

DeMorgan's Theorem is used in
two complementary forms

$$\overline{(A + B + C + \dots)} = \overline{A}.\overline{B}.\overline{C} \dots$$
$$\overline{A} + \overline{B} + \overline{C} + \dots = \overline{(A.B.C \dots)}$$

In Boolean algebra formalism

$$\overline{(A + B)} = \overline{A}.\overline{B}$$

Compl of Sums = Prod of Compl

$$\overline{A} + \overline{B} = \overline{(A.B)}$$

Sum of Compl = Compl of Prod

Postulate 4 defines complements,

$$A + \bar{A} = 1$$

$$A \cdot \bar{A} = 0$$

Prove first form of DeMorgan's theorem

$$\begin{aligned}A + B + \overline{A}.\overline{B} &= (A + B + \overline{A}).(A + B + \overline{B}) && \text{P2} \\ &= (B + 1).(A + 1) && \text{P4} \\ &= 1.1 && \text{T2} \\ &= 1\end{aligned}$$

and also

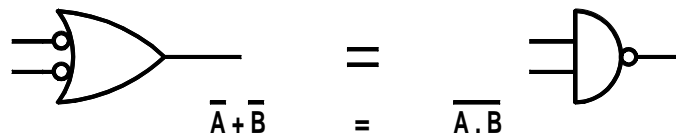
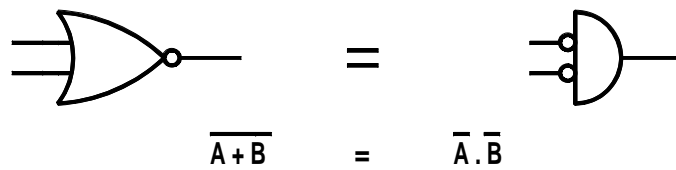
$$\begin{aligned}(A + B).\overline{A}.\overline{B} &= \overline{A}.\overline{B}.(A + B) && \text{P1} \\ &= \overline{A}.\overline{B}.A + \overline{A}.\overline{B}.B && \text{P2} \\ &= \overline{B}.\overline{A}.A + \overline{A}.\overline{B}.B \\ &= \overline{B}.0 + \overline{A}.0 \\ &= 0\end{aligned}$$

Prove second form of DeMorgan's Theorem

$$\begin{aligned}\overline{A} + \overline{B} + A.B &= (\overline{A} + \overline{B} + A).(\overline{A} + \overline{B} + B) && \text{P2} \\ &= (\overline{B} + 1).(\overline{A} + 1) && \text{T2} \\ &= 1\end{aligned}$$

and

$$\begin{aligned}(\overline{A} + \overline{B}).A.B &= \overline{A}.A.B + \overline{B}.A.B && \text{P2} \\ &= B.0 + A.0 && \text{P4} \\ &= 0\end{aligned}$$



Circuit statement of DeMorgan's theorem.
Allows interchange of NAND and NOR logic gates.

Enumerative truth table verification of the first form of the theorem:

A	B	$A + B$	$\overline{A + B}$	\overline{A}	\overline{B}	$\overline{A} \cdot \overline{B}$
0	0	0	1	1	1	1
0	1	1	0	1	0	0
1	0	1	0	0	1	0
1	1	1	0	0	0	0

Enumerative truth table verification of the second form of the theorem:

A	B	\bar{A}	\bar{B}	$\bar{A} + \bar{B}$	$A.B$	$\overline{A.B}$
0	0	1	1	1	0	1
0	1	1	0	1	0	1
1	0	0	1	1	0	1
1	1	0	0	0	1	0
