- An implicant corresponds to a group on a Karnaugh map which contains only 1 s.
- A prime implicant corresponds to a group on a Karnaugh map which is not contained wholly within a larger group.
- An essential prime implicant corresponds to a group on a Karnaugh map which contains terms which are not contained in any other group.

The Quine-McCluskey method is implemented in three stages.

- Reorder the Canonical terms in increasing order of number of 1 s that appear when the minterm numbers are expressed in binary.
- Reduce the number of terms by combining, starting with the smallest number of 1 s.
- Select the essential prime implicants, starting with the reduced terms containing the fewest number of literals.

Prime implicant. A prime implicant is a product term which cannot be combined with other product terms to generate a product term with fewer literals than the original term.

Prime numbers such as 5, 11, 29
or 1100001446613353 cannot be factorized.
No rule for identifying prime numbers.
No rule for generating prime numbers.

Similarly there is no easy rule for identifying prime implicants except to try all possible factors or reductions.

$$\begin{aligned} f(ABCD) &= \overline{A}.\overline{B}.\overline{C}.\overline{D} + \overline{A}.\overline{B}.\overline{C}.D + \overline{A}.B.\overline{C}.\overline{D} \\ &\quad + \overline{A}.B.\overline{C}.D + A.\overline{B}.\overline{C}.\overline{D} + \overline{A}.B.C.D \\ &\quad + A.B.\overline{C}.D + A.B.C.\overline{D} + A.B.C.D \\ &= \Sigma m(0,1,4,5,7,8,13,14,15) \end{aligned}$$

	$\overline{A}.\overline{B}$	$\overline{A}.B$	$A.B$	$A.\overline{B}$
$\overline{C}.\overline{D}$	1	1	0	1
$\overline{C}.D$	1	1	1	0
$C.D$	0	1	1	0
$C.\overline{D}$	0	0	1	0

$$f(ABCD) = \overline{A}.\overline{C} + B.D + A.B.C + \overline{B}.\overline{C}.\overline{D}$$

Prime implicants of the original function.

Essential prime implicant which covers at least one minterm which is not covered by any other prime implicant.

	$\overline{A}.\overline{B}$	$\overline{A}.B$	$A.B$	$A.\overline{B}$
$\overline{C}.\overline{D}$	0	0	1	0
$\overline{C}.D$	1	1	1	0
$C.D$	1	1	1	0
$C.\overline{D}$	0	0	1	0

	$\overline{A}.\overline{B}$	$\overline{A}.B$	$A.B$	$A.\overline{B}$
$\overline{C}.\overline{D}$	0	0	1	0
$\overline{C}.D$	1	1	1	0
$C.D$	1	1	1	0
$C.\overline{D}$	0	0	1	0

Quine-McCluskey tabular method of reduction based on

$$X.A + X.\overline{A} = X.(A + \overline{A}) = X.1 = X$$

Generates prime implicants

Don't cares represented by $-$

Minimize a Boolean function for which the minterm expression is:

$$f(x_4, x_3, x_2, x_1, x_0) = \Sigma m(0, 1, 4, 5, 9, 12, 14, 20, 29)$$

Step 1 Write down the tabulation of the minterms.

Minterm	x_4	x_3	x_2	x_1	x_0
0	0	0	0	0	0
1	0	0	0	0	1
4	0	0	1	0	0
5	0	0	1	0	1
9	0	1	0	0	1
12	0	1	1	0	0
14	0	1	1	1	0
20	1	0	1	0	0
29	1	1	1	0	1

Step 2. Reorder the rows so that the rows are in a form which makes it convenient to use $A + \overline{A} = 1$ by dividing the rows into groups containing 0, 1, 2, 3 etc 1 s, in each group.

	Minterm	x_4	x_3	x_2	x_1	x_0	
Group 0	0	0	0	0	0	0	$\Leftarrow$
Group 1	1	0	0	0	0	1	$\Leftarrow$
	4	0	0	1	0	0	$\Leftarrow$
Group 2	5	0	0	1	0	1	$\Leftarrow$
	9	0	1	0	0	1	$\Leftarrow$
	12	0	1	1	0	0	$\Leftarrow$
	20	1	0	1	0	0	$\Leftarrow$
Group 3	14	0	1	1	1	0	PI_1
Group 4	29	1	1	1	0	1	PI_2

Step 3 Search for rows in the next group which differ in only 1 bit position.
If it is not possible to combine a row with a row in the next group, then that row represents a prime implicant and is marked PI_n.

Minterm	x_4	x_3	x_2	x_1	x_0	
0,1	0	0	0	0	–	$\Leftarrow$
0,4	0	0	–	0	0	$\Leftarrow$
1,5	0	0	–	0	1	PI_3
1,9	0	–	0	0	1	PI_4
4,5	0	0	1	0	–	$\Leftarrow$
4,12	0	–	1	0	0	PI_5
4,20	–	0	1	0	0	PI_6
12,14	0	1	1	–	0	PI_7

Step 4. Repeat this row combining process for the rows marked with $\Leftarrow$

Minterm	x_4	x_3	x_2	x_1	x_0	
0,1,4,5	0	0	–	0	–	PI_8

Now left us with eight Prime Implicants
Original expression now reduced to:

$$f(x_4, x_3, x_2, x_1, x_0)$$

$$= \overline{x_4}.x_3.x_2.x_1.\overline{x_0} + x_4.x_3.x_2.\overline{x_1}.x_0$$

$$+\overline{x_4}.\overline{x_3}.\overline{x_1}.x_0 + \overline{x_4}.\overline{x_2}.\overline{x_1}.x_0 + \overline{x_4}.x_2.\overline{x_1}.\overline{x_0}$$

$$+\overline{x_3}.x_2.\overline{x_1}.\overline{x_0} + \overline{x_4}.x_3.x_2.\overline{x_0} + \overline{x_4}.\overline{x_3}.\overline{x_1}$$

Next reduce these Prime Implicants to set of Essential Prime Implicants.

Step 5 Reduction carried out by forming a Prime Implicant chart.
Tabulate the minterms against the prime implicants.

	0	1	4	5	9	12	14	20	29
PI_1							X		
$\rightarrow$ PI_2									X
PI_3		X		X					
$\rightarrow$ PI_4		X			X				
PI_5			X			X			
$\rightarrow$ PI_6			X					X	
PI_7						X	X		
$\rightarrow$ PI_8	X	X	X	X					

One X in a column means only one Prime Implicant covering that minterm and that Prime Implicant is an Essential Prime Implicant.

Step 6 Remove minterms covered by Essential Prime Implicants.

	12	14
PI_1		X
PI_5	X	
PI_7	X	X

All minterms covered by Prime Implicants PI_7

The succesful reduction is therefore

$$
\begin{aligned}
& f(x_4, x_3, x_2, x_1, x_0) \\
& \quad = PI_2 + PI_4 + PI_6 + PI_7 + PI_8 \\
& \quad = (11101) + (0-001) + (-0100) \\
& \qquad +(011-0) + (00-0-) \\
& \quad = x_4.x_3.x_2.\overline{x_1}.x_0 + \overline{x_4}.\overline{x_2}.\overline{x_1}.x_0 + \overline{x_3}.x_2.\overline{x_1}.\overline{x_0} \\
& \qquad + \overline{x_4}.x_3.x_2.\overline{x_0} + \overline{x_4}.\overline{x_3}.\overline{x_1}
\end{aligned}
$$

1. Use the Quine-McCluskey method to reduce the function specified by:

   $f_1(x_5, x_4, x_3, x_2, x_1, x_0)$

   $$= \Sigma m(5, 13, 18, 19, 26, 27, 39, 47, 49, 57)$$

2. Use the Quine-McCluskey method to reduce the function specified by:

   $f_2(x_5, x_4, x_3, x_2, x_1, x_0)$

   $$= \Sigma m(5, 6, 12, 21, 26, 27, 30, 31,$$

   $$37, 44, 45, 46, 47, 53, 58, 62)$$