

Any combination of resistors, capacitors and inductors, when driven by a sinusoidal signal at an angular frequency ω , can be combined to give a resultant impedance in the form:—

$$Z = R + jX$$

where R is the resistance and X is the reactance.

Resistors in series—resultant resistance is the sum of the individual resistances:—

$$R_S = R_1 + R_2 + R_3 + \dots$$

Resistors, capacitors and inductors in series—resultant impedance is the sum of the individual impedances:—

$$Z_S = R_1 + R_2 + \dots + \frac{1}{j\omega C_1} + \frac{1}{j\omega C_2} + \dots + j\omega L_1 + j\omega L_2 + \dots$$

Reduce using complex algebra

Complex impedance, Z , becomes the sum of a resistive component, R , and a reactive component, X .

$$Z_S = R_S + jX_S$$

The units of Z , R and X are Ohms.

Components in parallel—use equivalent of

$$\frac{1}{R_P} = \frac{1}{R_1} + \frac{1}{R_2} + \dots$$

with the R_x replaced by the component complex impedance.

Impedance of a capacitor is $\frac{1}{j\omega C}$

Impedance of an inductor is $j\omega L$.

Both of these terms depend on the frequency of the voltage across the component and therefore the impedance changes if the frequency changes

Also the waveform is presumed to be of sinusoidal form and to have been applied for long enough for any start up transients to die away and a steady state reached.

Calculate the impedance of a $0.1\mu F$ capacitor connected in series with an 820Ω resistance at a frequency of 1 kHz.

$$\begin{aligned} Z &= R + \frac{1}{j\omega C} \\ &= R + \frac{1}{j2\pi fC} \\ &= 820 + \frac{1}{j2\pi 1000 \times 0.1 \times 10^{-6}} \\ &= 820 - \frac{j}{0.000628} \\ &= 820 - j1592 \end{aligned}$$

Resistance $R = 820\Omega$

Reactance $X = -1592\Omega$

Impedance $Z = 820 - j1592$

Calculate the impedance of a $1M\Omega$ resistor in parallel with a $30pF$ capacitor at $40kHz$.
(Oscilloscope equivalent input impedance)

$$\begin{aligned}\frac{1}{Z} &= \frac{1}{R} + j\omega C = \frac{1}{10^6} + j2\pi 40 \times 10^3 \times 30 \times 10^{-12} \\ &= 10^{-6} + j7.54 \times 10^{-6} \\ \text{Therefore } Z &= \frac{1}{10^{-6} + j7.54 \times 10^{-6}} \\ &= \frac{10^{-6} - j7.54 \times 10^{-6}}{(10^{-6})^2 + (7.54 \times 10^{-6})^2} \\ &= \frac{10^{-6} - j7.54 \times 10^{-6}}{5.78 \times 10^{-11}} \\ &= 17k\Omega - j130k\Omega\end{aligned}$$
