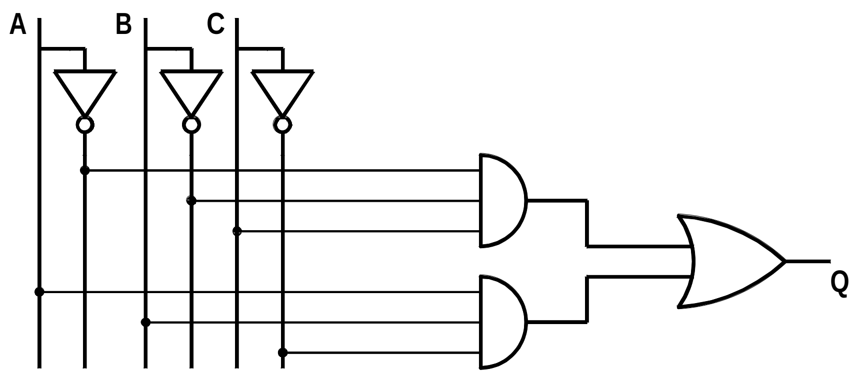
- The terms of the canonical form which represents a particular truth table can be manipulated using the Boolean algebra so as to obtain a circuit having:
  - a minimum level (two level minimization)
  - or a minimum no of gates
  - or a minimum no of interconnections
- The possibility of proving that a particular form of a reduced expression is truly a minimum is considered.

For n inputs or literals,
the truth table has 2^n rows.

If $Q = 1$ for m of the 2^n rows,
we require m n-input AND gates and
1 m-input OR gate for Sum of Products.

A	B	C	Q
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	0
1	0	0	1
1	0	1	0
1	1	0	1
1	1	1	0



Truth table for $Q = \overline{A}\overline{B}C + AB\overline{C}$
and the Sum of Products circuit.

Ideally we would be able to combine rows of the truth table and reduce the logic gate count and also the track count.

There are a number of ways of doing this which are all based ultimatly on Boolean minimization.

To be discussed in next five units.

Main tools for Boolean Minimization.

$$A + \overline{A} = 1$$

and T3

$$A + AB = A \qquad A(A + B) = A$$

and T4

$$A + \overline{A}.B = A + B \qquad A(\overline{A} + B) = A.B$$

Simplify the following canonical expression to a Sum of Products.

$$Q = A.B.C + A.B.\overline{C} + \overline{A}.B.C$$

Use $A + \overline{A} = 1$.

$$\begin{aligned} Q &= A.B.C + A.B.\overline{C} + \overline{A}.B.C \\ &= A.B.(C + \overline{C}) + \overline{A}.B.C \\ &= A.B.1 + \overline{A}.B.C \\ &= A.B + \overline{A}.B.C \end{aligned}$$

Sum of Products, as required.

The simplification is not always unique.

$$\begin{aligned} Q &= A.B.C + A.B.\overline{C} + \overline{A}.B.C \\ &= A.B.C + \overline{A}.B.C + A.B.\overline{C} \\ &= (A + \overline{A}).B.C + A.B.\overline{C} \\ &= B.C + A.B.\overline{C} \end{aligned}$$

This is just as simple
but is different from

$$Q = A.B + \overline{A}.B.C$$

It is not posible to prove that the simplest form has been found.

Convert to the minterm list form.
Simplify the expression, using Boolean algebra
Draw the circuit.

$$\begin{aligned} Q \;=\; & \overline{A}.\overline{B}.\overline{C}.\overline{D} + \overline{A}.\overline{B}.C.D + \overline{A}.B.C.D + A.\overline{B}.\overline{C}.\overline{D} \\ & + A.\overline{B}.C.\overline{D} + A.\overline{B}.C.D + A.B.C.\overline{D} + A.B.C.D \end{aligned}$$

Convert the literals to binary,
Convert the binary number to decimal
and obtain the minterm list directly.

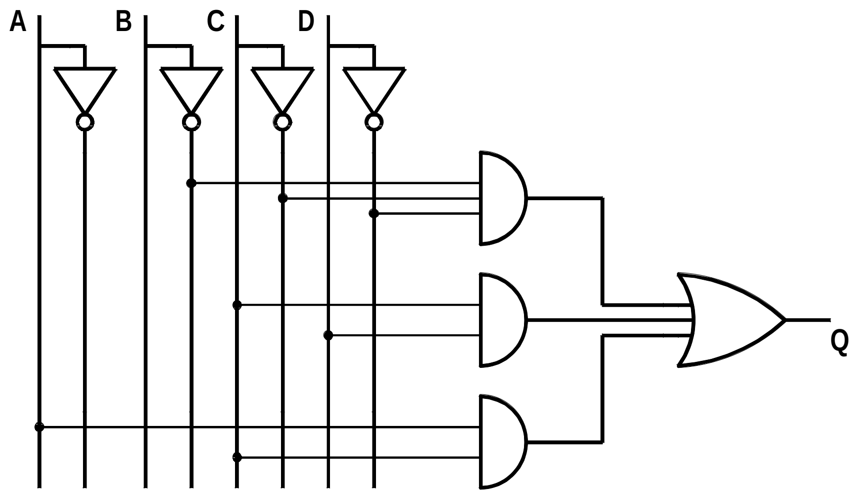
$$\begin{aligned} Q &= \Sigma m(0000,\ 0011,\ 0111,\ 1000, \\ & \quad 1010,\ 1011,\ 1110,\ 1111) \\ &= \Sigma m(0,3,7,8,10,11,14,15) \end{aligned}$$

Convert to the minterm list form.
Simplify the expression, using Boolean algebra
Draw the circuit.

A Boolean simplification proceeds as follows:

$$
\begin{aligned}
Q &= \overline{A}.\overline{B}.\overline{C}.\overline{D} + \overline{A}.\overline{B}.C.D + \overline{A}.B.C.D + A.\overline{B}.\overline{C}.\overline{D} \\
&\quad + A.\overline{B}.C.\overline{D} + A.\overline{B}.C.D + A.B.C.\overline{D} + A.B.C.D \\
&= (\overline{A} + A).\overline{B}.\overline{C}.\overline{D} + \overline{A}.(\overline{B} + B).C.D \\
&\quad + A.\overline{B}.C(\overline{D} + D) + A.B.C.(\overline{D} + D) \\
&= \overline{B}.\overline{C}.\overline{D} + \overline{A}.C.D + A.\overline{B}.C + A.B.C \\
&= \overline{B}.\overline{C}.\overline{D} + \overline{A}.C.D + A.C.(\overline{B} + B) \\
&= \overline{B}.\overline{C}.\overline{D} + \overline{A}.C.D + A.C \\
&= \overline{B}.\overline{C}.\overline{D} + C(\overline{A}.D + A) \\
&= \overline{B}.\overline{C}.\overline{D} + C(D + A) \text{ by T4} \\
&= \overline{B}.\overline{C}.\overline{D} + C.D + A.C
\end{aligned}
$$

Convert to the minterm list form.
Simplify the expression, using Boolean algebra
Draw the circuit.



$$
\begin{aligned}
Q &= \overline{A}.\overline{B}.\overline{C}.\overline{D} + \overline{A}.\overline{B}.C.D + \overline{A}.B.C.D + A.\overline{B}.\overline{C}.\overline{D} \\
&\quad + A.\overline{B}.C.\overline{D} + A.\overline{B}.C.D + A.B.C.\overline{D} + A.B.C.D \\
&= \overline{B}.\overline{C}.\overline{D} + C.D + A.C
\end{aligned}
$$

1. Simplify the following expression

$$F1 = AB\overline{C} + A\overline{BC} + \overline{AB}C + \overline{A}BC$$

2. Simplify the following expression

$$F2 = AB\overline{C} + \overline{A}BC + ABC$$

3. Simplify the following expression

$$F3 = \overline{A}B\overline{C} + AB\overline{C} + \overline{A}BC + A\overline{B}C$$

1. Simplify the following expression

$$\begin{aligned} F4 &= \overline{ABCD} + \overline{A}B\overline{CD} + \overline{ABC}D + \overline{A}B\overline{C}D \\ &\quad + AB\overline{C}D + \overline{A}BCD + ABCD + A\overline{B}C\overline{D} \end{aligned}$$

2. Show, by use of the postulates and theorems of Boolean algebra and by direct calculation using a truth table, that the following identity is true.

$$\begin{aligned} F5 &= C.B + A.\overline{B}.C.D + C.\overline{D} \\ &\quad + A.\overline{C} + \overline{A}.B.\overline{C} + \overline{A}.\overline{B}.\overline{C}.\overline{D} \\ &= = A + B + \overline{D} \end{aligned}$$

Draw the logic gate diagram which represents each side of the identity.