- A **prime implicant** is a product term which cannot be further simplified by combination with other terms.

- The Gray code ordering of Karnaugh map rows and columns implies that the canonical forms of the sums of products in any adjacent row or column differ only in one bit position.

- The Boolean reduction

  $$F = A.B + A.\overline{B} = A.(B + \overline{B}) = A.1 = A$$

  can then be repeatedly applied to reduce groups to prime implicants and give a minimized expression.

A	B	Q
0	0	0
0	1	0
1	0	1
1	1	1

	$\overline{B}$	B
$\overline{A}$	0	0
A	1	1

$$Q = A.\overline{B} + A.B = A.(\overline{B} + B) = A.1 = A$$

$$
\begin{aligned}
Q &= \overline{A}\,\overline{B}\,\overline{C}\,\overline{D} + \overline{A}\,\overline{B}\,\overline{C}\,D + A\,\overline{B}\,C\,\overline{D} + A\,B\,C\,\overline{D} \\
&= \overline{A}\,\overline{B}\,\overline{C} + A\,C\,\overline{D}
\end{aligned}
$$

Prime implicants are terms which cannot be further simplified.

	$\overline{A}\,\overline{B}$	$\overline{A}\,B$	$A\,B$	$A\,\overline{B}$
$\overline{C}\,\overline{D}$	1	0	0	0
$\overline{C}\,D$	1	0	0	0
$C\,D$	0	0	0	0
$C\,\overline{D}$	0	0	1	1

	$\overline{A}.\overline{B}$	$\overline{A}.B$	$A.B$	$A.\overline{B}$
$\overline{C}.\overline{D}$	0	0	1	0
$\overline{C}.D$	0	0	0	0
$C.D$	1	0	0	1
$C.\overline{D}$	0	0	1	0

Wraparound property of the Gray code number system.
Edge pairs have been identified by half ellipses.

Karnaugh map gives reduced Boolean expression

$$Q = \overline{B}.C.D + A.B.\overline{D}$$

	$\overline{A}.\overline{B}$	$\overline{A}.B$	$A.B$	$A.\overline{B}$
$\overline{C}.\overline{D}$	0	0	0	0
$\overline{C}.D$	0	0	0	0
$C.D$	1	1	1	1
$C.\overline{D}$	0	0	0	0

	$\overline{A}.\overline{B}$	$\overline{A}.B$	$A.B$	$A.\overline{B}$
$\overline{C}.\overline{D}$	0	0	1	1
$\overline{C}.D$	0	0	1	1
$C.D$	0	0	0	0
$C.\overline{D}$	0	0	0	0

Quads or groups of four inputs or literals

	$\overline{A}.\overline{B}$	$\overline{A}.B$	$A.B$	$A.\overline{B}$
$\overline{C}.\overline{D}$	0	0	0	0
$\overline{C}.D$	1	1	1	1
$C.D$	1	1	1	1
$C.\overline{D}$	0	0	0	0

Groups of eight or **octets** can also be identified and combined.

	$\overline{A}.\overline{B}$	$\overline{A}.B$	$A.B$	$A.\overline{B}$
$\overline{C}.\overline{D}$	0	0	0	0
$\overline{C}.D$	1	1	1	1
$C.D$	0	1	0	0
$C.\overline{D}$	0	0	0	0

Overlapping groups are permissible.
and give simplifications.

$$Q = \overline{C}.D + \overline{A}.B.C.D$$

Quad and singlet simplifies to

$$Q = \overline{C}.D + \overline{A}.B.D$$

When systems which contain "don't care" states are specified the usual form is:

$$f(ABCD) = \Sigma m(0, 1, 4, 6, 8) + d(3, 12, 14)$$

where states 3, 12 and 14 are the "don't care" states.

These dont care states can be included or not included in Karnaugh maps.

Aim for most simple result.

	$\overline{A}.\overline{B}$	$\overline{A}.B$	$A.B$	$A.\overline{B}$
$\overline{C}.\overline{D}$	1	1	X	1
$\overline{C}.D$	1	0	0	0
$C.D$	X	0	0	0
$C.\overline{D}$	0	1	X	0

Represent the "don't care" terms in the Karnaugh map by X s instead of the 0 s and 1 s used normally.
Consideration of 1's only gives the simplification.

$$Q = \overline{A}.\overline{B}.\overline{C} + \overline{A}.\overline{C}.\overline{D} + A.\overline{B}.\overline{C}.\overline{D} + \overline{A}.B.C.\overline{D}$$

	$\overline{A}.\overline{B}$	$\overline{A}.B$	$A.B$	$A.\overline{B}$
$\overline{C}.\overline{D}$	1	1	X	1
$\overline{C}.D$	1	0	0	0
$C.D$	X	0	0	0
$C.\overline{D}$	0	1	X	0

Use the "don't care" states as if they were logic 1s.

$$Q = \overline{C}.\overline{D} + \overline{A}.\overline{B}.\overline{C} + B.C.\overline{D}$$

	$\overline{A}.\overline{B}$	$\overline{A}.B$	$A.B$	$A.\overline{B}$
$\overline{C}.\overline{D}$	1	1	X	1
$\overline{C}.D$	1	0	0	0
$C.D$	X	0	0	0
$C.\overline{D}$	0	1	X	0

Wraparound gives a group of four.

$$Q = \overline{C}.\overline{D} + B.\overline{D} + \overline{A}.\overline{B}.\overline{C}$$

	$\overline{A}.\overline{B}$	$\overline{A}.B$	$A.B$	$A.\overline{B}$
$\overline{C}.\overline{D}$	0	4	12	8
$\overline{C}.D$	1	5	13	9
$C.D$	3	7	15	11
$C.\overline{D}$	2	6	14	10

A template for preparing a Karnaugh map from the minterm list is obtained by writing the minterm number into the Karnaugh map at the appropriate position for that minterm.

	$\overline{A}.\overline{B}$	$\overline{A}.B$	$A.B$	$A.\overline{B}$
$\overline{C}.\overline{D}$	0	0	1	0
$\overline{C}.D$	1	1	1	1
$C.D$	0	1	0	0
$C.\overline{D}$	0	0	1	0

Simplify.

	$\overline{A}.\overline{B}$	$\overline{A}.B$	$A.B$	$A.\overline{B}$
$\overline{C}.\overline{D}$	0	0	1	0
$\overline{C}.D$	1	1	1	1
$C.D$	0	1	0	0
$C.\overline{D}$	0	0	1	0

	$\overline{A}.\overline{B}$	$\overline{A}.B$	$A.B$	$A.\overline{B}$
$\overline{C}.\overline{D}$	0	0	1	0
$\overline{C}.D$	1	1	1	1
$C.D$	0	1	0	0
$C.\overline{D}$	0	0	1	0

$$\begin{aligned} Q &= \overline{C}.D + \overline{A}.B.D + A.B.\overline{D} \\ &= \overline{C}.D + \overline{A}.B.C.D + A.B.\overline{D} \end{aligned}$$

	$\overline{A}.\overline{B}$	$\overline{A}.B$	$A.B$	$A.\overline{B}$
$\overline{C}.\overline{D}$	1	1	1	1
$\overline{C}.D$	0	1	1	1
$C.D$	0	1	1	1
$C.\overline{D}$	1	1	1	1

Simplify the Karnaugh map.

Unit 14 Karnaugh maps

	$\overline{A}.\overline{B}$	$\overline{A}.B$	$A.B$	$A.\overline{B}$
$\overline{C}.\overline{D}$	1	1	1	1
$\overline{C}.D$	0	1	1	1
$C.D$	0	1	1	1
$C.\overline{D}$	1	1	1	1

A possible simplification which represents

$$Q = C.B + A.\overline{B}.C.D + C.\overline{D} + A.\overline{C} + \overline{A}.B.\overline{C} + \overline{B}.\overline{C}.\overline{D}$$

	$\overline{A}.\overline{B}$	$\overline{A}.B$	$A.B$	$A.\overline{B}$
$\overline{C}.\overline{D}$	1	1	1	1
$\overline{C}.D$	0	1	1	1
$C.D$	0	1	1	1
$C.\overline{D}$	1	1	1	1

But better is:

$$Q = A + B + \overline{D}$$