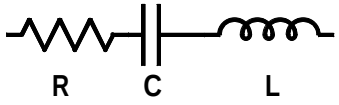
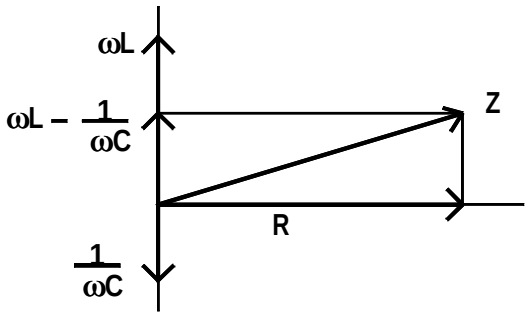
- The impedance of components can be represented on a complex impedance diagram by vectors drawn from the origin.

- When components are connected in series, the resultant impedance is obtained from the vector sum of the impedances of the individual components.

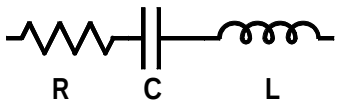


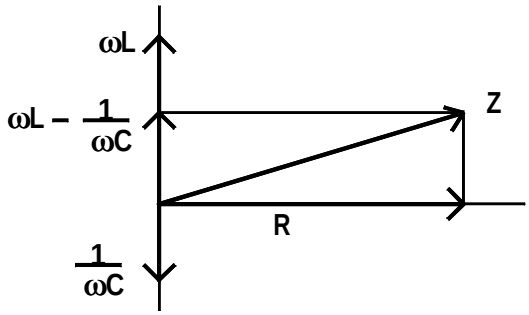


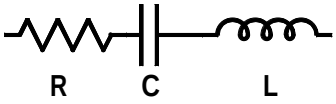
- A resistance is represented by a vector of magnitude R Ohms drawn along the positive x axis.

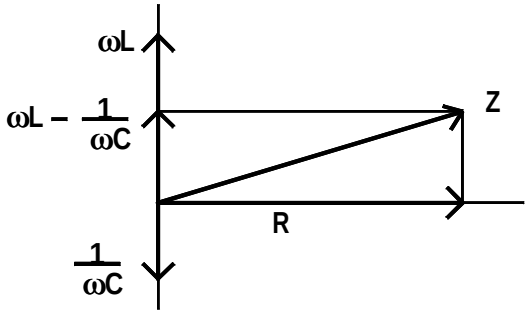
- A capacitor is represented by a vector of magnitude $\frac{1}{\omega C}$ Ohms drawn along the negative y axis.

- An inductance is represented by a vector of magnitude ωL Ohms drawn along the positive y axis.







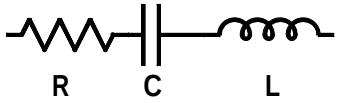


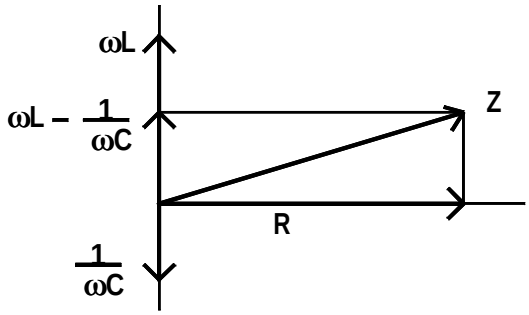
The resultant impedance, Z, of the three components connected in series is shown in the diagram.
The magnitude of the impedance of the resultant is obtained from:—

$$|Z| = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}$$

The phase angle, ϕ, is obtained from:—

$$\phi = \tan^{-1}\left(\frac{\omega L - \frac{1}{\omega C}}{R}\right)$$

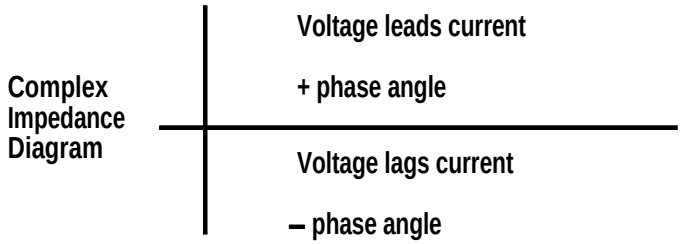




We now have the result that the voltage and the current are related by:—

$$\begin{aligned} V &= ZI \\ &= |Z|\, e^{j\phi} I \\ \text{or} \quad V_o e^{j\omega t} &= |Z|\, I_o e^{j(\omega t+\phi)} \end{aligned}$$

The sign of the phase angle can be determined by using the equation for ϕ or by using the diagram shown



The current waveform is taken as the reference and the sign of the phase angle is + when the resultant impedance lies in the top half of the diagram and – when the resultant impedance lies in the bottom half of the diagram.

If $I = I_0 \sin(2\pi ft)$ then $V = V_0 \sin(2\pi ft + \phi)$.

Example
Calculate the complex impedance of $100nF$ in series with 820Ω at a frequency of 3.5kHz.

The magnitude of the impedance is given by:–

$$\begin{aligned} |Z| &= \sqrt{820^2 + \left(\frac{1}{2\pi 3.5 \times 10^3 \times 100 \times 10^{-9}}\right)^2} \\ &= 938 \text{ Ohms} \end{aligned}$$

and the phase angle by:—

$$\begin{aligned} \phi &= \tan^{-1}\left(\frac{\frac{-1}{2\pi fC}}{R}\right) \\ &= \tan^{-1}\left(\frac{-1}{2\pi fCR}\right) \\ &= -29 \text{ degrees or } -0.51 \text{ radians} \end{aligned}$$