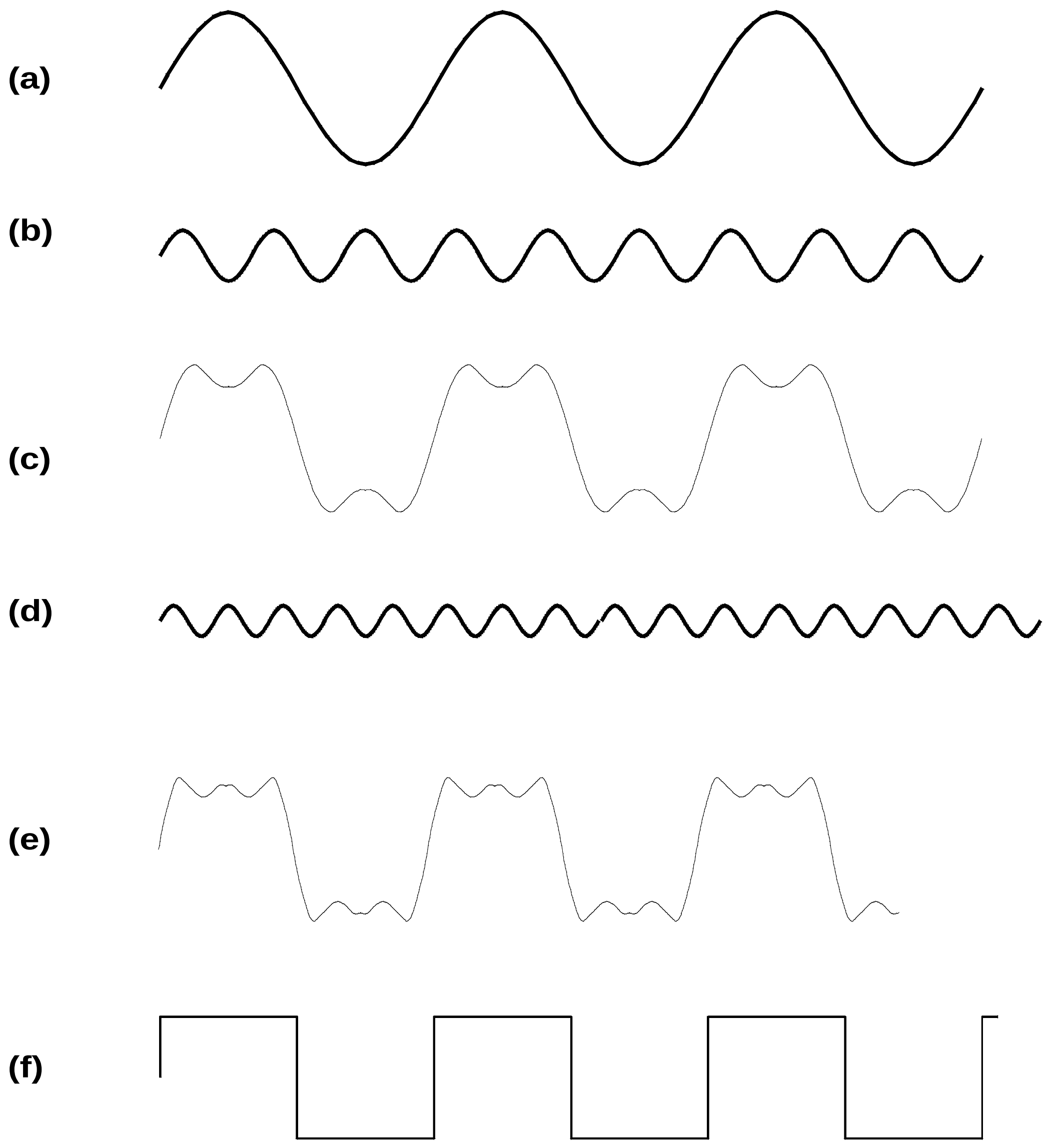
- Any repetitive waveform can be synthesised from the sum of sinusoidal waves of appropriate amplitude and phase.

- The frequencies of the Fourier components are the fundamental frequency and integer multiples of this frequency.

- The sharper the corners in the original waveform, the greater will be the amplitudes of the higher frequency components of the waveform.

- The response of any filter to a repetitive waveform is obtained by summing the response for each of the Fourier components of the input waveform.


(a)
(b)
(c)
(d)
(e)
(f)

Take a sinusoidal wave of fundamental frequency, f_0 and amplitude A
Plus $3f_0$ and amplitude 33% of A
Plus $5f_0$ and amplitude 20% of A
Plus ...
Continue this process for all of the odd harmonics of f_0 given by $f = (2n + 1)f_0$ and having amplitudes $\frac{1}{2n+1}$, where n is an integer,
eventually giving to the composite synthesised square waveform in (f)

As higher harmonics are added in, the corners of the square waveform are sharpened up.

A square waveform can be considered as the sum of sinusoidal waveform of fundamental frequency $f_0 = \frac{1}{T}$ where T is the period of the square wave combined with the sinusoids at the odd harmonics of this fundamental frequency.

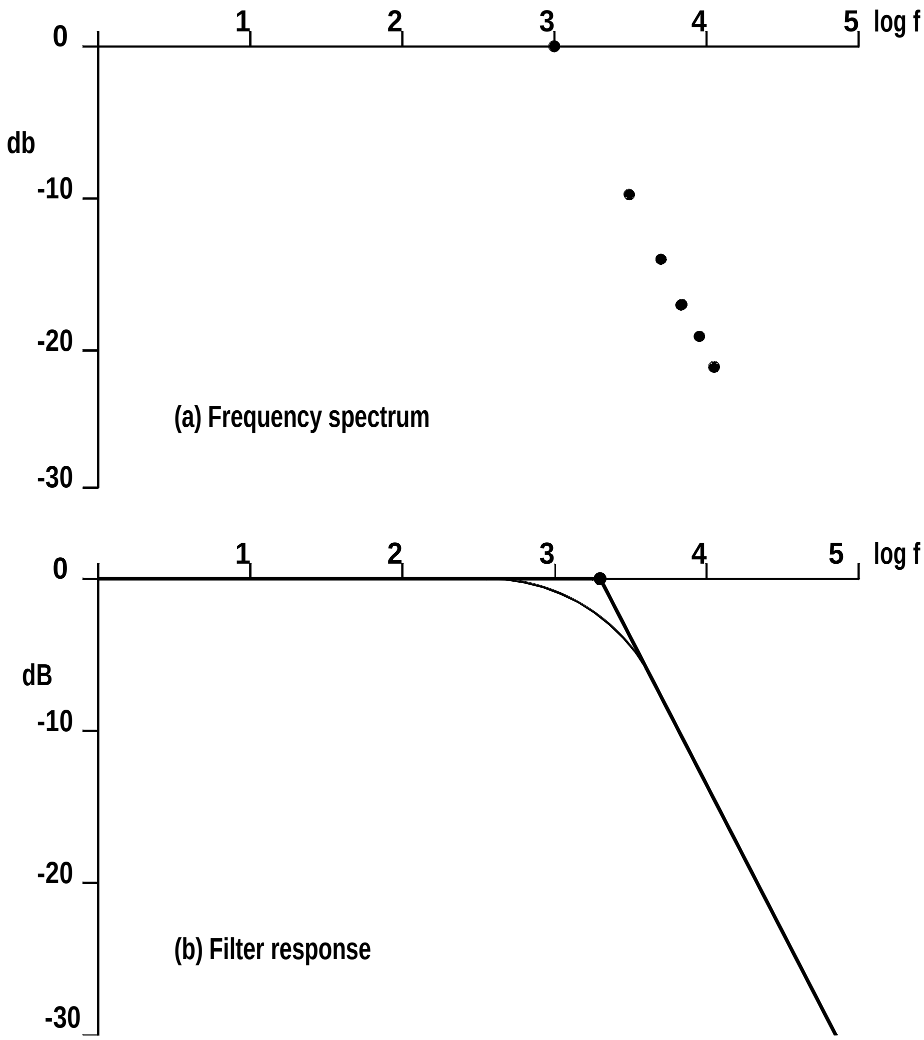
In our example we have taken the phase shifts of the harmonics to be zero, that is all of the harmonics are zero at times 0, $\frac{T}{2}, T, \frac{3T}{2}, \ldots$.
If the phase of the harmonics is not zero then the waveform synthesised can be quite different even though the amplitudes of the harmonics are unchanged.

Synthesis of filter response

To determine the effect of a filter on an arbitrary repetitive waveform:—

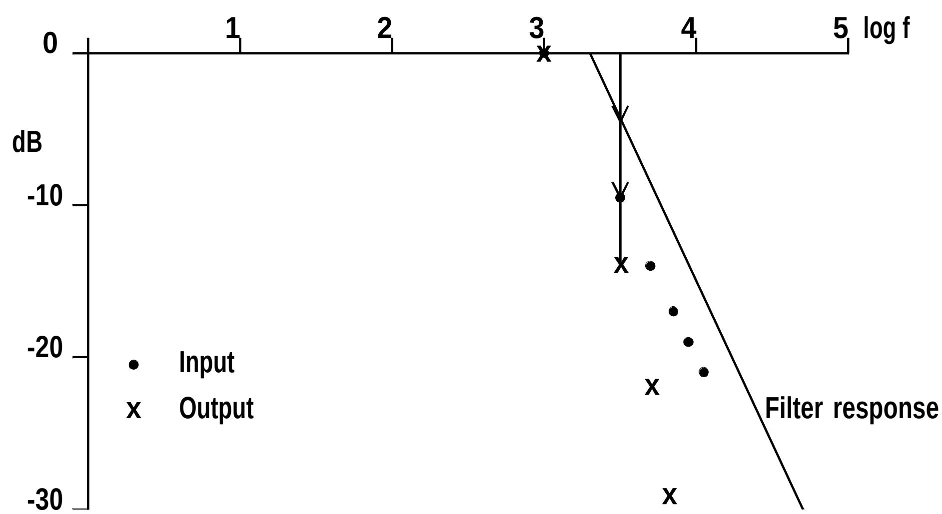
- Obtain the Fourier spectrum of the input waveform.
- Calculate the effect of the filter on each of the Fourier components.
- Combine the modified components to obtain the output waveform.

This prodedure can be carried out numerically but often a graphical method will permit rapid estimation of the output waveform



(a) Fourier spectrum (b) Filter response

In this example the lower frequency components emerge unchanged but the higher frequencies are attenuated
At each frequency we have multiplied the amplitude of the Fourier component at that frequency by the magnitude of the attenuation of the filter to get the magnitude of the output.
This is done by adding the logs

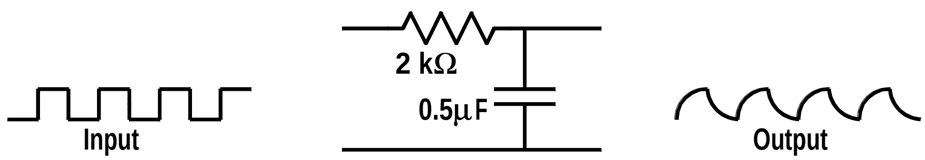


For example, at a frequency of 3kHz or at 3.48 on the log f axis, the filter response is -4dB and the amplitude of the Fourier component of the square wave is -9dB which gives

$$-9\text{dB} - 4\text{dB} = -13\text{dB}$$

as indicated by the arrows
Remember that the amplitudes are smaller than the reference and therefore the dB values are negative.
Each Fourier component is attenuated by the filter response at that frequency

The RC low pass filter has a corner frequency which is close to the fundamental frequency of the square wave.
The higher frequency harmonics associated with the sharp edges of the square waveform have been attenuated by the filter to leave a much smoother output waveform.



Phase shifts occur in the filter and they may affect the output waveform.
If the filter response is such that the phase delay in the filter is constant over the pass band of the filter then the distortion of the waveform due to phase changes will be minimised.

Finally, one rule of thumb:—

> The sharper the corners in the original waveform, the greater will be the amplitudes of the higher frequency components of the waveform.