- For the successful operation of the Karnaugh map technique, it is necessary that:
  - Adjacent terms in the map differ in only one literal.
  - All possible adjacencies be discovered.
- The Karnaugh map method has to be modified for five inputs and does not work for more than five inputs.
- An alternative mapping method for minimization of systems having more than four inputs is presented.

Karnaugh map only works for two literals on axis.
It can fail to identify $A+\overline{A}=1$ in some cases when there are three or more literals.

$\overline{x_2 x_1 x_0}$	$\overline{x_2 x_1} x_0$	$\overline{x_2} x_1 x_0$	$\overline{x_2} x_1 \overline{x_0}$	$x_2 x_1 \overline{x_0}$	$x_2 x_1 x_0$
0	1	0	0	0	0

No adjacent pairs so no simplification but $\overline{x_2}.\overline{x_1}.x_0$ and $x_2.\overline{x_1}.x_0$, while not adjacent, can be combined to give the simplification:

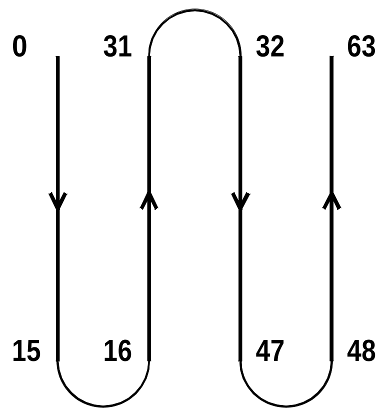
$$\overline{x_2 x_1} x_0 + x_2 \overline{x_1} x_0 = \overline{x_1} x_0$$

Heavy duty computer programs available for very large problems:

- Quine-McCluskey
- Espresso program suite
- Binary Decision Diagram

But we need method for medium sized problems.
Extend the Karnaugh map method.

Fold numbers from 0 to 63 into array.



Folded ordering of table positions.
Convert minterm number to binary then to Gray code.
Write in the minterm number at the location given by Gray code number interpreted as a binary number.

minterm 55 is 0110111 in binary
which converts to Gray code 100101 or position 37.

0	16	48	32
1	17	49	33
3	19	51	35
2	18	50	34
6	22	54	38
7	23	55	39
5	21	53	37
4	20	52	36
12	28	60	44
13	29	61	45
15	31	63	47
14	30	62	46
10	26	58	42
11	27	59	43
9	25	57	41
8	24	56	40

$$f_1(x_5, x_4, x_3, x_3, x_1, x_0)$$

$$= \Sigma m(5, 6, 12, 21, 26, 27, 30, 31,$$

$$37, 44, 45, 46, 47, 53, 58, 62)$$

0	16	48	32
1	17	49	33
3	19	51	35
2	18	50	34
6	22	54	38
7	23	55	39
5	21	53	37
4	20	52	36
12	28	60	44
13	29	61	45
15	31	63	47
14	30	62	46
10	26	58	42
11	27	59	43
9	25	57	41
8	24	56	40

Pairs of terms occur symmetrically on either side of lines dividing the table into halves, quarters or eights.

Terms (31, 30, 26, 27) form a quad as do (5, 21, 53, 37), (30, 62, 26, 58) and also (44, 45, 47, 46).

$$f_1(x_5, x_4, x_3, x_2, x_1, x_0)$$

$$= \Sigma m((5, 21, 53, 37), (31, 30, 26, 27),$$

$$(30, 62, 26, 58), (44, 45, 47, 46), (12, 44), 6)$$

$$= \overline{x_3}x_2\overline{x_1}x_0 + \overline{x_5}x_4x_3x_1 + x_4x_3x_1\overline{x_0} +$$

$$x_5\overline{x_4}x_3x_2 + \overline{x_4}x_3x_2\overline{x_1}\overline{x_0} + \overline{x_5}\overline{x_4}\overline{x_3}x_2x_1\overline{x_0}$$

This tabular map can be extended to the case of 7 inputs by preparing a larger mapping table as shown:

0	16	48	32	96	112	80	64
1	17	49	33	97	113	81	65
3	19	51	35	99	115	83	67
2	18	50	34	98	114	82	66
6	22	54	38	102	118	86	70
7	23	55	39	103	119	87	71
5	21	53	37	101	117	85	69
4	20	52	36	100	116	84	68
12	28	60	44	108	124	92	76
13	29	61	45	109	125	93	77
15	31	63	47	111	127	95	79
14	30	62	46	110	126	94	78
10	26	58	42	106	122	90	74
11	27	59	43	107	123	91	75
9	25	57	41	105	121	89	73
8	24	56	40	104	120	88	72

1. Use the method described in this unit to minimize the function

   $f(x_3, x_2, x_1, x_0) = \Sigma m(1, 5, 7, 9, 12, 13, 14)$

   Verify that you obtain the same simplified function that was obtained in Example 14.1.

2. How many columns will there be in the table for the case of n different inputs?

3. Simplify the expression:

   $f_1(x_3, x_2, x_1 x_0) = \Sigma m(1, 2, 3, 6, 8, 10, 12, 13, 14)$

1. Simplify the expression:

$$f_2(x_5, x_4x_3, x_2, x_1, x_0)$$

$$= \Sigma m(19, 23, 27, 31, 55, 59, 63)$$

2. Simplify the expression:

$$f_3(x_5, x_4, x_3x_2, x_1, x_0)$$

$$= \Sigma m(5, 13, 16, 17, 18, 19, 21, 25, 27, 29,$$

$$33, 37, 45, 48, 49, 50, 51, 53, 57, 59, 61)$$

1. Simplify the expression:

$$f_3(x_7, x_6, x_5, x_4, x_3, x_2, x_1, x_0)$$

$$\Sigma m(3, 7, 11, 13, 19, 27, 39, 47, 49, 53, 67, 71, 75, 79, 83, 91, 103, 111, 113, 117)$$