

The general form of Ohm's law is

$$V = Z \times I$$

where Z is called the complex impedance.

- The impedance of a resistor is R .
- The impedance of a capacitor is $\frac{1}{j\omega C}$.
- The impedance of an inductor is $j\omega L$.

The units of impedance are Ohms.

For a resistor the voltage and the current are in phase.

For a capacitor or for an inductor the current and voltage are out of phase by 90° or $\frac{\pi}{2}$.

Representation by trigonometric functions is cumbersome

A more elegant approach is to use complex numbers

Use the relationship

$$e^{j\theta} = \cos \theta + j \sin \theta$$

where $j = \sqrt{-1}$.

Note that in electronics we use j rather than i to represent $\sqrt{-1}$ because of the possibility of confusion with i when it is used to represent a current.

A sinusoidally varying voltage can then be represented by the imaginary part of:–

$$\begin{aligned} V &= V_o e^{j\omega t} \\ \text{and then } \frac{dV}{dt} &= V_o j\omega e^{j\omega t} \\ &= j\omega V \end{aligned}$$

$$\begin{aligned} \text{So for capacitors } I &= C \frac{dV}{dt} \\ &= j\omega CV \end{aligned}$$

$$\text{giving } V = \frac{1}{j\omega C} I$$

$$\text{For inductances } L \frac{dI}{dt} = V$$

$$\begin{aligned} \text{By integration } LI &= \int V dt \\ &= \frac{1}{j\omega} V \end{aligned}$$

$$\text{Giving } V = j\omega LI$$

Generalise Ohm's Law to get:—

$$V = ZI$$

where $Z =$ Complex Impedance

and $Z_R = R$ for a resistance

$$Z_C = \frac{1}{j\omega C} \quad \text{for a capacitor}$$

and $Z_L = j\omega L$ for an inductor

Calculate the impedance of a $0.1\mu F$ capacitor at a frequency of 19kHz.

The impedance of a capacitor is:—

$$\begin{aligned}Z_C &= \frac{1}{j\omega C} \\&= \frac{1}{j2\pi fC} \\&= \frac{-j}{2\pi fC} \\&= \frac{-j}{2\pi 19 \times 10^3 \times 0.1 \times 10^{-6}} \\&= \frac{-j}{0.01193} \\&= -83.8j \text{ Ohms}\end{aligned}$$

In the equation

$$Z_C = -83.8j \text{ Ohms}$$

The 83.8 gives the numerical relationship between the magnitude or amplitude of the voltage and current waveforms.

The j indicates that the voltage and current sinusoidal waveforms are 90° out of phase with each other

The $-$ sign indicates that the current waveform leads the voltage waveform.
