

- Noise signals present within a bandwidth  $B$  are specified in units of

Volts per  $\sqrt{Hz}$  or Amps per  $\sqrt{Hz}$

- The thermal or white noise from a resistor,  $R$ , at temperature  $T$  within a bandwidth  $B$  is:—

$$V_{noise} = \sqrt{4kTRB}$$

- The shot noise associated with a DC current  $I$  is:—

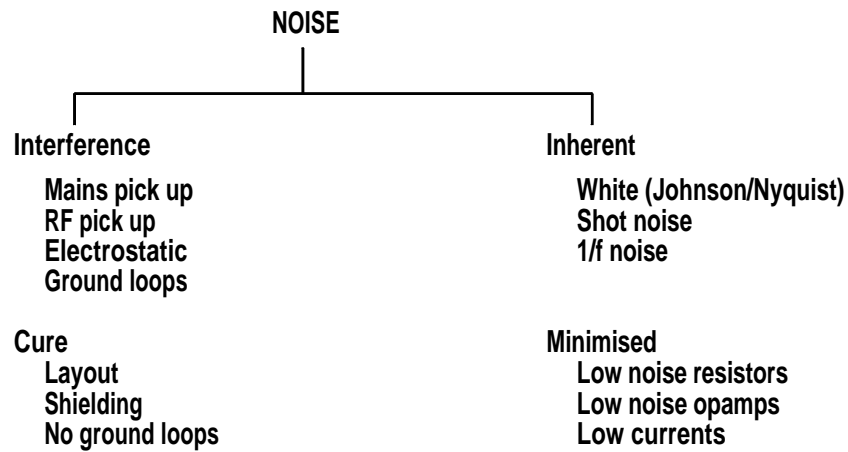
$$I_{noise} = \sqrt{2eIB}$$

- Flicker noise or one over  $F$  noise is comparable in magnitude to thermal noise at about  $100Hz$  and has a spectrum which varies as  $\frac{1}{F}$ .
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Turn up the gain of an amplifier, with no signal present at the input,  
A hiss will be heard.

Also on older TV sets snow on the screen.

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Noise sources.

EMI Electromagnetic interference must be reduced.

Standards are set for equipment.

Electrically quiet location.

Some noise is inherent.

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Units in which noise is measured.

We have a noise power spectrum.

Light can be white or pink or blue

We have the concept of white noise having a power distribution spread equally over all frequencies. We also have the concept of spectral density.

If the power in a frequency range  $\delta F = F_1 - F_2$  is measured, we can then plot the power per unit frequency as a function of frequency. We measure voltage but

$$P = \frac{V^2}{R}$$

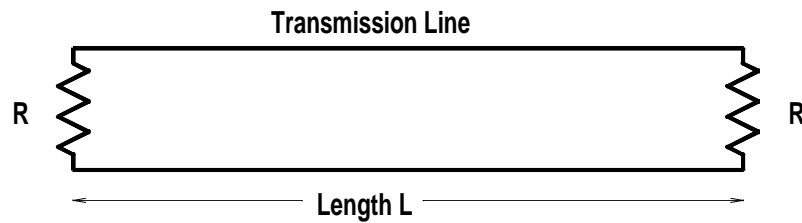
$$V \propto \sqrt{P}$$

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Therefore if we take the square root of the power spectral density we get the units of the noise voltage spectral density as

Volts per  $\sqrt{Hz}$

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Transmission line carrying noise.

White noise analyzed by Nyquist and experimentally measured by Johnson.

Resistor at a temperature  $T$  K.

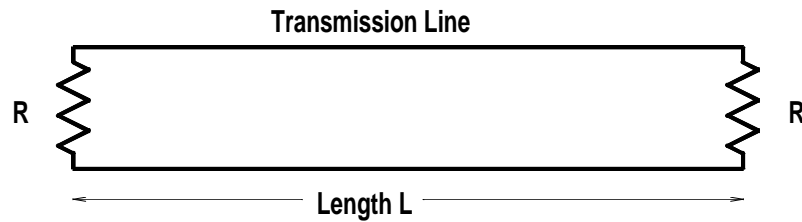
Electrons move randomly with a kinetic energy appropriate to  $kT$  where  $k$  is Boltzmann's constant.

Connect a resistor at each end of a matching transmission line

TV co-axial cable  $75\Omega$

Noiseless loss free line.

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Electrons in the resistor radiate electromagnetic signals into the transmission line

Resistor absorbs any signal traveling down the line to the resistor.

Thermal equilibrium is reached

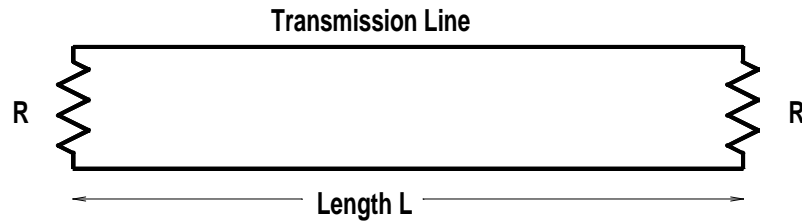
Spectral power distribution on the line is the same as the spectral distribution emitted by the resistor.

Cut the resistors from the line and leave the electromagnetic signal trapped on the line, bouncing back and forth between the ends.

What is the energy distribution of this signal on the transmission line?

Modes of the line

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Modes will be integer multiples of the fundamental,  $F_0 = \frac{C}{2L}$ .

$N$  modes between the frequency  $F_1 = p\frac{C}{2L}$  and the frequency  $F_2 = q\frac{C}{2L}$ , where  $p$  and  $q$  are integers

$$N = p - q = \frac{F_1}{\frac{C}{2L}} - \frac{F_2}{\frac{C}{2L}} = (F_1 - F_2)\frac{2L}{C}$$

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Each mode has an energy  $kT$  associated with it

Energy,  $W$ , on the line in the frequency range  $F_1$  to  $F_2$  is

$$W = NkT = (p - q)kT = (F_1 - F_2)\frac{2L}{C}kT$$

Energy was delivered into the line by the two resistors in one line transit time  $\frac{L}{C}$  and therefore the power from each resistor is

$$P = \frac{W}{\frac{2L}{C}} = \frac{(F_1 - F_2)\frac{2L}{C}kT}{\frac{2L}{C}} = (F_1 - F_2)kT = \overline{i^2}R$$

where  $\overline{i^2}$  is the mean square noise current.

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The mean square noise voltage which drives this current through the two resistors in series is then (by squaring Ohm's Law and making a substitution for  $\overline{i^2}R$ )

$$\overline{v^2} = \overline{i^2}(2R)^2 = 4\overline{i^2}RR = 4(F_1 - F_2)kTR = 4kTRB$$

where  $B = F_1 - F_2$  is the bandwidth.

The noise voltage is then

$$v_{noise} = \sqrt{4kTRB}$$

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**Shot noise.**

$$I = N \times q$$

$N$  electrons on average flow in a given time interval then the statistical fluctuation is  $\sqrt{N}$ . A measurement will be in the range  $N \pm \sqrt{N}$ . If the bandwidth of the measuring amplifier is  $B$  then the sample time is given by  $\Delta t = \frac{1}{B}$ .

$$I = \frac{Ne}{\Delta t} = NeB$$

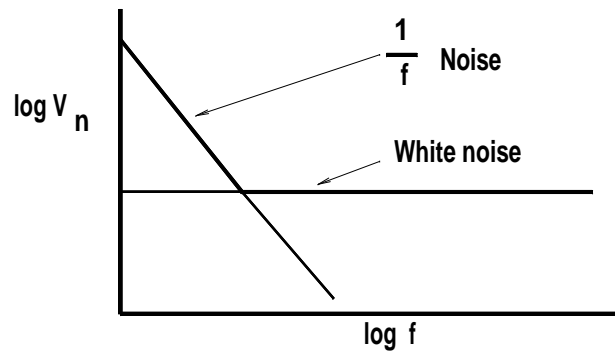
The fluctuation in the current is

$$\Delta I = eB\sqrt{N} = eB\sqrt{\frac{I}{eB}} = \sqrt{eIB}$$

More detailed calculation gives

$$\Delta I = \sqrt{2eIB}$$

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### **Flicker noise or $\frac{1}{F}$ noise.**

This mechanism is most significant at low frequencies when  $\frac{1}{F}$  is small.

The noise voltages associated with flicker noise are masked by shot noise at frequencies above some corner frequency  $F_0$ .

Typically this corner frequency is at about  $100\text{Hz}$ .

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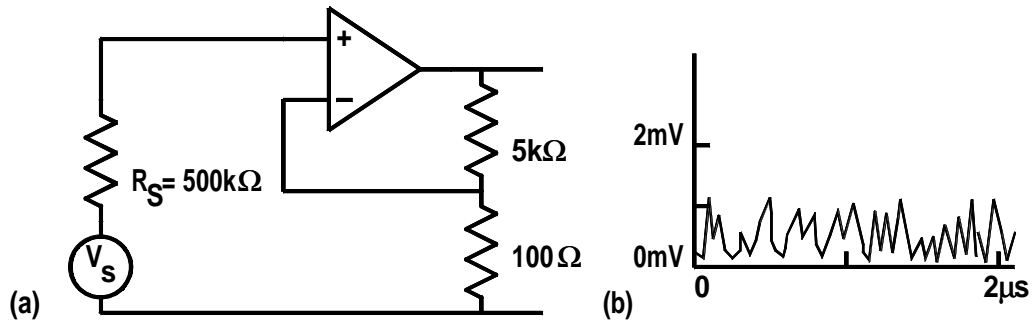
Mechanism of flicker noise not fully understood

Some models use Chaos theory and Intermittency.

It is a problem with DC amplifiers and very low frequency amplifiers

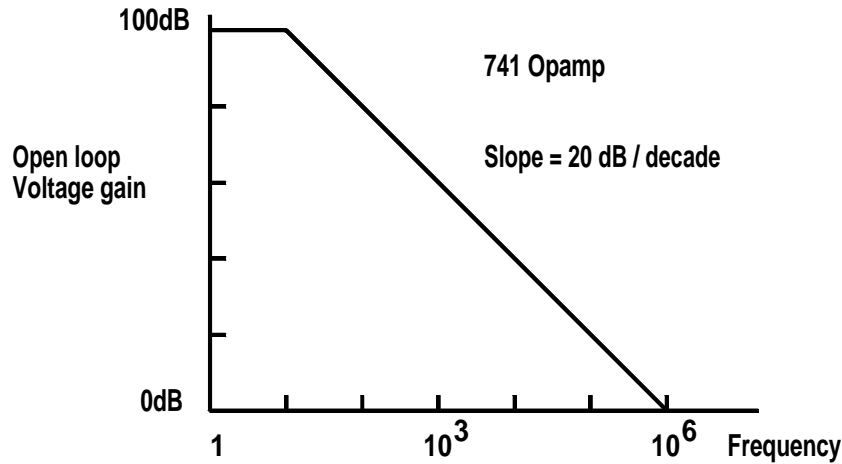
Minimize or avoid flicker noise by chopping the signal at say  $1kHz$  and thus shift the the signal up in frequency and out of the flicker noise domain.

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Calculate the thermal noise from the  $500\text{k}\Omega$  resistor at  $20^\circ\text{C}$  which would be observed on an oscilloscope connected to the output of the circuit

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Gain is  $A = 1 + \frac{5000}{100} = 51$  or  $34dB$ . Plot the  $34dB$  line and the bandwidth is obtained to be about  $30kHz$ . The thermal noise from the  $500k\Omega$  resistor is given by

$$\begin{aligned}
 V_n &= \sqrt{4kTR_s} \times \sqrt{\text{Bandwidth}} \\
 &= \sqrt{4 \times 1.67 \times 10^{-23} \times 293 \times 5 \times 10^5 \times 3 \times 10^4} \\
 &= 17\mu V_{RMS}
 \end{aligned}$$

$$V_{out} = 51 \times 17\mu V_{RMS} = 0.87mV_{RMS}$$

Grass!

Calculate the voltage shot noise which would be observed with an oscilloscope having a  $15\text{MHz}$  bandwidth due to a current of  $0.8\text{A}$  flowing through a  $50\Omega$  resistor.

The shot noise current is

$$\begin{aligned}i_{noise} &= \sqrt{2eI_{DC}B} \\&= \sqrt{2 \times 1.6 \times 10^{-19} \times 0.8 \times 15 \times 10^6} = 2 \times 10^{-6}\text{A}\end{aligned}$$

Noise voltage across the  $50\Omega$  is

$$V_{noise} = 2 \times 10^{-6} \times 50 = 0.1\text{mV}$$

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