- In a Boolean expression a **literal** is an input variable or its complement.
- A Boolean function is in canonical Sum of Products form when each product term contains each of the literals or inputs.
- A Boolean function is in canonical Product of Sums form when each of the Sum terms contains each of the literals or inputs.

We need a proceedure for translating from real world to boolean expression to computer compatible format for safe design.

Truth table enumerates all possibilities
Every literal appears in the truth table.
A truth table row is a product of literals in either complemented or uncomplemented forms.

A	B	C	Q	
0	0	0	0	
0	0	1	1	
0	1	0	0	
0	1	1	1	$\Leftarrow$
1	0	0	0	
1	0	1	0	
1	1	0	1	
1	1	1	1	

Arrowed row, $\Leftarrow$,
$A = 0$, $B = 1$ and $C = 1$.
If $A = 0$ then $\overline{A} = 1$
Boolean identity for this row,
$\overline{A}.B.C = 1$.

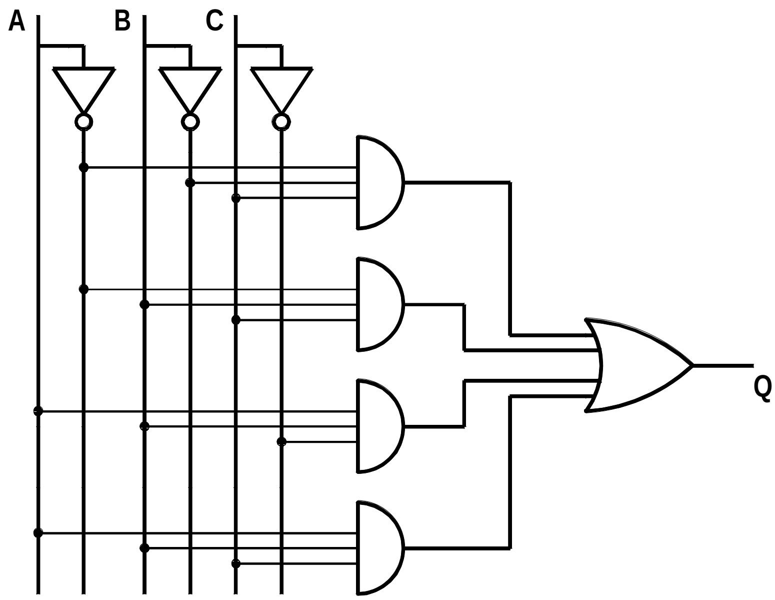
A	B	C	Q	
0	0	0	0	
0	0	1	1	
0	1	0	0	
0	1	1	1	$\Leftarrow$
1	0	0	0	
1	0	1	0	
1	1	0	1	
1	1	1	1	

Boolean expression for this truth table

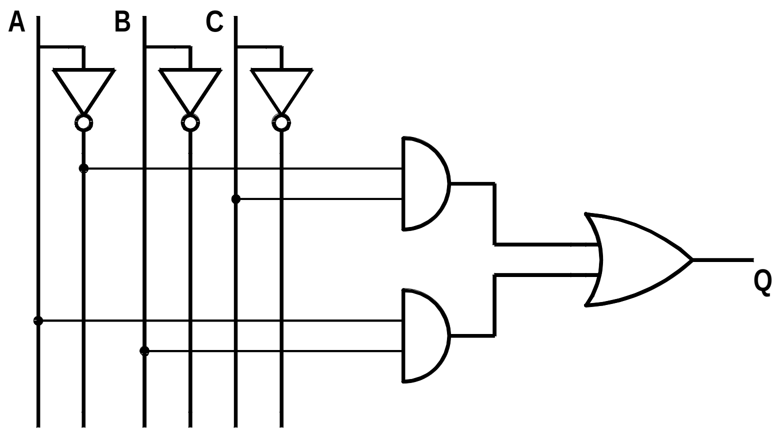
$$Q = \overline{A}.\overline{B}.C + \overline{A}.B.C + A.B.\overline{C} + A.B.C$$

Simplify using Boolean algebra

$$\begin{aligned} Q &= \overline{A}.\overline{B}.C + \overline{A}.B.C + A.B.\overline{C} + A.B.C \\ &= \overline{A}.C.(\overline{B} + B) + A.B.(\overline{C} + C) \\ &= \overline{A}.C.1 + A.B.1 \\ &= \overline{A}.C + A.B \end{aligned}$$



(a)



(b)

$$
\begin{aligned}
Q &= \overline{A}.\overline{B}.C + \overline{A}.B.C + A.B.\overline{C} + A.B.C \\
&= \overline{A}.C + A.B
\end{aligned}
$$

Truth table is a two level AND-OR circuit or a Boolean algebraic Sum of Products.
Standardize: **Canonical Form** of the Boolean expression.
Each of the input variables or literal appears once in each of the product terms of the expression
and also the expression contains a product term for each logical 1 in the truth table output.
Uncomplemented or complemented form, A or $\overline{A}$

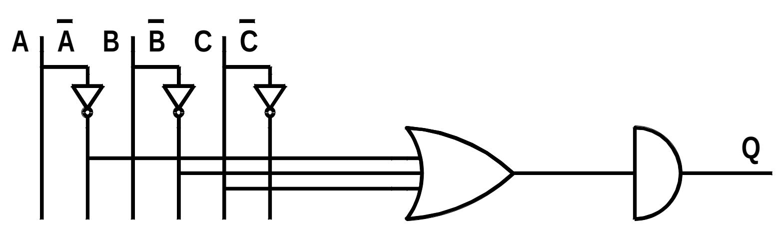
$$Q = \overline{A}.\overline{B}.C + \overline{A}.B.C + A.B.\overline{C} + A.B.C$$

The canonical form can be either
Sum of Products form (SOP)
OR
Product of Sums form.

A	B	C	Q	$\overline{Q}$
0	0	0	1	0
0	0	1	1	0
0	1	0	1	0
0	1	1	1	0
1	0	0	1	0
1	0	1	1	0
1	1	0	0	1
1	1	1	1	0

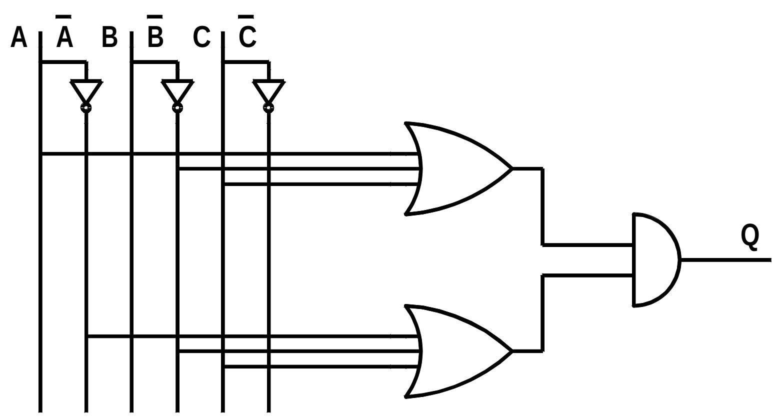
Obtain the OR-AND form for table.
Extra column inserted for $\overline{Q}$
Only one 1 in the $\overline{Q}$ column $\overline{Q} = AB\overline{C}$
DeMorgan's theorem gives:

$$Q = \overline{AB\overline{C}} = \overline{A} + \overline{B} + \overline{\overline{C}} = \overline{A} + \overline{B} + C$$



A	B	C	Q	$\overline{Q}$
0	0	0	1	0
0	0	1	1	0
0	1	0	0	1
0	1	1	1	0
1	0	0	1	0
1	0	1	1	0
1	1	0	0	1
1	1	1	1	0

$$
\begin{aligned}
Q &= \overline{\overline{A}B\overline{C} + AB\overline{C}} \\
&= (\overline{\overline{A}B\overline{C}})(\overline{AB\overline{C}}) \\
&= (A + \overline{B} + C)(\overline{A} + \overline{B} + C)
\end{aligned}
$$



A	B	C	Q
0	0	0	1
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	1
1	0	1	0
1	1	0	0
1	1	1	1

Write down the Sum of Products, canonical Boolean expression which represents this truth table.

Convert the Sum of Products

$$Q = \overline{A}.\overline{B}.C + \overline{A}.B.C + A.B.\overline{C} + A.B.C$$

to a Product of Sums form.

Convert the Product of Sums

$$Q = (\overline{A}+\overline{B}+\overline{C})(\overline{A}+B+\overline{C})(A+\overline{B}+C)(A+B+C)$$

to a Sum of Products form.

A	B	C	Q
0	0	0	0
0	0	1	1
0	1	0	1
0	1	1	0
1	0	0	1
1	0	1	1
1	1	0	1
1	1	1	0

Write down the canonical Boolean Product of Sums expression which represents this truth table.

A	B	C	Q
0	0	0	1
0	0	1	1
0	1	0	0
0	1	1	0
1	0	0	1
1	0	1	1
1	1	0	0
1	1	1	1

Draw the OR-AND circuit which implements this truth table.