The basic postulates of Boolean algebra are:

P1. Commutation operations
$A + B = B + A$
$A.B = B.A$

P2. Distribution operations
$A + (B.C) = (A + B).(A + C)$
$A.(B + C) = A.B + A.C$

P3. Identity elements exist
$A + 0 = A$
$A.1 = A$

P4. Complements exist
$A + \overline{A} = 1$
$A.\overline{A} = 0$

Some useful theorems are:

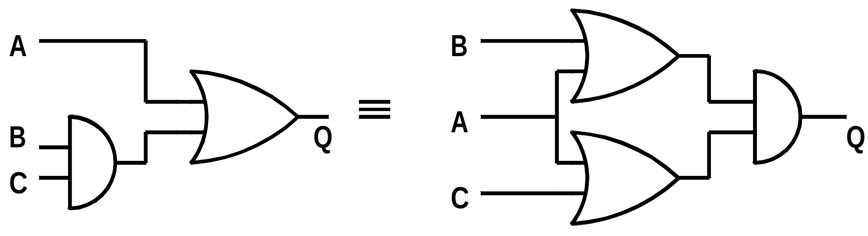
$$\begin{array}{lrcl}
\text{T1} & A + A & = & A \\
 & A.A & = & A \\
\text{T2} & A + 1 & = & 1 \\
 & A.0 & = & 0 \\
\text{T3} & A + A.B & = & A \\
 & A.(A + B) & = & A \\
\text{T4} & A + \overline{A}.B & = & A + B \\
 & A.(\overline{A} + B) & = & A.B
\end{array}$$

Demonstration of reasonableness of these postulates by enumeration.

Postulate P4, $A + \overline{A} = 1$

A	$\overline{A}$	$A + \overline{A}$
0	1	1
1	0	1

Unit 9 Boolean algebra



Logic circuit for Postulate 2; $A + (B.C) = (A + B).(A + C)$.

A	B	C	$B.C$	$A + B.C$	$A + B$	$A + C$	$(A + B).($
0	0	0	0	0	0	0	0
0	0	1	0	0	0	1	0
0	1	0	0	0	1	0	0
0	1	1	1	1	1	1	1
1	0	0	0	1	1	1	1
1	0	1	0	1	1	1	1
1	1	0	0	1	1	1	1
1	1	1	1	1	1	1	1

Some frequently used theorems.

T1	$A + A = A$	$A.A = A$
T2	$A + 1 = 1$	$A.0 = 0$
T3	$A + A.B = A$	$A.(A + B) = A$
T4	$A + \overline{A}.B = A + B$	$A.(\overline{A} + B) = A.B$

Prove the idempotent relationship,

$$A + A = A$$

$$\begin{aligned} A + A &= (A + A).1 && \text{by P3.} \\ &= (A + A).(A + \overline{A}) && \text{by P4.} \\ &= A + A.\overline{A} && \text{by P2a reversed.} \\ &= A + 0 && \text{by P4.} \\ &= A \end{aligned}$$

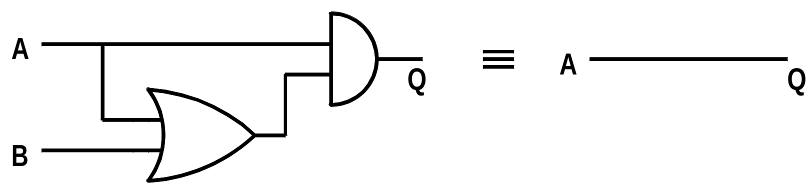
Similarly for the absorption Theorem,

T3, $A.(A+B) = A$

$$\begin{aligned} A.(A+B) &= (A+0).(A+B) && \text{by P3.} \\ &= A+0.B && \text{by P2.} \\ &= A+0 && \text{by T2.} \\ &= A && \text{by P3.} \end{aligned}$$

Or using the enumeration approach

A	B	$(A+B)$	$A.(A+B)$
0	0	0	0
0	1	1	0
1	0	1	1
1	1	1	1



Logic gate circuit representing Theorem 3b; $A.(A+B) = A$.

A	B	$(A+B)$	$A.(A+B)$
0	0	0	0
0	1	1	0
1	0	1	1
1	1	1	1