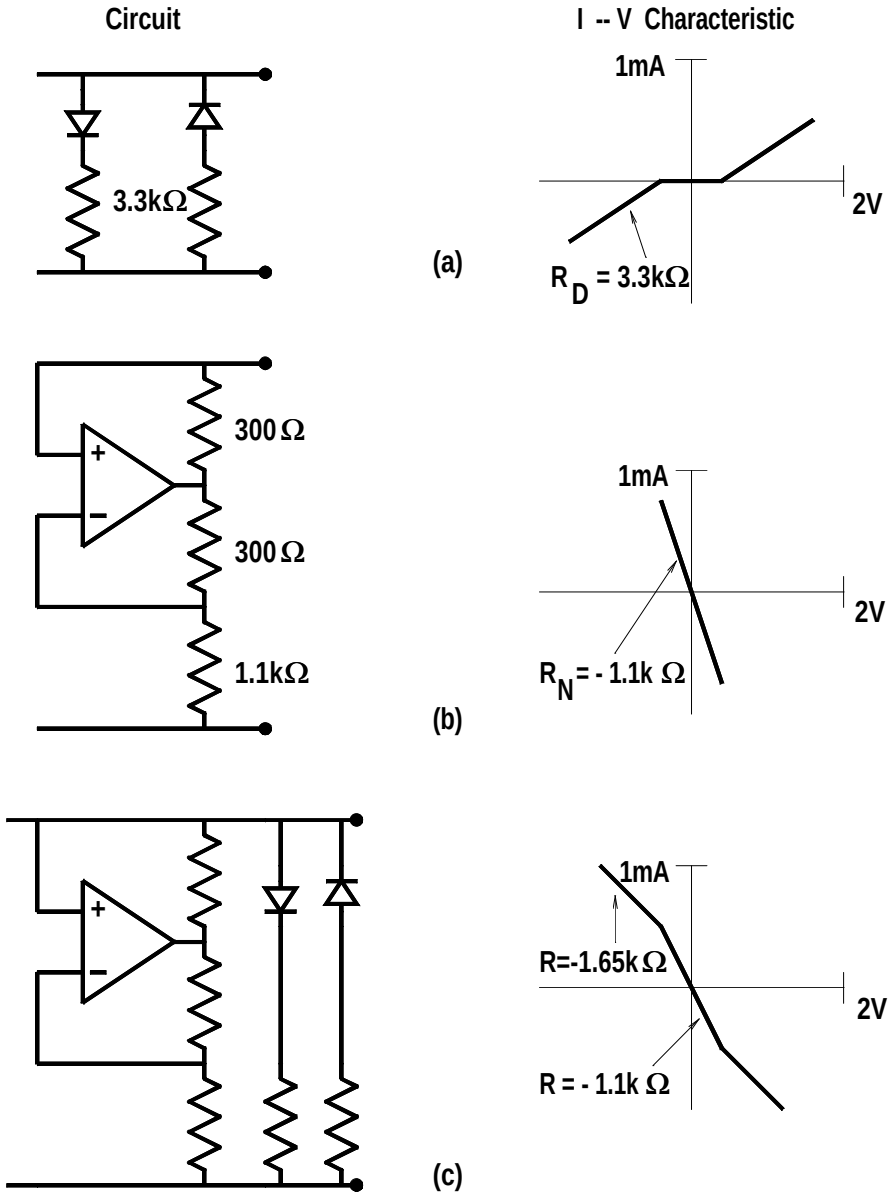
- Chua diode is essentially a nonlinear negative resistance.

- A passive negative resistance is impossible

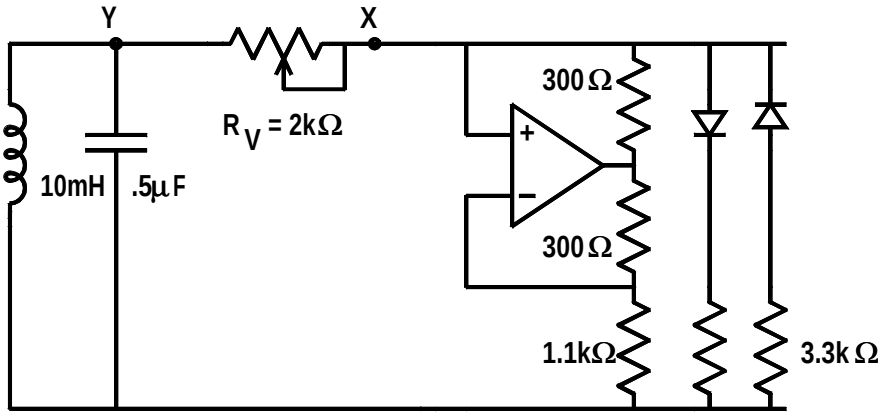
- A negative resistance requires an energy source.

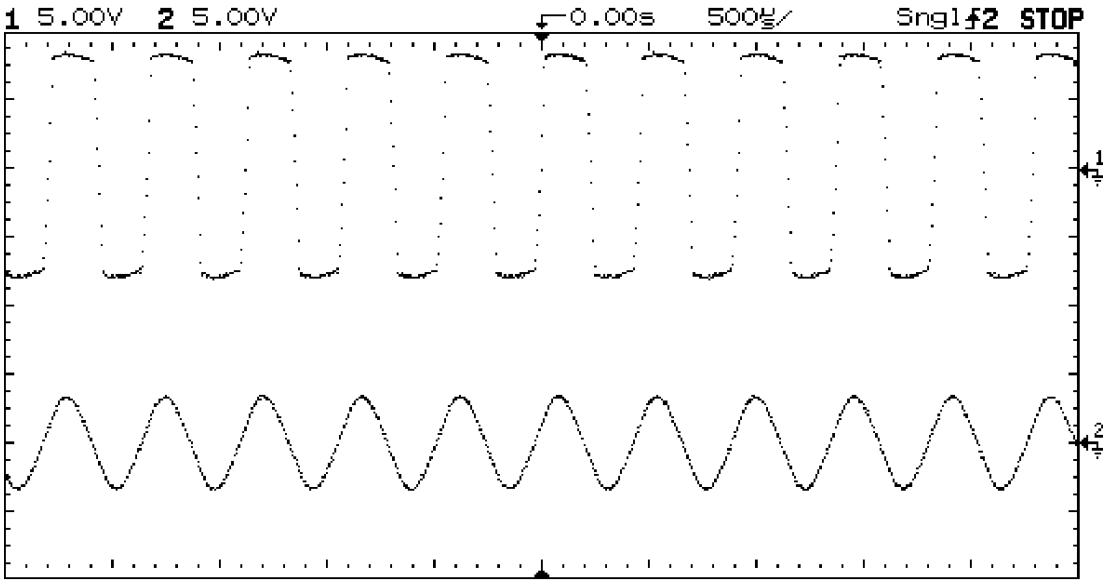
Photodiode characteristic.



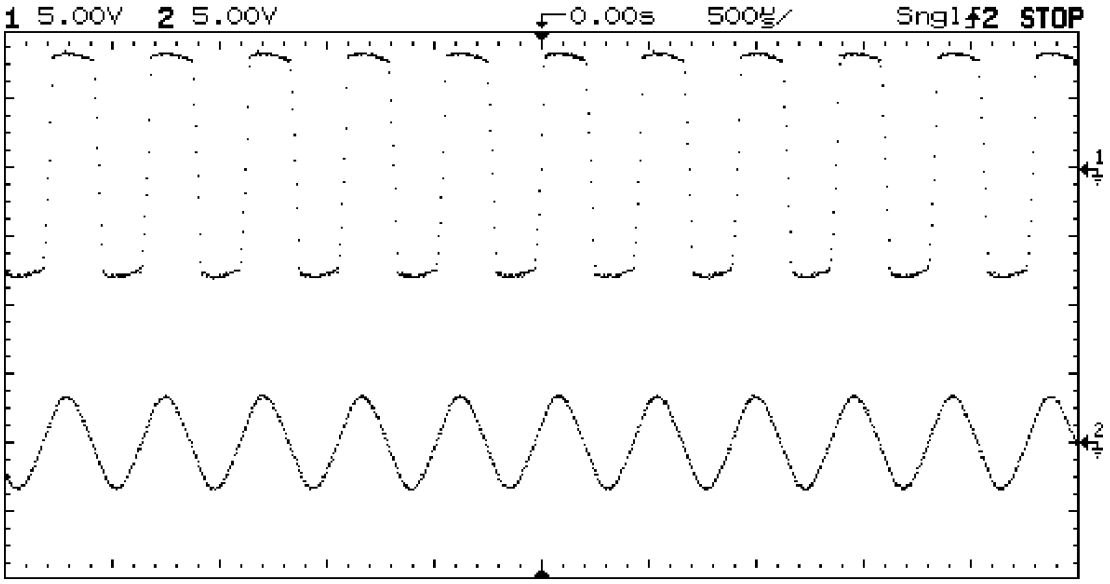


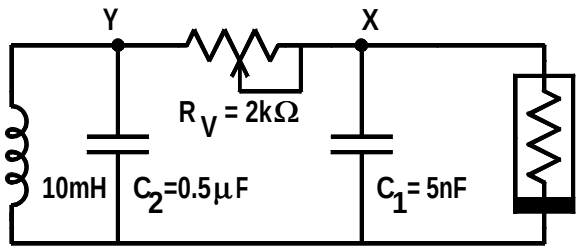
Operation of Chua's diode

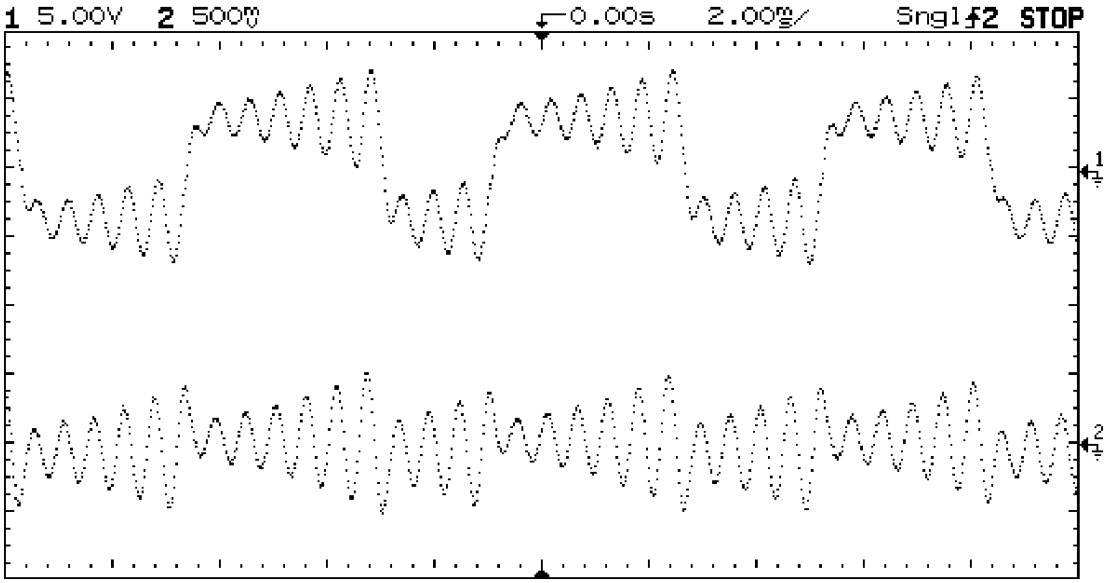




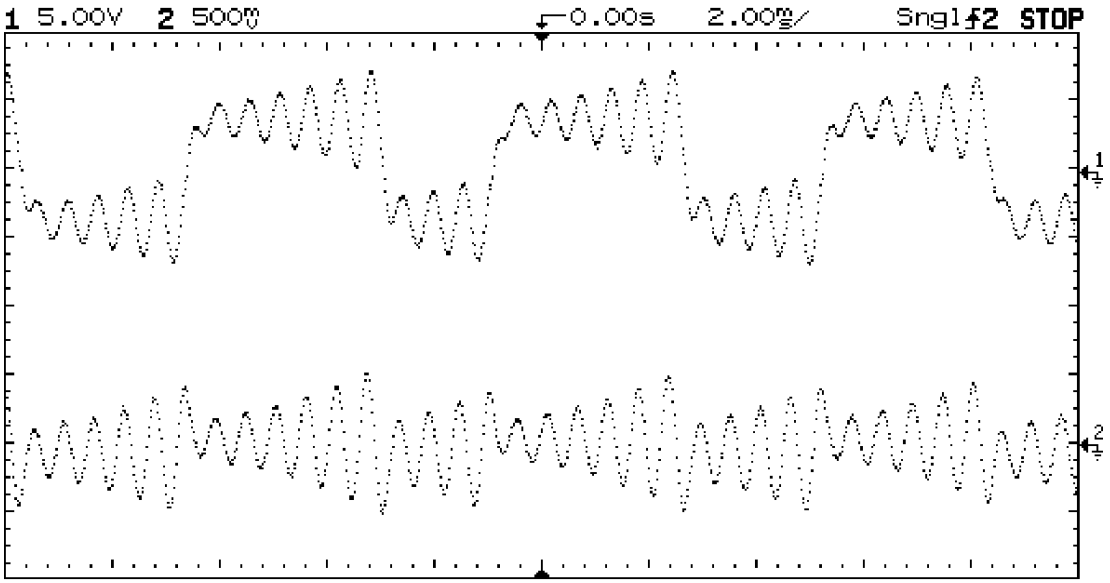
Chua's oscillator circuit.

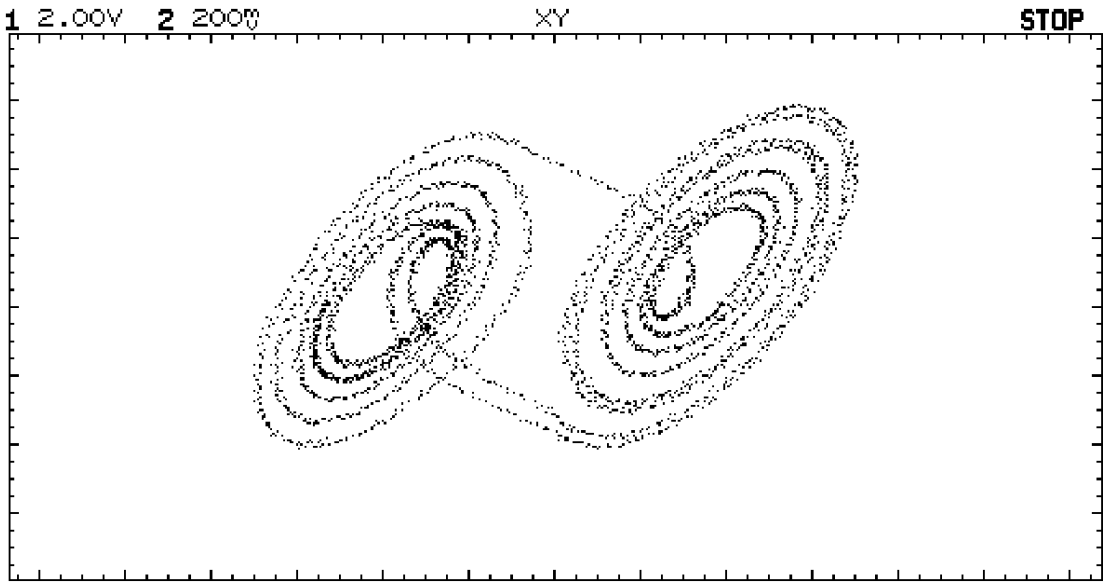

1 5.00V 2 5.00V
0.00s
500µs/
Sngl
2 STOP



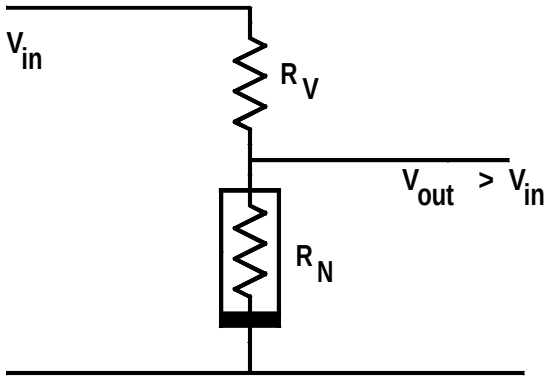


Chua's chaotic oscillator





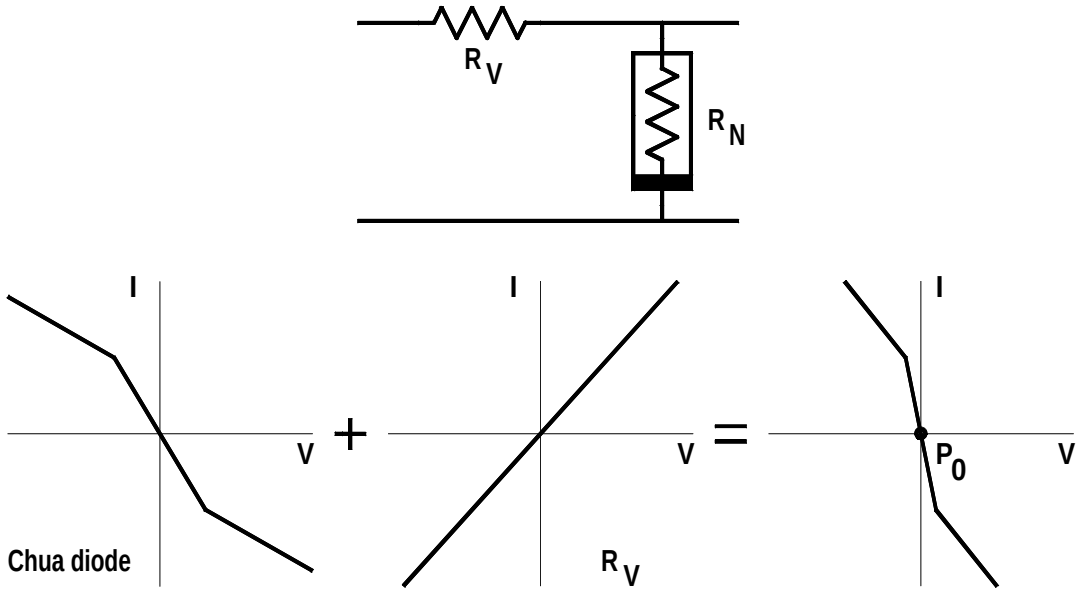
Double scroll chaotic strange attractor.



Negative resistance amplifier.

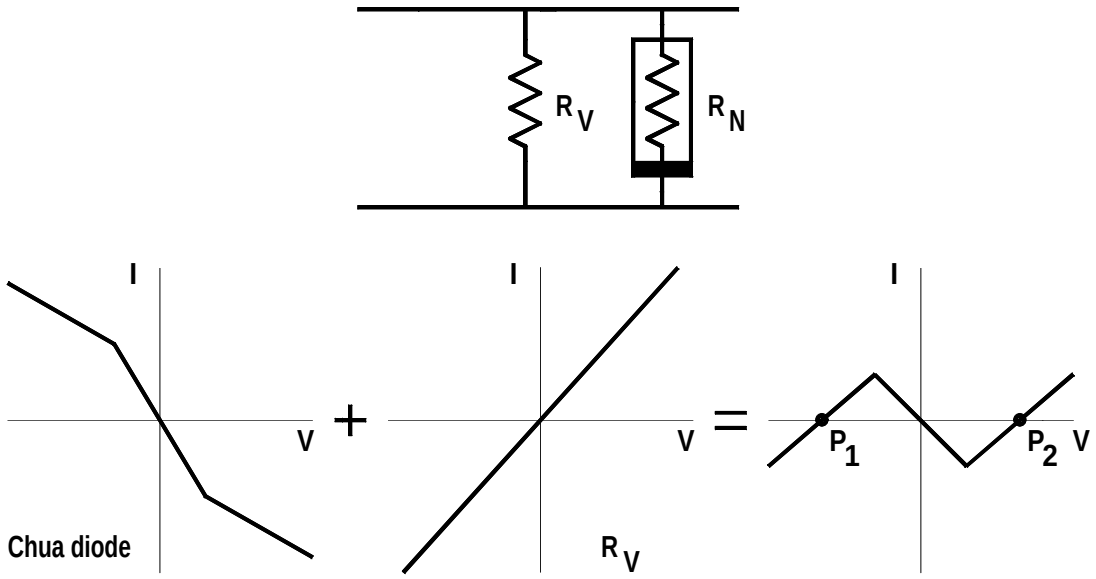
$$V_{out} = \frac{R_N}{R_V + R_N} \times V_{in}$$

But R_N is negative



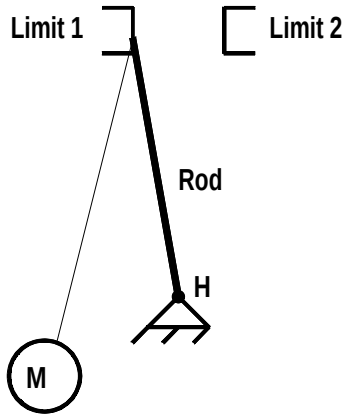
Series circuit of the R_V and the R_N of the Chua diode.

Add the voltages horizontally

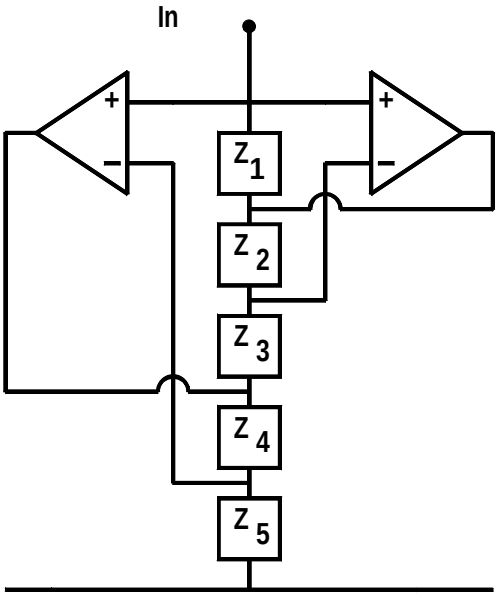


I–V characteristics for parallel R_V and R_N.

Add the currents vertically



Resonantly driven sloppy pendulum.



Riordan gyrator for which the inductance is given by:

$$Z_{in} = \frac{Z_1 Z_3 Z_5}{Z_2 Z_4}$$