Unit 12 Minterms and Maxterms.

- The minterm list contains the numbers of the rows of the truth table for which the output $Q = 1$ and represents a Sum of Products of canonical form.
- The Maxterm list contains the numbers of the rows of the truth table for which the output $Q = 0$ and represents a Product of Sums of canonical form.
- The Principle of Duality states that the dual of a function is obtained by exchanging operators and identity elements, that is exchange $+$ for . and A for $\overline{A}$.

When a problem is initially specified, it is necessary to prepare a truth table containing all possible input combinations and the required outputs. It has been seen that even for the three input truth table, the representation is cumbersome. When the number of independent inputs becomes large, we then have truth tables of 2^n rows which cannot easily be handled.

It is therefore necessary to have a more concise representation of the truth table. There are two complementary approaches to representing the truth tables. In the first, a set of **minterms** is assigned which designate which rows of the truth table have outputs for which $Q = 1$. The procedure is to prepare the truth table with the inputs or literals in order as shown in the table. These input combinations are then converted to decimal numbers to obtain the row number. The minterm list is then the list of the numbers of the canonical product terms for which the output is $Q = 1$.

k	A	B	C	Q
0	0	0	0	1
1	0	0	1	0
2	0	1	0	1
3	0	1	1	0
4	1	0	0	1
5	1	0	1	0
6	1	1	0	0
7	1	1	1	1

In the table, we see that rows numbered 0, 2, 4 and 7 have outputs of $Q = 1$. The minterm list formed from this truth table is therefore:

$$f(ABC) = m_0 + m_2 + m_4 + m_7$$

where m_x is the particular minterm. By convention this is then abbreviated to:

$$f(ABC) = \Sigma m(0, 2, 4, 7)$$

or the minterm list is the Sum of Products of the literals (inputs) for which the output $Q = 1$.

Sometimes in large truth tables, it is more convenient to specify the terms for which $Q = 0$ because there are fewer terms to be enumerated. In that case the Maxterm list form is more convenient. The Maxterm list is prepared from the numbered rows of the truth table but is given as a Product of Sums. For example, the numbering sequences would be:

	A	B	C	$\overline{A}$	+	$\overline{B}$	+	$\overline{C}$	Q
0	0	0	0	1		1		1	1
1	0	0	1	1		1		0	0
2	0	1	0	1		0		1	0
3	0	1	1	1		0		0	1
4	1	0	0	0		1		1	0
5	1	0	1	0		1		0	1
6	1	1	0	0		0		1	0
7	1	1	1	0		0		0	0

For the table above, the minterm list is:

$$f(ABC) = \Sigma m(0, 3, 5)$$

and the Maxterm list is:

$$f(ABC) = \Pi M(1, 2, 4, 6, 7)$$

The conversion from a minterm list to a Maxterm list is an example of the application of DeMorgan's theorem. Take row number 2 from the table which is represented by the Boolean expression $\overline{A}.B.\overline{C}$. Apply DeMorgan's theorem to this expression and we get:

$$\overline{\overline{A}.B.\overline{C}} = \overline{\overline{A}} + \overline{B} + \overline{\overline{C}} = A + \overline{B} + C$$

So that we then obtain the expression for the terms for which the output is 0.

This conversion from minterm to maxterm form is formalized in The Principle of Duality which states that the dual of a function is obtained by exchanging operators and identity elements, that is exchange + for . and A for $\overline{A}$.

It can be easily seen that, given the minterm list, the Maxterm list is obtained as all of the term numbers which are not in the minterm list. For instance

$$\Sigma m(2,6,7) \quad \text{is equivalent to } \Pi M(0,1,3,4,5)$$

Alternatively, the minterm list is the set of row numbers for which $Q = 1$ and the Maxterm list is the set of row numbers for which $Q = 0$ taken from the truth table of product terms.

Note that it is important that the Boolean expression which describes the truth table be in the canonical form so it is necessary to expand the expression to the canonical form if the expression has already been simplified. This is done by using the identities:

$$A + \overline{A} = 1 \quad \text{and} \quad A.\overline{A} = 0$$

So, if we have the noncanonical expression:

$$Q = A.B + \overline{A}.B.\overline{C}$$

this can be expanded as follows:

$$\begin{aligned} Q &= A.B.1 + \overline{A}.B.\overline{C} \\ &= A.B.(C + \overline{C}) + \overline{A}.B.\overline{C} \\ &= A.B.C + A.B.\overline{C} + \overline{A}.B.\overline{C} \\ &= \Sigma m(7,6,2) = \Sigma m(2,6,7) \end{aligned}$$

Similarly a Product of Sums could be expanded to the canonical form:

$$\begin{aligned} Q &= (A + B)(\overline{A} + C) \\ &= (A + B + 0)(\overline{A} + 0 + C) \\ &= (A + B + C.\overline{C})(\overline{A} + B.\overline{B} + C) \\ &= (A + B + C)(A + B + \overline{C})(\overline{A} + B + C)(\overline{A} + \overline{B} + C) \\ &= \Pi M(0,1,4,6) \end{aligned}$$

where the expansion:

$$(A + B + C.\overline{C}) = (A + B + C)(A + B + \overline{C})$$

is obtained by the application of P2 of Unit 9.

12.1 Problems

12.1 Prepare the minterm list from the following truth table.

A	B	C	Q
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	1
1	0	1	0
1	1	0	0
1	1	1	1

12.2 Prepare the maxterm list from the truth table in Problem 12.1.

12.3 Prepare the truth table for the minterm list given by:

$$f(ABCD) = \Sigma m(1, 4, 7, 10, 14, 15)$$

12.4 Prepare the truth table from the Maxterm list given by:

$$f(ABCD) = \Pi M(3, 5, 10, 13)$$

12.5 Calculate the minterm list equivalent for $\Pi M(1, 5, 6, 7, 13, 15)$.

12.6 Obtain the maxterm list from the minterm list

$$f(ABC) = \Sigma m(0, 1, 3, 5, 6)$$

and draw the OR—AND gate corresponding to this maxterm list.

12.7 Show that the minterm list or Sum of Products (SOP) is equivalent to an AND/OR configuration, that is a layer of AND gates all feeding into a single OR gate.

12.8 Show that the maxterm list description or Product of Sums (POS) is equivalent to a NOR/NAND configuration, that is a layer of NOR gates all feeding into a single NAND gate.